EMA 202, Fall 2014 **Name: ______________________________**

Midterm #2, closed book/notes, 90 min.

Instructions

- Write your name on every sheet.
- Show all of your work.
- To maximize opportunities for partial credit, solve problems symbolically to the extent possible and substitute numerical values at the end.
- Include free body diagrams for all equilibrium equations.

- This is a closed book examination.
- The only notes allowed are the equations provided with this exam.
- Calculators are allowed.
- The use of cell phones is prohibited.

- The instructors and the University of Wisconsin expect the highest standards of honesty and integrity in the academic performance of its students. It is important that the work submitted on this examination is yours and yours alone.
- Receiving or giving aid in an examination or using notes on a closed note exam will be considered cheating and will result in a grade of F and the case being reported to the Dean of Students Office.

Circle Your Discussion Section:

301	(Tu 1:20, David)
302	(Tu 2:25, David)
303	(W 3:30, David)
304	(Th 8:50, Matt)
305	(Th 12:05, Matt)
306	(Th 1:20, Matt)

Grading:

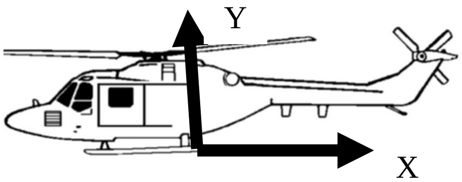
Q1	/25
Q2	/25
Q3	/25
Q4	/25
Total	/100

EMA 202, Fall 2014 **Name: ______________________________**
Midterm #2, closed book/notes, 90 min.

Question 1 (25 points)

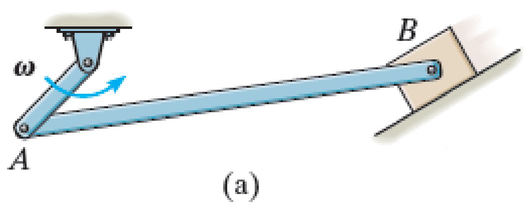
Conceptual questions: please include a few sentences or equations to justify your answers

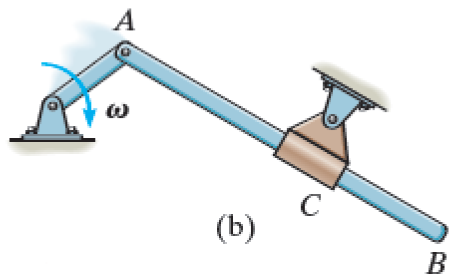
1A (7 points) A 1000kg helicopter starts from rest at t = 0 sec. Given the force components below, find the velocity vector of the helicopter after 10 seconds.



$$\sum F_x = 720t$$
$$\sum F_y = 2160 - 360t$$
$$\sum F_z = 0$$

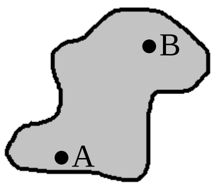
1B (6 points) Draw the instantaneous center of velocity of link AB for the two cases below



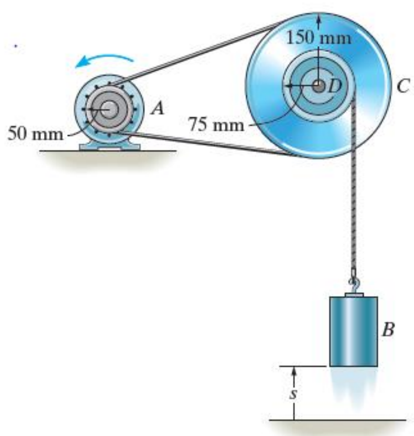


EMA 202, Fall 2014 **Name: ______________________________**
Midterm #2, closed book/notes, 90 min.

1C (6 points) For a given instant, a rigid body has velocity at point A parallel to the velocity at point B. Does this guarantee that the angular velocity of body is zero at this instant?



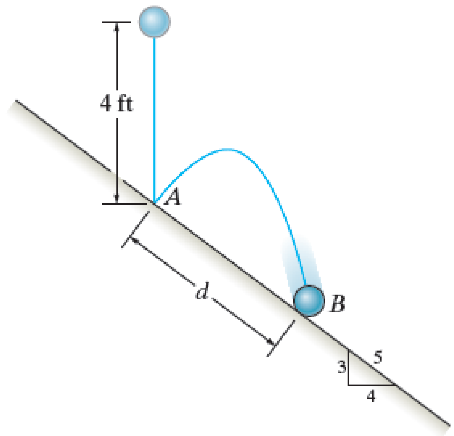
1D (6 points) At this instant, pulley A has an angular acceleration of 6 rad/s^2. What is the acceleration of block B?



EMA 202, Fall 2014 **Name: ______________________________**
Midterm #2, closed book/notes, 90 min.

Question 2 (25 points)

A 1 lb ball is dropped from rest and falls a distance of 4ft before striking the smooth plane at A. If e = 0.8, determine the distance *d* to where the ball again strikes the plane at B.



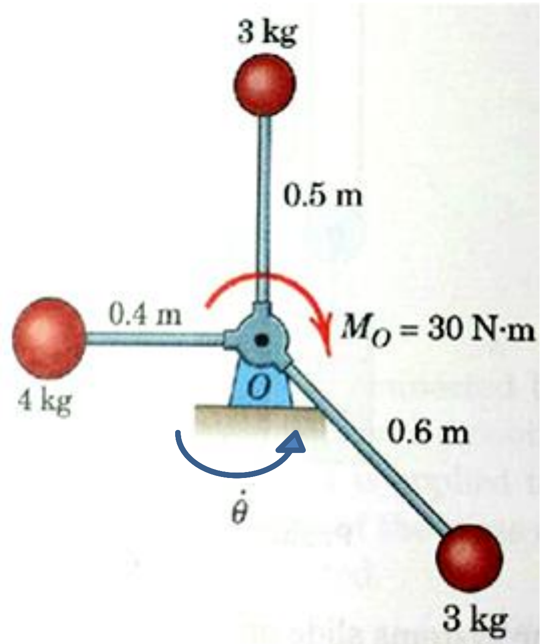
EMA 202, Fall 2014 **Name: ______________________________**
Midterm #2, closed book/notes, 90 min.

Question 3 (25 points)

Three small spheres are welded to the light rigid frame which is rotating in a horizontal plane about point O with an angular velocity of 20 rad/s in the counter clockwise direction. If a moment $M_o = 30$ Nm in the clockwise direction is applied to the frame for 5 seconds,

A) compute the angular impulse vector

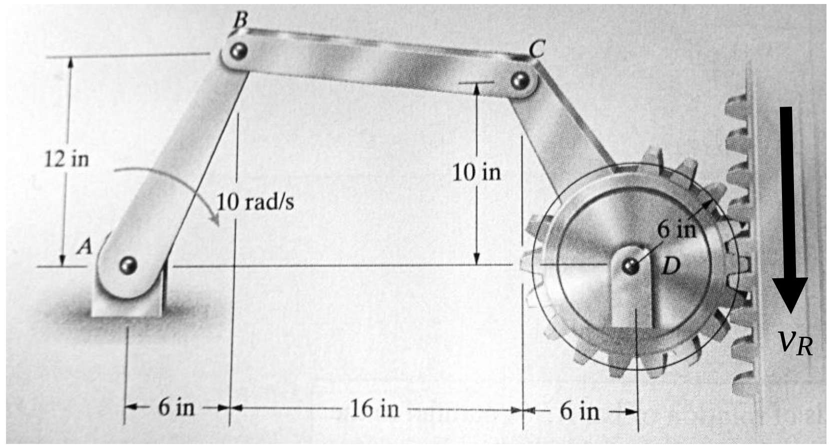
B) compute the new angular velocity after 5 seconds



EMA 202, Fall 2014 **Name: ______________________________**
Midterm #2, closed book/notes, 90 min.

<u>Question 4 (25 points)</u>

Bar AB rotates with a clockwise angular velocity of 10 rad/sec.



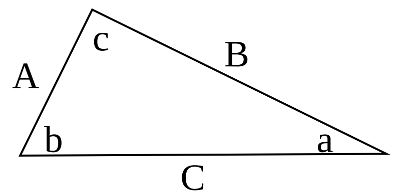
Find

a) velocity vector for pt B
b) velocity vector for pt C
c) angular velocity of bar BC
d) velocity vector of rack, $\boldsymbol{v_R}$

EMA 202, Fall 2014 **Name: ______________________________**
Midterm #2, closed book/notes, 90 min.

Geometry:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$
$$C^2 = A^2 + B^2 - 2AB\cos c$$
$$\cos^2\theta + \sin^2\theta = 1$$



Particle Rectilinear Motion:

$$a = \frac{dv}{dt} \qquad v = \frac{ds}{dt} \qquad ads = vdv$$

For the special case of constant acceleration (a_c), and assuming initial conditions are specified at t = 0:

$$v(t) = v_o + a_c t \qquad s(t) = s_o + v_o t + \frac{1}{2}a_c t^2 \qquad v^2 = v_o^2 + 2a_c(s - s_o)$$

Particle Curvilinear Motion:

Cartesian Results: $\vec{\mathbf{v}} = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}} + \dot{z}\hat{\mathbf{k}}$ $\qquad \vec{\mathbf{a}} = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}}$

Normal/Tangential Results:

$$\vec{\mathbf{v}} = \dot{s}\hat{\mathbf{u}}_t$$
$$\vec{\mathbf{a}} = \dot{v}\hat{\mathbf{u}}_t + \frac{v^2}{\rho}\hat{\mathbf{u}}_n \qquad \text{where} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

Polar Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\theta}\,\hat{\mathbf{u}}_\theta$$
$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{u}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\mathbf{u}}_\theta$$

Spherical Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\phi}\,\hat{\mathbf{u}}_\phi + r\dot{\theta}\sin\phi\,\hat{\mathbf{u}}_\theta$$
$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\sin^2\phi)\,\hat{\mathbf{u}}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi} - r\dot{\theta}^2\sin\phi\cos\phi)\,\hat{\mathbf{u}}_\phi$$
$$+ (r\ddot{\theta}\sin\phi + 2\dot{r}\dot{\theta}\sin\phi + 2r\dot{\phi}\dot{\theta}\cos\phi)\hat{\mathbf{u}}_\theta$$

General Time Derivatives of a Vector:

$$\dot{\hat{\mathbf{u}}}(t) = \vec{\boldsymbol{\omega}}_u \times \hat{\mathbf{u}}$$
$$\dot{\vec{\mathbf{A}}}(t) = \dot{A}\hat{\mathbf{u}}_A + \vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}} \qquad \ddot{\vec{\mathbf{A}}}(t) = \ddot{A}\hat{\mathbf{u}}_A + 2\vec{\boldsymbol{\omega}}_A \times \dot{A}\hat{\mathbf{u}}_A + \dot{\vec{\boldsymbol{\omega}}}_A \times \vec{\mathbf{A}} + \vec{\boldsymbol{\omega}}_A \times (\vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}})$$

Principle of Work and Energy: $\Sigma U_{1\to2} = T_2 - T_1$ where $U_{1\to2} = \int_1^2 \vec{\mathrm{F}} \cdot d\vec{\mathrm{r}}$ and $T = \frac{1}{2}mv^2$

Conservation of Energy (assumes only conservative forces): $T_1 + V_1 = T_2 + V_2$

(For rigid bodies, these results are unchanged except for the fact that $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$)

EMA 202, Fall 2014 **Name: ______________________________**
Midterm #2, closed book/notes, 90 min.

V is potential energy and takes different forms depending on the source. For terrestrial gravity, $V = mgy$, (mgy_G for a rigid body) while for a linear elastic spring, $V = \frac{1}{2}k\delta^2$

Power: $P = \vec{F} \cdot \vec{v}$

Efficiency: $\varepsilon = \dfrac{\text{power output}}{\text{power input}}$

Linear Impulse and Momentum: $\sum \vec{\mathrm{I}}_{1\to 2} = \vec{\mathrm{p}}_2 - \vec{\mathrm{p}}_1$ where $\vec{\mathrm{I}}_{1\to 2} = \int_1^2 \vec{\mathrm{F}}dt$ and $\vec{\mathrm{p}} = m\vec{\mathrm{v}}$
(For rigid bodies, the linear impulse and momentum expressions are identical but with $\vec{\mathrm{v}} = \vec{\mathrm{v}}_G$)

Coefficient of restitution: $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum: $\sum \vec{\mathrm{I}}_{M\,1\to 2} = \vec{\mathrm{H}}_{O2} - \vec{\mathrm{H}}_{O1}$ where
$\vec{\mathrm{I}}_{M\,1\to 2} = \int_1^2 \vec{\mathrm{M}}_O dt$ and $\vec{\mathrm{H}} = \vec{r} \times m\vec{\mathrm{v}}$

(For rigid bodies, this principle holds as long as O is either the center of gravity G or a point of fixed rotation. If the center of gravity, $H_G = I_G\omega$; if a fixed axis, $H_O = I_O\omega$.)

Relative General Plane Motion:

Translating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \vec{\mathbf{v}}_{B/A} = \vec{\mathbf{v}}_A + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \vec{\mathbf{a}}_{B/A} = \vec{\mathbf{a}}_A + \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}_{B/A} - \omega^2 \vec{\mathbf{r}}_{B/A}$$

Rotating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \left(\vec{\mathbf{v}}_{B/A}\right)_{xy} + \vec{\Omega} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \left(\vec{\mathbf{a}}_{B/A}\right)_{xy} + \dot{\vec{\Omega}} \times \vec{\mathbf{r}}_{B/A} + \Omega \times \left(\Omega \times \vec{\mathbf{r}}_{B/A}\right) + 2\Omega \times \left(\vec{\mathbf{v}}_{B/A}\right)_{xy}$$

Equations of Motion: $\sum F_x = m(a_G)_x$; $\sum F_y = m(a_G)_y$
or $\sum F_t = m(a_G)_t$; $\sum F_n = m(a_G)_n$

$\sum M_G = I_G\alpha$ *or* $\sum M_O = I_O\alpha$ *or* ... if you must ...
$\sum M_P = -\bar{y}m(a_G)_x + \bar{x}m(a_G)_y + I_G\alpha$

Mass Moment of Inertia: Definition, and in terms of radius of gyration, k: $I = \int r^2 dm = k^2 m$
Parallel axis theorem: $I = I_G + md^2$