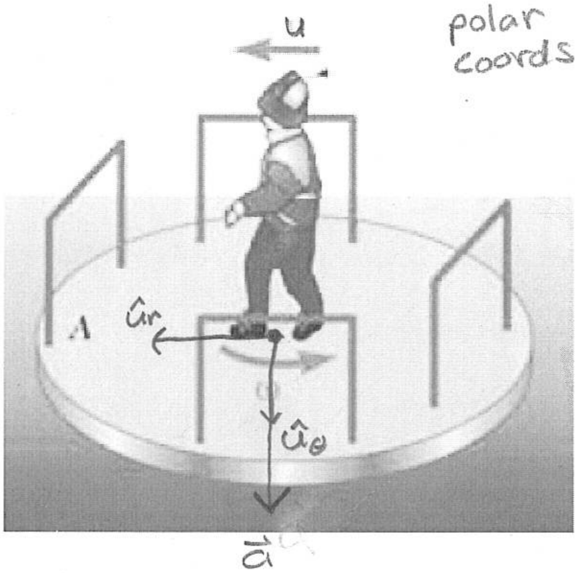
EMA 202, Spring 2015 **Name:** ______________________________

Midterm #1, closed book/notes, 90 min.

Question 1 (10 points)

1A (5 points) A child walks across a merry-go-round with constant speed *u* relative to the platform. The merry-go-round is rotating about its center at a constant angular velocity, ω, in the direction shown. When the child is at the center of the platform, write an expression for the acceleration vector and draw the acceleration vector (with coordinate system) on the figure below.



polar coords $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$

given $\omega = c$, so $\ddot{\theta} = 0$

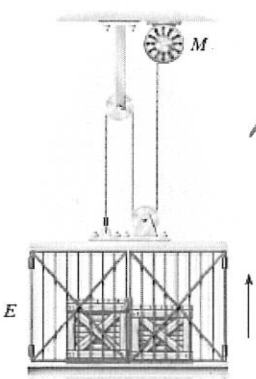
$u = \dot{r} = c$, so $\ddot{r} = 0$

find $\vec{a}$ @ $r = 0$

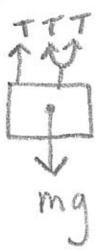
$\vec{a} = (0 - 0\cdot\omega^2)\hat{u}_r + (0\cdot 0 + 2u\omega)\hat{u}_\theta$

$\boxed{\vec{a} = 2u\omega\,\hat{u}_\theta}$

1B (5 points)
A 500kg elevator starts from rest and travels upward with a constant acceleration of 2m/s^2. Determine the power output of motor M at t = 2 seconds. $P = \vec{F}\cdot\vec{V}$



FBD



$\Sigma F_y = 3T - mg = ma_y$

$T = \dfrac{m(a_y + g)}{3}$

$T = \dfrac{500kg(2\,m/s^2 + 9.81\,m/s^2)}{3}$

$T = 1968.3\,N$

Velocity

$V_E = \cancel{V_0}^{0} + a_y t = 2\,m/s^2(3\,sec)$

$V_E = 6\,m/s$

Power

$P = \vec{F}\cdot\vec{V} = 3T\hat{j}\cdot 6\,m/s\,\hat{j}$ $\boxed{P = 35.4\,kW}$

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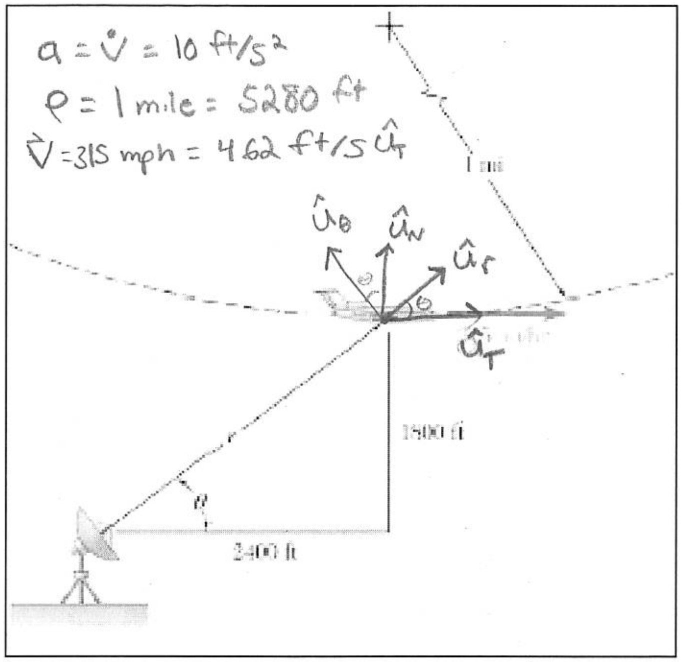
<u>Question 2 (30 points)</u>

At the bottom of a loop in the vertical plane, an airplane has a horizontal velocity of 315mph and is accelerating at a rate of 10 ft/s^2. The radius of curvature of the loop is 1 mile. The plane is being tracked by radar at O. What are the values for $\dot{r},\ \ddot{r}, \dot{\theta}, and\ \ddot{\theta}$ at this instant?



accel in NT

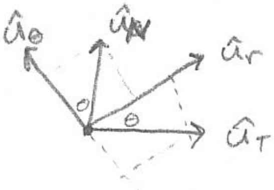
$V = 315$ MPH $= 462$ ft/s

$$\vec{a} = \dot{v}\,\hat{u}_T + \frac{v^2}{\rho}\hat{u}_N$$

$$\vec{a} = 10\ \text{ft/s}^2\,\hat{u}_T + \frac{(462\ \text{ft/s})^2}{5280\ \text{ft}}\hat{u}_n$$

$a_T = 10$ ft/s^2

$a_N = 40.425$ ft/s^2



polar coords

$$r = \sqrt{1800\text{ft}^2 + 2400\text{ft}^2} = 3000\ \text{ft}$$

$$\theta = \tan^{-1}\left(\frac{1800}{2400}\right) = 36.9°$$

write $\hat{u}_N$ & $\hat{u}_T$ in terms of $\hat{u}_r, \hat{u}_\theta$

$\hat{u}_T = \cos\theta\,\hat{u}_r - \sin\theta\,\hat{u}_\theta$

$\hat{u}_N = \sin\theta\,\hat{u}_r + \cos\theta\,\hat{u}_\theta$

$$\vec{V} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{u}_\theta$$

convert to r, θ

$$\vec{V} = 462\,\hat{u}_T = 462\cos\theta\,\hat{u}_r - 462\sin\theta\,\hat{u}_\theta$$

$$\vec{a} = 10\,\hat{u}_T + 40.425\,\hat{u}_N = 10\cos\theta\,\hat{u}_r - 10\sin\theta\,\hat{u}_\theta + 40.425\sin\theta\,\hat{u}_r + 40.425\cos\theta\,\hat{u}_\theta$$

$$\vec{a} = (10\cos\theta + 40.425\sin\theta)\,\hat{u}_r + (40.425\cos\theta - 10\sin\theta)\,\hat{u}_\theta$$

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(Additional workspace for Question 2)

$v_r)$ $\dot{r} = 462 \cos\theta$ w/ $\theta = 36.9°$

$\boxed{\dot{r} = 369.5 \text{ ft/s}}$

$v_\theta)$ $r\dot{\theta} = -462 \sin\theta$ w/ $r = 3000$ ft

$\dot{\theta} = \dfrac{-462 \sin\theta}{r}$

$\boxed{\dot{\theta} = -.092 \text{ r/s}}$

$a_r)$ $\ddot{r} - r\dot{\theta}^2 = 10\cos\theta + 40.425 \sin\theta$

$\ddot{r} = 10\cos\theta + 40.425 \sin\theta + r\dot{\theta}^2$

$\boxed{\ddot{r} = 57.7 \text{ ft/s}^2}$

$a_\theta)$ $r\ddot{\theta} + 2\dot{r}\dot{\theta} = 40.425\cos\theta - 10\sin\theta$

$\ddot{\theta} = \dfrac{1}{3000}\left(40.425\cos\theta - 10\sin\theta - 2(\dot{r})\dot{\theta}\right)$

$\boxed{\ddot{\theta} = .0314 \text{ r/s}^2}$

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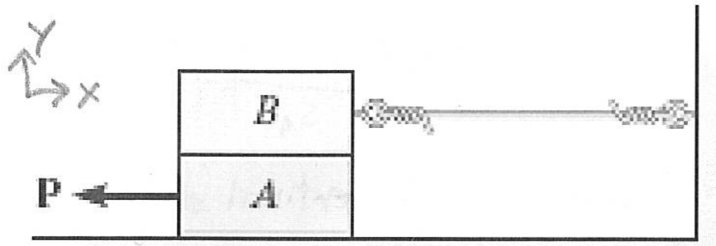
<u>Question 3 (30 points)</u>

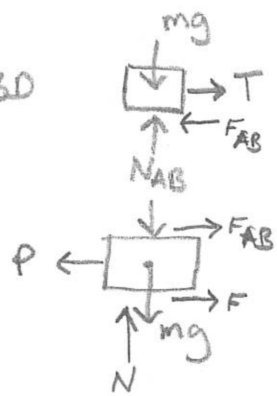
Each of the two blocks has a mass m. The coefficient of kinetic friction on all surfaces of contact is μ. If a horizontal force P moves the bottom block, determine the acceleration of the bottom block in cases (a) and (b).

Additionally, solve for the force P, in which case (a) and (b) have the same acceleration.



(a)

A) FBD

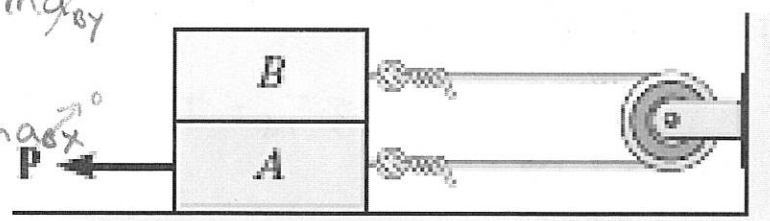


$+\uparrow \Sigma F_{YB} = N_{AB} - mg = m\cancel{a_{BY}}^{0}$

$N_{AB} = mg$

$\xrightarrow{+} \Sigma F_{XB} = T - F_{AB} = m\cancel{a_{BX}}^{0}$

$T = F_{AB}$



(b)

$$+\uparrow \Sigma F_{YA} = N - N_{AB} - mg = m\cancel{a_{AY}}^{0}$$

$$N = N_{AB} + mg = 2mg$$

$$\xrightarrow{+} \Sigma F_{XA} = -P + F + F_{AB} = m a_{AX} \quad w/ \quad F = \mu N \;\; \& \;\; F_{AB} = \mu N_{AB}$$

$$-P + \mu N + \mu N_{AB} = m a_{AX}$$

$$-P + \mu 2mg + \mu mg = m a_{ax}$$

$$\boxed{a_{ax} = \frac{-P}{m} + 3\mu g}$$

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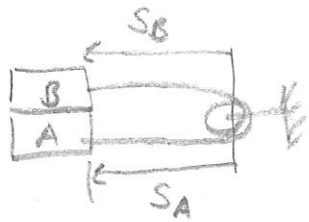
(Additional workspace for Question 3)

B)



rope length

$L = S_A + S_B$

$\dot{L} = 0 = V_A + V_B$

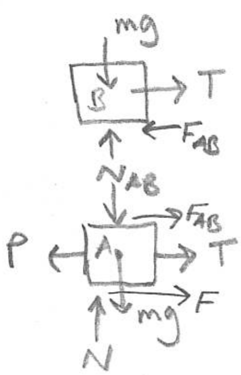
$\ddot{L} = 0 = a_A + a_B$

$\boxed{a_A = -a_B}$

From part A vertical dir.

$N = 2mg$

$N_{AB} = mg$



$\xrightarrow{+} \sum F_{xB} = T - F_{AB} = m a_{Bx} \Rightarrow T = F_{AB} - m a_{Ax}$

$\xrightarrow{+} \sum F_{xA} = T + F_{AB} + F - P = m a_{ax}$

$F_{AB} - m a_x + F_{AB} + F - P = m a_x$

$2 F_{AB} + F - P = 2 m a_x$ w/ $F = \mu N = \mu 2mg$

$F_{AB} = \mu N_{AB} = \mu mg$

$2(\mu mg) + 2\mu mg - P = 2 m a_x$

$$\boxed{a_x = \frac{-P}{2m} + 2\mu g}$$

C) find P so accel for (a) & (b) are equal

(a) (b)

$-\frac{P}{m} + 3\mu g = \frac{-P}{2m} + 2\mu g$

$3\mu g - 2\mu g = \frac{-P}{2m} + \frac{P}{m}$

$\mu g = \frac{P}{2m}$

$\boxed{P = 2m\mu g}$

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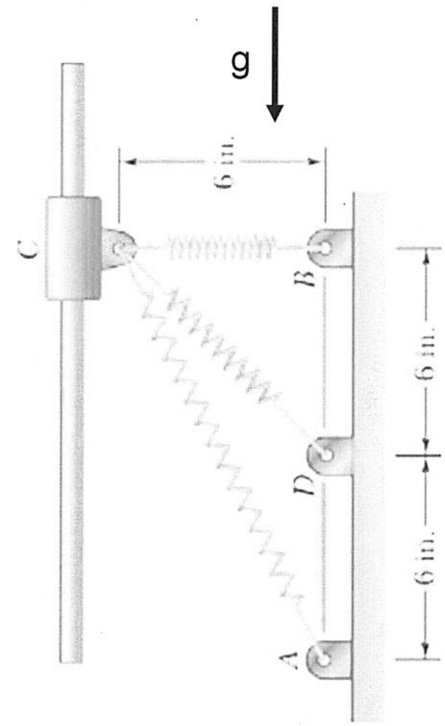
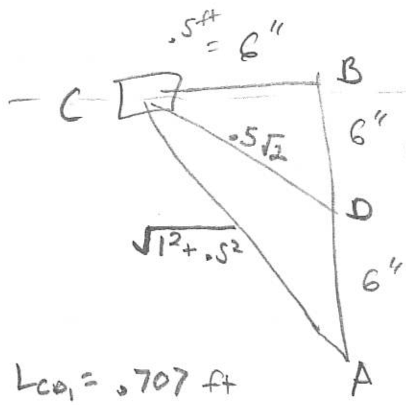
Question 4 (30 points)



A 3-lb collar C may slide without friction along the vertical rod. It is attached to 3 springs, each with spring constant k = 2 lb/in and an undeformed length of 6 inches. Given that the collar is released from rest, determine the speed of the collar after it has traveled 6 inches down.

$k = 2\ \text{lb/in} = 24\ \text{lb/ft}$



$T_1 + V_1 = T_2 + V_2$

① $T_1 = \frac{1}{2} m v_1^2 \to 0$ released from rest

$V_1 = V_{1sBC} + V_{1sDC} + V_{1sAC} + \cancel{mgh} \to 0$

$L_{CD_1} = .707\ \text{ft}$

$L_{AC_1} = 1.12\ \text{ft}$

$V_{1sBC} = \frac{1}{2} k \delta^2 = \frac{1}{2} k (L - L_0)^2 = \underline{0}$

$V_{1sDC} = \frac{1}{2} k \delta^2 = \frac{1}{2} k (L_{CD_1} - .5\ \text{ft})^2 = \frac{1}{2}(24\ \text{lb/ft})(.707 - .5)^2$

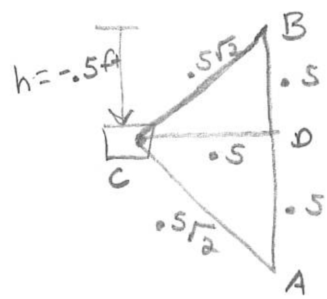
$\underline{V_{1sDC} = .514\ \text{ft·lb}}$

$V_{1sAC} = \frac{1}{2} k (L_{AC_1} - .5\ \text{ft})^2 = \underline{4.613\ \text{ft·lb}}$

$V_1 = 0 + .514 + 4.613\ \text{ft-lb} = \underline{5.13\ \text{ft-lb}}$

<u>State 2</u>

$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}\left(\frac{3\ \text{lb}}{32.2\ \text{ft/s}^2}\right) v_2^2$



$V_2 = V_{2sBC} + V_{2sCD} + V_{2sAC} - mgh$

$V_{2sBC} = V_{2sAC} = \frac{1}{2} k (.5\sqrt{2} - .5)^2 = .514\ \text{ft-lb}$

$V_{2CD} = \frac{1}{2} k (.5 - .5)^2 = 0$

$V_2 = 2 \cdot V_{2sBC} - mgh = 2(.514\ \text{ft·lb}) - 3\ \text{lb}(.5\ \text{ft})$

$\boxed{V_2 = -.472\ \text{lb-ft}}$

$$\cancel{T_1}^{\,0} + V_1 = T_2 + V_2$$

$$5.13 \text{ ft-lb} = \frac{1}{2}\left(\frac{3 \text{ lb}}{32.2 \text{ ft/s}^2}\right)v_2^{\,2} + (-.472 \text{ lb ft})$$

$$v_2^{\,2} = 120.25 \text{ ft-lb}$$

$$\boxed{v_2 = 10.97 \text{ ft/s}} \downarrow$$