**ME 240 / EMA 202, Fall 2017**
**Midterm #2, closed book/notes, 60 min.**

**Name:** Solutions

## Instructions

- Show all of your work.
- To maximize opportunities for partial credit, solve problems symbolically to the extent possible and substitute numerical values at the end.
- Include free body diagrams for all equilibrium equations.

- The only notes allowed are the equations provided with this exam.
- The use of cell phones is prohibited.
- The instructors and the University of Wisconsin expect the highest standards of honesty and integrity in the academic performance of its students. It is important that the work submitted on this examination is yours and yours alone.
- Receiving or giving aid in an examination or using notes on a closed note exam will be considered cheating and will result in a grade of F and the case being reported to the Dean of Students Office.

## Circle Your Discussion Section:

| EMA 202 | Time | TA |
|---|---|---|
| 301 | 8:50 | Peter Grimmer |
| 302 | 9:55 | Peter Grimmer |
| 303 | 11:00 | Peter Grimmer |
| 304 | 12:05 | Aaron Wright |
| 305 | 1:20 | Aaron Wright |
| 306 | 2:25 | AJ Gross |
| 307 | 12:05 | Shu Wang |
| **ME 240** | | |
| 301 | 8:50 | AJ Gross |
| 302 | 9:55 | Shu Wang |
| 303 | 11:00 | Jenna Thorp |
| 304 | 12:05 | Jenna Thorp |
| 305 | 1:20 | Guannan Guo |
| 306 | 2:25 | Guannan Guo |

## Grading:

| | |
|---|---|
| Q1 | /34 |
| Q2 | /33 |
| Q3 | /33 |
| Total | /100 |

**ME 240 / EMA 202, Fall 2017** **Name:** ______________________________
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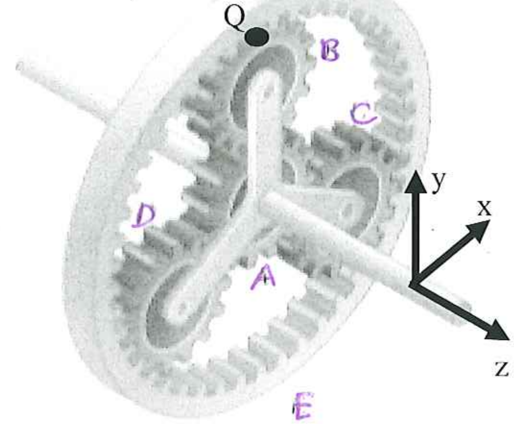
## Question 1 (34 points)

In the planetary gear system shown, the radius of gears A, B, C, and D is 30 mm and the radius of the outer gear E is 90 mm. Knowing that gear E has a constant angular velocity of 180 rpm clockwise and that the central gear A has a constant angular velocity of 240 rpm clockwise, determine




(a) the angular velocity of the planetary gears B, C, and D

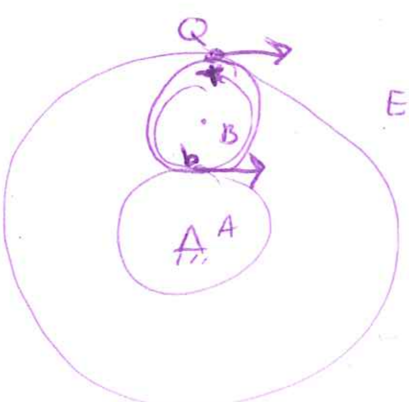
(b) the angular velocity of the spider connecting the planetary gears.

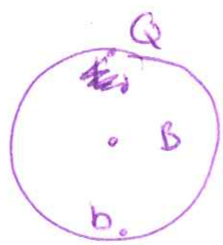
(c) the acceleration of point Q. Point Q is attached at the top of gear B where it meets gear E.

$\vec{\omega}_E = -180 \frac{rev}{min} \hat{k} \left(\frac{2\pi\, rad}{1\, rev}\right)\left(\frac{1\, min}{60\, sec}\right) = -6\pi \hat{k}$

$\vec{\omega}_A = -240 \frac{rev}{min} \hat{k} \left(\frac{2\pi\, rad}{1\, rev}\right)\left(\frac{1\, min}{60\, sec}\right) = -8\pi \hat{k}$

A) Find $\omega_B$




write velocity of pt b & Q then solve for $\omega_B$

Gear A, pt. b

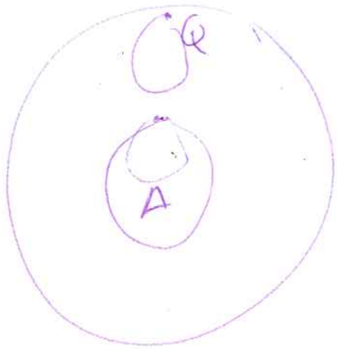
$\vec{V}_b = \cancel{\vec{V}_A} + \vec{\omega}_A \times \vec{r}_{b/A}$

0 pinned, $+30mm\,\hat{j}$

$\vec{V}_b = -8\pi \hat{k} \times 30\,mm\,\hat{j}$

$\vec{V}_b = 240\pi\ mm/s \uparrow$

Gear E, pt Q




$\vec{V}_Q = \vec{\omega}_E \times \vec{r}_E = -6\pi \hat{k} \times 90\,mm\,\hat{j}$

$\underline{\vec{V}_Q = 540\pi \ \frac{mm}{s} \ \hat{i}}$

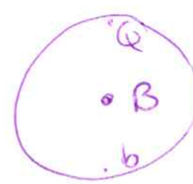
<u>Gear B</u> w/ points b & Q

$\vec{V}_b = \vec{V}_Q + \vec{\omega}_B \times \vec{r}_{b/Q}$

$240\pi \hat{i} = 540\pi \hat{i} + \omega_B \hat{k} \times -60\,mm\,\hat{j}$

$-300\pi \hat{i} = +60\,mm\,\omega_B \hat{i}$

$\boxed{\vec{\omega}_B = -5\pi \ r/s \ \hat{k}}$ same for C & D

B) spider  $\vec{\omega}_s = ?$ 

find $\vec{V}_B$ then $\vec{\omega}_s$

<u>gear B</u>

$\vec{V}_B = \vec{V}_b + \vec{\omega}_B \times \vec{r}_{B/b}$

$\vec{V}_B = 240\pi \hat{i} + (-5\pi \hat{k} \times 30\,mm\,\hat{j})$

$\boxed{\vec{V}_B = 240\pi \hat{i} + 150\pi \hat{i} = 390\pi \ mm/s \ \hat{i}}$

<u>spider</u>

$\vec{V}_B = \vec{\omega}_S \times \vec{r}_{B/A}$

$390\pi \hat{i} = \omega_S \hat{k} \times 60\,mm\,\hat{j} = -60\,\omega_S \hat{i} \rightarrow \boxed{\vec{\omega}_S = -6.5\pi \ \frac{r}{s} \ \hat{k}}$

$= -60\,\omega_S \hat{i}$

c) Find $\vec{a}_Q$, constant $\omega$'s

1, p3

~~spider gear~~ spider B

$$\vec{a}_B = \underbrace{\vec{a}_A}_{0} + \underbrace{\vec{\alpha}_{s/spider}}_{0} \times \vec{r}_{B/A} - \omega_s^2 \, \vec{r}_{B/A}$$

$$\vec{a}_B = -(-6.5\pi)^2 \, 60\,mm \, \hat{j}$$

$$\vec{a}_B = -7963.94 \; mm/s^2 \; \hat{j}$$

gear B

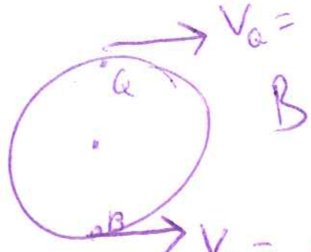
$$\vec{a}_Q = \vec{a}_B + \underbrace{\vec{\alpha}_B}_{0} \times \vec{r}_{Q/B} - \omega_B^2 \, \vec{r}_{Q/B}$$

$$\vec{a}_Q = -7963.94 \, mm/s^2 \, \hat{j} \underbrace{- (5\pi)^2 \, 30\,mm \, \hat{j}}_{-7402.20 \, \hat{j}}$$

$$\boxed{\vec{a}_Q = -15366.14 \; mm/s^2 \; \hat{j}}$$

# IC method

$\vec{V}_Q = 90mm\underbrace{\left(180\,\frac{rev}{min}\right)\left(\frac{2\pi}{rad}\right)\left(\frac{1\,min}{60\,sec}\right)}_{6\pi} = 1696.46\,\frac{mm}{s}$

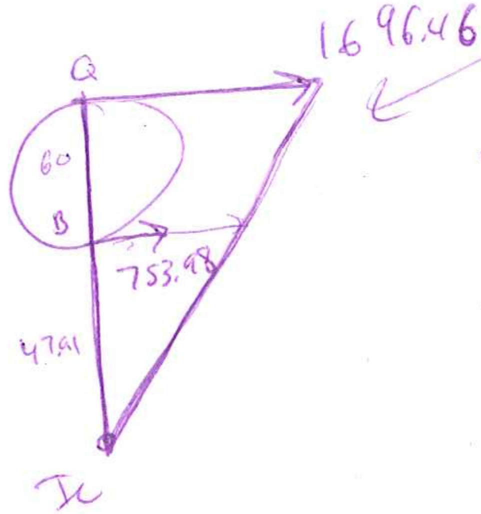



$V_B = 30mm\overbrace{\left(240\,\frac{rev}{min}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)\left(\frac{1\,min}{60\,sec}\right)}^{8\pi}$

$\vec{V}_B = 753.98\ mm/s$

$$\boxed{\tan\theta = \frac{1696.46}{r_{Q/IC}} = \frac{753.98}{r_{B/IC}}}$$

$r_{Q/IC} = (r_{B/IC} + 60mm)$




$\vec{V}_Q = \vec{\omega}_B \times \vec{r}_{Q/IC}$

$\vec{V}_B = \vec{\omega}_B \times \vec{r}_{B/IC}$


Q) $1696.46 = \omega_B \hat{k} \times r_{Q/IC}\hat{j} = -\omega_B r_{QIC}\hat{i}$

B) $753.98 = \omega_B \hat{k} \times r_{B/IC}\hat{j} = -\omega_B r_{B/IC}\hat{i}$

$$\boxed{\omega_B = \frac{-753.98}{r_{B/IC}}}$$

$$\boxed{\vec{\omega}_B = 15.7\ r/s\ \hat{k}}$$

$r_{B/IC} = \frac{753.98}{1696.46} r_{Q/IC}$

$r_{B/IC} = \frac{753.98}{1696.46}(r_{B/IC} + 60mm)$

$(1-(.444))\,r_{B/IC} = \frac{753.98}{1696.46}\,60mm$

$\boxed{r_{B/IC} = 47.91}$

## Spider gear w/ IC

$$\vec{V}_S = \vec{\omega}_B \times \vec{r}_{S/IC}$$

$$= -15.7 \, r/s \, \hat{k} \times (47.91 + 30mm)\hat{j}$$

$$V_S = 1223.8 \ mm/s$$

$$\vec{V}_S = \vec{\omega}_S \times \vec{r}_{S/A}$$

$$1223.8 = \omega_S \hat{k} \times 60 \hat{j}$$

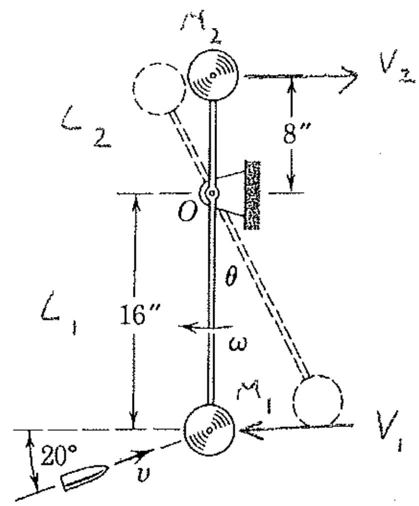
$$\omega_S = 20.39 \ r/s$$

**EMA 202, [redacted]** **Name: ______________________________**

**Midterm #2, closed book/notes, 90 min.**

**Question 2 (33 points)**

The pendulum consists of two 8.0 lb concentrated weights positioned as shown on a light but rigid bar. The pendulum is swinging through the vertical position with a clockwise angular velocity $\omega = 6.0$ rad/sec when a 2.0 ounce bullet, traveling with a velocity of v = 1000 ft/sec in the direction shown, strikes the lower weight and is embedded in it. Calculate the angular velocity $\omega_2$ that the pendulum has immediately after impact, and find the maximum angular deflection $\theta$ of the pendulum.




$W_M = 8$ lb. $\qquad W_B = \frac{1}{8}$ lb.

$\omega_o = 6$ r/s $\qquad V = 1000$ f/s

System — Angular momentum conserved during impact about point O. $\Rightarrow H_{o1} = H_{o2}$

Before Impact

$$L_1\omega_o = V_1 = \frac{16}{12}\omega_o \qquad V_2 = \frac{8}{12}\omega_o = L_2\omega_o$$

$$\circlearrowleft+ \Rightarrow H_{o1} = -L_1 M_1 V_1 - L_2 M_2 V_2 + L_1 M_B V\cos 20$$

$$\text{or} \quad H_{o1} = -\frac{8}{32.2}(6)\left(\frac{16}{12}\right)^2 - \frac{8}{32.2}(6)\left(\frac{8}{12}\right)^2 + \left(\frac{16}{12}\right)\left(\frac{\frac{1}{8}}{32.2}\right)1000\cos 20$$

$$\Rightarrow H_{o1} = 1.551 \text{ lb-ft-s}$$

After Impact $\qquad V_1 = L_1\omega_2 \qquad V_2 = L_2\omega_2$

$$H_{o2} = L_1^2(M_B + M_1)\omega_2 + L_2^2 M_2\omega_2$$

$$\text{or} \quad H_{o2} = \left(\frac{16}{12}\right)^2\left[\frac{\frac{1}{8}+8}{32.2}\right]\omega_2 + \left(\frac{8}{12}\right)^2\frac{8}{32.2}\omega_2$$

$$= (0.559)\,\omega_2$$

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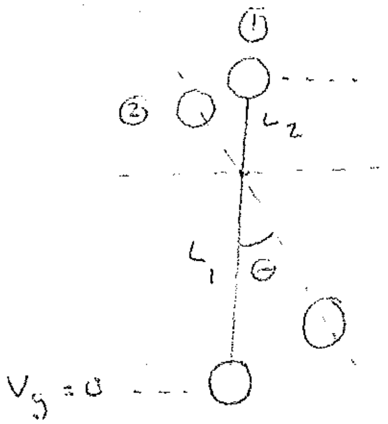
**Midterm #2, closed book/notes, 90 min.**

(Additional workspace for Question 2)

$H_{o1} = H_{o2} \Rightarrow \omega_2 = 2.77\ r/s$ ↺

<u>Pendulum - Bullet Work - Energy</u> After Impact




Energy conservation:

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = \frac{1}{2}(M_B + M_1)v_1^2 + \frac{1}{2}M_2 v_2^2$$

$$T_1 = \frac{1}{2}\left(\frac{\frac{1}{8}+8}{32.2}\right)\left(\frac{16}{12}\right)^2\omega_2^2 + \frac{1}{2}\frac{8}{32.2}\left(\frac{8}{12}\right)^2\omega_2^2$$

$$T_1 = 2.152 \text{ lb-ft}$$

$$T_2 = 0 \qquad V_1 = 0 \qquad V_2 = (W_1 + W_B)L_1(1-\cos\theta)$$
$$- W_2 L_2 (1-\cos\theta)$$

$$V_2 = \left[\left(\frac{1}{8}+8\right)\left(\frac{16}{12}\right) - 8\left(\frac{8}{12}\right)\right](1-\cos\theta)$$

$$V_2 = (5.500)(1-\cos\theta)$$

$$\Rightarrow \quad 2.152 = (5.5)(1-\cos\theta)$$

$$\Rightarrow \quad \cos\theta = 0.609 \Rightarrow \theta = 52.5°$$

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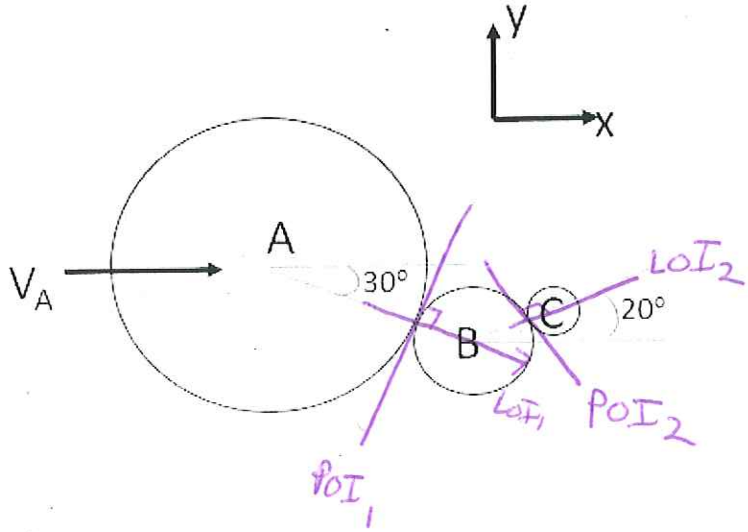
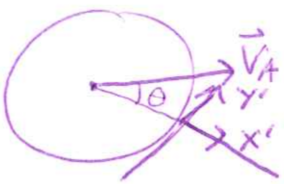
**Question 3 (33 points)**

Ball A is traveling purely in the x-direction with velocity $\mathbf{V_A}$ when it impacts ball B. Subsequently, ball B impacts ball C. Assume that both collisions have a coefficient restitution **e**.

Determine the magnitude of the velocity of ball C after it is impacted by ball B in terms of $\mathbf{m_A}$, $\mathbf{m_B}$, $\mathbf{m_C}$, **e**, $\mathbf{V_A}$, and the angles shown.




Impact A & B




$\vec{V}_A = V_A\hat{i} = V_a\cos 30\,\hat{i}' + V_A\sin 30\,\hat{j}'$

LOI in x'-direction

x') Conserv. momentum

$$\underbrace{m_A(V_a\cos 30)}_{V_{Ax'}^-} + m_B(\cancel{V_{Bx'}^-}) = m_A(V_{Ax'}^+) + m_B(V_{Bx'}^+)$$

0, @ rest

COR. $e = \frac{V_{Bx'}^+ - V_{Ax'}^+}{V_{Ax'}^- - \cancel{V_{Bx'}^-}}$ ↘ 0   Solve for $V_{Ax'}^+$, want $V_{Bx'}^+$

$V_{Ax'}^+ = V_{Bx'}^+ - V_{Ax'}^- e$   plug into momentum eqn

$$m_A V_A\cos 30 = m_A(V_{Bx'}^+ - \overset{V_A\cos 30}{V_{Ax'}^-}e) + m_B V_{Bx'}^+$$

Solve for $V_{Bx'}^+$

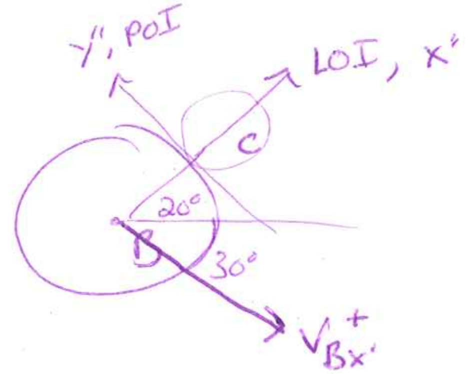
$$m_A V_A \cos 30 + m_A e V_A \cos 30 = (m_A + m_B) V_{Bx'}^+$$

$$\boxed{V_{Bx'}^+ = \frac{m_A}{(m_A+m_B)} V_A \cos 30 \, (1+e)}$$

POI in y' direction

$$V_{By'}^- = V_{By'}^+ = \underline{0}$$

Impact @ B & C




rotate $V_{Bx'}^+$ into x'' & y'' C.S.

$$\vec{V}_{Bx'}^+ = \underbrace{V_B^+ \cos 50 \, \hat{i}''}_{V_{Bx''}^+} \underbrace{\pm V_B^+ \sin 50 \, \hat{j}''}_{V_{By''}^+}$$

*notation (++) is after 2nd collision

X''-direction

$$m_B V_{Bx''}^+ + m_C \underbrace{\cancel{V_{Cx''}^+}}_{0,\ @\text{rest}} = m_B V_{Bx''}^{++} + m_C V_{Cx''}^{++}$$

COR

$$e = \frac{V_{Bx''}^{++} - V_{Cx''}^{++}}{\underbrace{\cancel{V_{Cx''}^+}}_{0} - V_{Bx''}^+}$$

want $V_{Cx''}^{++}$, solve for $V_{Bx''}^{++}$

$$\boxed{V_{Bx''}^{++} = V_{Cx''}^{++} - e \underbrace{V_{Bx''}^+}_{V_B^+ \cos 50°}}$$

sub into momentum, solve for $V_C^{++}$

$$m_B V_B^+ \cos 50^\circ = m_B\left(V_{Cx''}^{++} - e V_B^+ \cos 50\right) + m_C V_{Cx''}^{++}$$

$$m_B(1+e)V_B^+ \cos 50^\circ = (m_B + m_C) V_{Cx''}^{++}$$

$$V_{Cx''}^{++} = \left(\frac{m_B}{m_B+m_C}\right)(1+e)\, V_B^+ \cos 50^\circ$$

$$\frac{m_A}{m_A+m_B} V_A \cos 30^\circ (1+e)$$

velocity of B after 1st impact

$$\boxed{V_{Cx''}^{++} = \left(\frac{m_B}{m_B+m_C}\right)\left(\frac{m_A}{m_A+m_B}\right)(1+e)^2\, V_A \cos 30^\circ \cos 50^\circ}$$