ME 240 / EMA 202, Fall 2016 **Name:** Solutions

Midterm #1, closed book/notes, 90 min.

Instructions

- Show all of your work.
- To maximize opportunities for partial credit, solve problems symbolically to the extent possible and substitute numerical values at the end.
- Include free body diagrams for all equilibrium equations.

- The only notes allowed are the equations provided with this exam.
- The use of cell phones is prohibited.
- The instructors and the University of Wisconsin expect the highest standards of honesty and integrity in the academic performance of its students. It is important that the work submitted on this examination is yours and yours alone.
- Receiving or giving aid in an examination or using notes on a closed note exam will be considered cheating and will result in a grade of F and the case being reported to the Dean of Students Office.

Circle Your Discussion Section:

EMA 202	Time	TA
301	8:50	Peter Grimmer
302	9:55	Aswin Rajendram Muthukumar
303	11:00	Aswin Rajendram Muthukumar
304	12:05	Zz Riford
305	1:20	Peter Grimmer
306	2:25	Peter Grimmer
307	12:05	Aaron Wright
ME 240		
301	8:50	Jenna Lynne
302	9:55	Jenna Lynne
303	11:00	Chembian Parthiban
304	12:05	Chembian Parthiban
305	1:20	Aaron Wright
306	2:25	Zz Riford

Grading:

Q1	/10
Q2	/30
Q3	/30
Q4	/30
Total	/100

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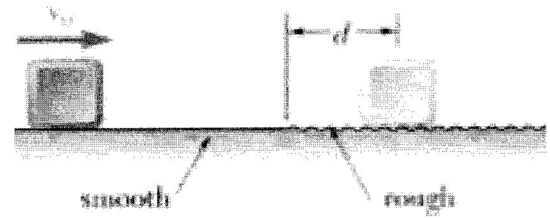
Question 1 (10 points)

For the following short answer problems, please include any relevant calculations and/or a brief explanation for your answer.

1A (5 points)

A block is traveling with a speed v_o on a smooth surface when the surface suddenly becomes rough with a coefficient of friction of μ causing the block to stop after a distance *d*. If the block were traveling twice as fast ($2v_o$), how far will it travel on the rough surface before stopping?



Work-Energy

$T_1 + U_{12} = \cancel{T_2}^{\,0}$ $\quad v_2 = 0$

$\downarrow mg$, $\leftarrow F_f = \mu_k mg$, $\uparrow N$

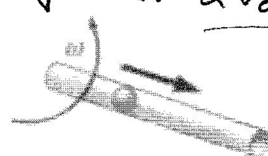
$\frac{1}{2} m v_1^2 - F_f d = 0$

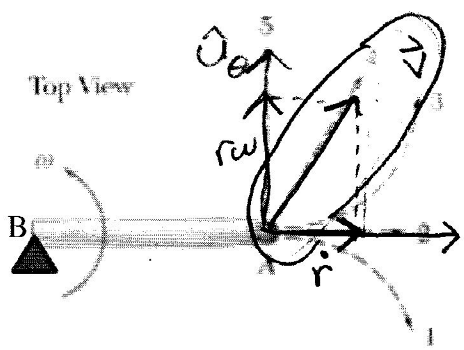
$\frac{1}{2} \cancel{m} v_1^2 - \mu_k \cancel{m} g d = 0 \Rightarrow \boxed{d = \frac{v^2}{2\mu_k g}}$

Sliding distance is a function of v^2 $\therefore$ $\underline{2v_o \rightarrow 4d}$ sliding distance

1B (5 points)

Marble A is placed in a hollow tube that is pinned at point B. The tube is swung in a horizontal plane causing the marble to be thrown from the end of the tube. As viewed from the top, circle the trajectory 1-5 that best describes the path of the marble after leaving the tube?





polar coordinates

$\vec{v} = \underbrace{\dot{r}\,\hat{u}_r}_{\text{ball traveling down tube}} + \underbrace{r\omega\,\hat{u}_\theta}_{\text{tube rotating}}$

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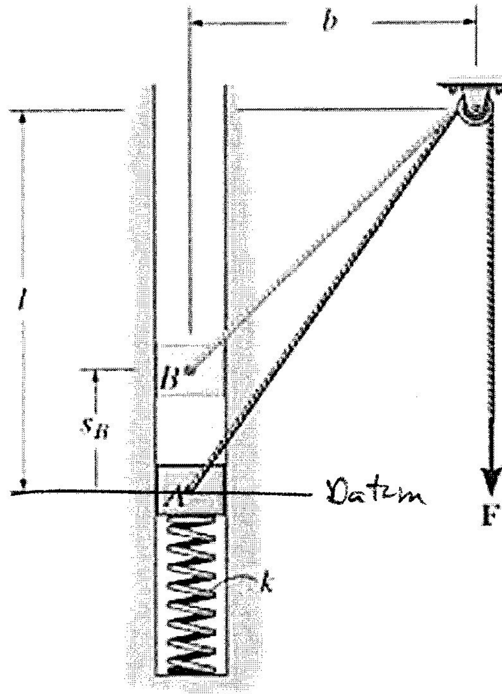
Question 2 (30 points)

The block has a mass M = 0.8kg and moves within the smooth vertical slot. It starts from rest when the **attached** spring is in the un-stretched position at A.

Determine the ***constant*** vertical force **F** which must be applied to the cord so that the block attains a speed V_B = 2.5 m/s when it reaches S_B = 0.15m. *Note that the spring is still attached to the block at position B.*



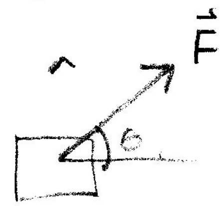
Given:

M = 0.8 kg	l = 0.4 m	V_B = 2.5 m/s
b = 0.3 m	S_B = 0.15 m	k = 100 N/m

Work - Energy

$$\cancel{T_1} + \cancel{V_1} + U_{12} = T_2 + V_2$$

0, @ rest   0, Datum



$$T_2 = \frac{1}{2} m V_2^2$$

$$V_2 = V_{2g} + V_{2s} = mgS_B + \frac{1}{2} k S_B^2$$

$$U_{12} = \int_1^2 \vec{F} \cdot d\vec{r} = \int_0^{S_B} F_y \, dy = \int_0^{S_B} F \sin\theta \, dy$$

$$\sin\theta = \frac{(L-y)}{\sqrt{(L-y)^2 + b^2}}$$

$$U_{12} = \int_0^{S_B} \frac{F(L-Y)}{\sqrt{(L-y)^2+b^2}} \, dy$$

$$u = (L-Y)^2 + b^2$$

$$du = -2(L-Y)\, dy$$

$$u_0 = L^2 + b^2$$

$$u_f = (L-S_B)^2 + b^2$$

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$$U_{12} = \frac{-F}{2} \int_{L^2+b^2}^{(L-S_B)^2+b^2} \frac{du}{\sqrt{u}} = \frac{-F}{2} \frac{\sqrt{u}}{1/2} \Big|_{L^2+b^2}^{(L-S_B)^2+b^2}$$

$$U_{12} = -F\left[\sqrt{(L-S_B)^2+b^2} - \sqrt{L^2+b^2}\right]$$

$$U_{12} = F\underbrace{\left[\sqrt{L^2+b^2} - \sqrt{(L-S_B)^2+b^2}\right]}_{\Delta L} = F\Delta L =$$

Back to W-E

$$F\cancel{\Delta L} = \frac{\frac{1}{2}mv_2^2 + mgS_B + \frac{1}{2}kS_B^2}{\Delta L}$$

$$F = \frac{\frac{1}{2}(.8\,kg)(2.5\,m/s)^2 + .8\,kg(9.81\,m/s^2)(.15\,m) + \frac{1}{2}(100\,N/m)(.15\,m)^2}{0.1095\,m}$$

$$\boxed{F = 43.9\,N}$$

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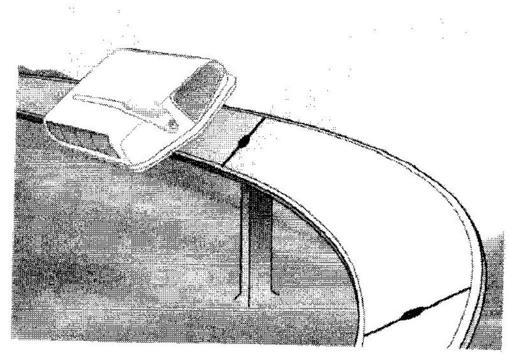
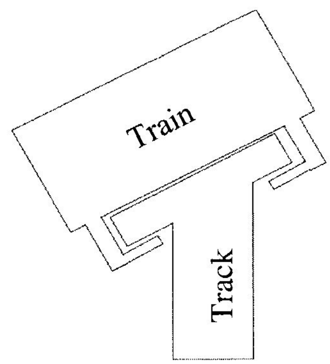
Question 3 (30 points)

The train in the figure is supported by magnetic repulsion forces exerted in the direction perpendicular to the track. Motion of the train in the transverse direction is prevented by lateral supports. The 20,000-kg train is traveling at 30 m/s on a curved segment of track described by the equation $y=.0033x^2$ where x and y are in meters. The bank angle of the track is 40° when x=0. When the train is at this point in the curve:

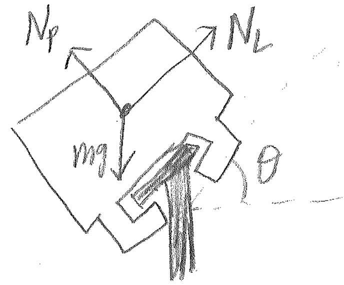


a) Draw the forces acting on the train in the outline below.



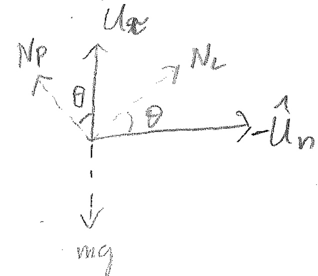
c) What force must the magnetic levitation system exert to support the train **and** what force is exerted by the lateral supports?

b) For what speed would the lateral force at this point of the curve be zero? (This is the optimum speed for the train to travel on the banked track. If you were a passenger, you would not need to exert any lateral force to remain in place in your seat.)



P = perpendicular
L = longitudinal

given $m = 20000$ kg
$V = 30$ m/s
$y = .0033x^2$ (m)
$\theta = 40°$ at $x = 0$



$$\rho ? \rightarrow \rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left(1 + (.0066x)^2\right)^{3/2}}{.0066} \text{ m}$$

$$\rho(x=0) = \frac{1}{.0066} = \underline{151.52 \text{ m}}$$

$$\Sigma F_t: N_P \cos\theta + N_L \sin\theta - mg = 0$$

$$\Sigma F_n: N_P \sin\theta - N_L \cos\theta = m\frac{v^2}{\rho}$$

$$N_L = \frac{mg - N_P \cos\theta}{\sin\theta}$$

$$N_P \sin\theta - \frac{mg - N_P\cos\theta}{\sin\theta}\cos\theta = m\frac{v^2}{\rho}$$

$$\vec{N}_P = \frac{m\frac{v^2}{\rho} + mg\cot\theta}{\sin\theta + \cot\theta\cos\theta} = 2.866 \times 10^5 \text{ N}$$
$$= \boxed{286.58 \text{ kN}}$$

$$\vec{N}_L = -36610.9 \text{ N}$$
$$= -36.61 \text{ kN}$$
$$\boxed{N_L = 36.61 \text{ kN}}$$

c) $N_L = 0$

$$N_P \sin\theta - 0 = m\frac{v^2}{\rho}$$

$$N_P \cos\theta + 0 - mg = 0$$

$$N_P = \frac{mg}{\cos\theta}$$

$$v = \sqrt{\frac{\cancel{m}g\rho\tan\theta}{\cancel{m}}}$$

$$\boxed{v = 35.30 \frac{m}{s}}$$

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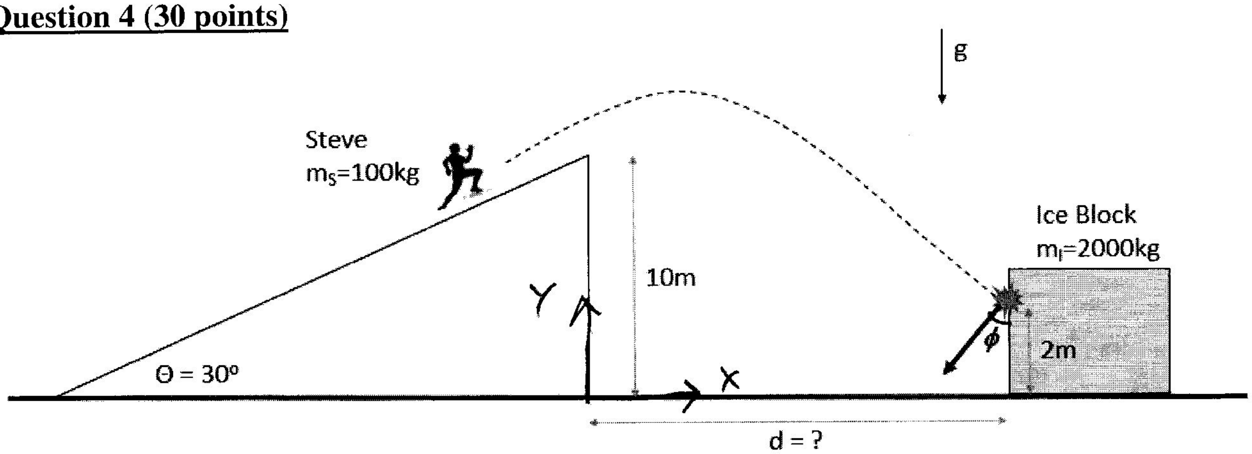
Question 4 (30 points)



For an amazing stunt, Steve runs off a 10m tall ramp with a 30° incline at a speed of 10m/s. He hopes to jump over a pit and impact a block of ice at a height of 2m.

A) At what distance, d, should the ice block be placed from the end of the ramp?
B) Assume that the 2000 kg ice block slides without friction on the ground. Given the coefficient of restitution between Steve and the ice block is 0.2, at what angle ϕ will Steve bounce off of the wall?
C) Given that the impact between Steve and the ice block takes 0.25 seconds. What is the average force felt by Steve during the impact?

given
$X_0 = 0$
$Y_0 = 10m$
$X_f = d$
$Y_f = 2m$
$\theta = 30°$
$V_0 = 10 m/s$

A) X-dir
$a_x = 0$
$V_x = V_0 \cos\theta$
$X(t) = V_0 \cos\theta \, t + X_0$
① $t = \dfrac{X}{V_0 \cos\theta}$

Y-dir
$a_y = -g$
$V_y = -gt + V_0 \sin\theta$
② $Y(t) = \dfrac{-g}{2} t^2 + V_0 \sin\theta t + Y_0$

Sub. ① into ②

$$Y_f = \frac{-g}{2}\left(\frac{X}{V_0 \cos\theta}\right)^2 + V_0 \sin\theta \left(\frac{X}{V_0 \cos\theta}\right) + Y_0$$

$$0 = \frac{-9.81 \, m/s^2}{2}\left(\frac{X^2}{(10 m/s)^2 \cos^2(30°)}\right) + \tan(30°) X + (10m - 2m)$$

$$\boxed{X = d = 16.32 m}$$

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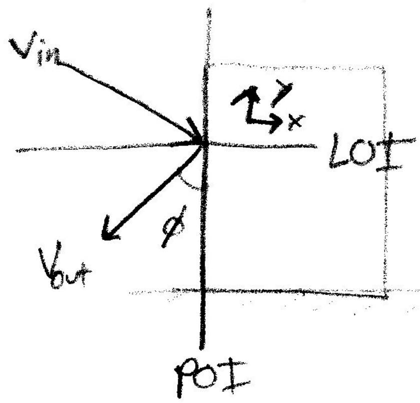
B) Find ϕ given $e = .2$, $m_S = 100kg$, $m_I = 2000kg$



$\vec{V}_{in} = V_{x_f}\hat{i} + V_{y_f}\hat{j}$

$\vec{V}_{in} = V_0\cos\theta\,\hat{i} + (-gt + V_0\sin\theta)\hat{j}$

From eq (1) $t = \frac{x}{V_0\cos\theta} = \frac{16.32m}{10 m/s \cos 30}$

$t = 1.88$ sec

$\vec{V}_{in} = 10 m/s \cos 30° \hat{i} + [-9.81 m/s^2 (1.88 sec) + 10 m/s \sin 30°]\hat{j}$

$\vec{V}_{in} = 8.66 m/s \hat{i} - 13.44 m/s \hat{j} \Rightarrow V_x^- = 8.66 m/s \quad V_y^- = -13.44 m/s$

Plane of impact direction - particle conservation of momentum

$V_y^- = V_y^+ = -13.44 m/s$

Line of impact - system momentum & coef. of restitution

$$m_S V_{Sx}^- + m_I \underbrace{\cancel{V_{Ix}^-}}_{0,\text{ Ice not moving}} = m_S V_{Sx}^+ + m_I V_{Ix}^+$$

$$e = \frac{V_{Ix}^+ - V_{Sx}^+}{V_{Sx}^- - \cancel{V_{Ix}^-}_{0}} \Rightarrow V_{Ix}^+ = eV_{Sx}^- + V_{Sx}^+$$

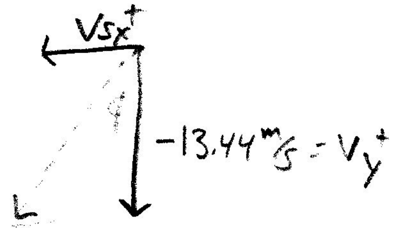
$$m_S V_{Sx}^- = m_S V_{Sx}^+ + m_I(eV_{Sx}^- + V_{Sx}^+)$$

$$\boxed{V_{Sx}^+ = -1.24 m/s}$$

$$m_S V_{Sx}^- - m_I e V_{Sx}^- = (m_S + m_I)V_{Sx}^+$$

$$V_{Sx}^+ = \frac{(m_S - m_I e)}{m_S + m_I} V_{Sx}^- = \frac{(100kg - 2000kg(.2))}{100 + 2000kg}(8.66 m/s)$$

Find angle ϕ



$$\tan\phi = \frac{V_{sx}^{+}}{V_{sy}^{+}} = \frac{1.24}{13.44}$$

$$\boxed{\phi = 5.27^\circ}$$

c) Force = ? if impact takes .25 sec.

Impulse - momentum x-direction

$$m_s V_{sx}^{-} + I_x = m_s V_{sx}^{+}$$

$$I_x = m_s(V_{sx}^{+} - V_{sx}^{-}) = 100\,kg\,(-1.24\,m/s - 8.66\,m/s)$$

$$I_x = -990\ N\text{-}s$$

$$I_x = F_{AVE}t = -990\,N\text{-}s$$

$$F_{AVE} = \frac{-990\,N\text{-}s}{.25\,s}$$

$$\vec{F}_{AVE} = -3960\,N \uparrow$$