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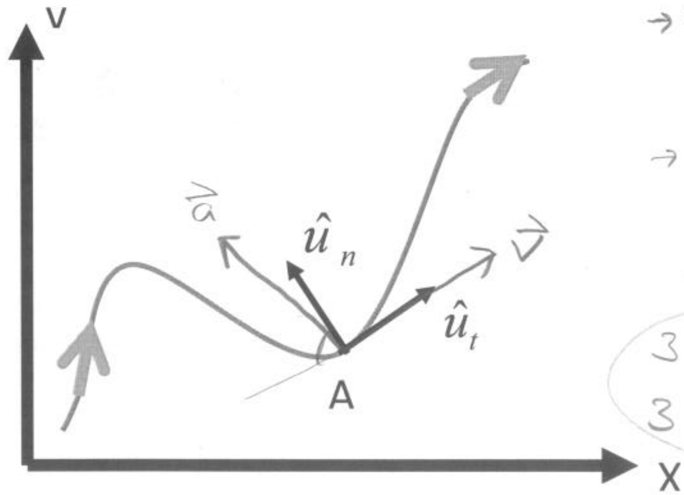
Midterm #1, closed book/notes, 90 min.

Question 1 (25 points)

Conceptual questions: please include a few sentences or equations to justify your answers

1A (6 points) The trajectory of a particle is shown on the figure below. At point A the speed of the particle is decreasing. Draw the acceleration and velocity vectors at point A (magnitude is not important)



→ velocity is tangent to the path

→ $\vec{a} = \underbrace{-a_t \hat{u}_t}_{\text{slowing down}} + a_n \hat{u}_n$

3 pts for $\vec{v}$
3 pts for $\vec{a}$

1B (5 points) A windmill sits in an inertial coordinate system. Its blades are rotating at a constant rate ω. Is the acceleration of point A equal to 0?

polar

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$

$\dot{r} = \ddot{r} = \ddot{\theta} = 0$

$\vec{a} = -r\dot{\theta}^2 \hat{u}_r \neq 0$

centripetal acceleration is nonzero

or in N-T

$\vec{a} = \dot{v}\hat{u}_T + \frac{v^2}{\rho}\hat{u}_n \quad , \quad \dot{v} = 0$

$\vec{a} = \frac{v^2}{\rho}\hat{u}_n \neq 0$

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1C (8 points) Determine the equation for the trajectory of a projectile, y(x). Assume x(t=0) = 0 and y(t=0) = 0. (hint: your equation should look like $Y = C_1X+C_2X^2$, Find C1 and C2)

2 pts $X(t) = V_0 \cos\theta \, t$ $\Rightarrow$ Solve for $t(x)$ $t = \frac{X}{V_0 \cos\theta}$ 2 pts

2 pts $Y(t) = V_0 \sin\theta \, t - \frac{g}{2} t^2$

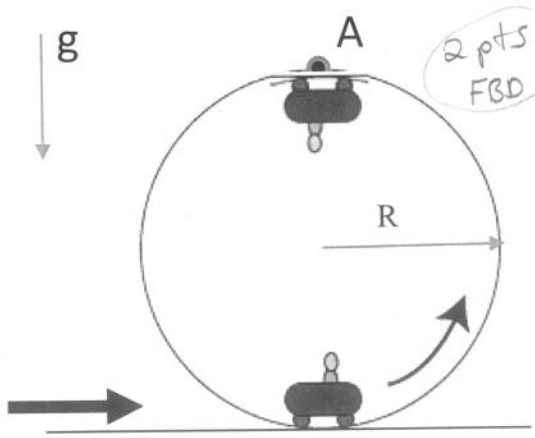
plug $t(x)$ into $Y(t)$

$$Y = V_0 \sin\theta \left(\frac{X}{V_0 \cos\theta}\right) - \frac{g}{2}\left(\frac{X}{V_0 \cos\theta}\right)^2$$

$C_1 = \tan\theta$

2 pts $\boxed{Y = \tan\theta \, X - \left(\frac{g}{2V_0^2 \cos^2\theta}\right) X^2}$ $C_2 = \frac{g}{2V_0^2\cos^2\theta}$

1D (6 points) A toy car is riding a track with a circular loop of radius R. What is the minimum speed that the car must be going at point A to maintain contact with the track? Let m=car mass



2 pts FBD mg N $\hat{u}_t$ $\hat{u}_n$ - for minimum speed let $N \to 0$

$$\Sigma F_n = mg - \cancel{N}^{0} = m a_n \quad \text{2pts}$$

$$a_n = \frac{v^2}{R}$$

$$\cancel{m} g = \cancel{m} \frac{v^2}{R}$$

$\boxed{V = \sqrt{gR}}$ 2pts

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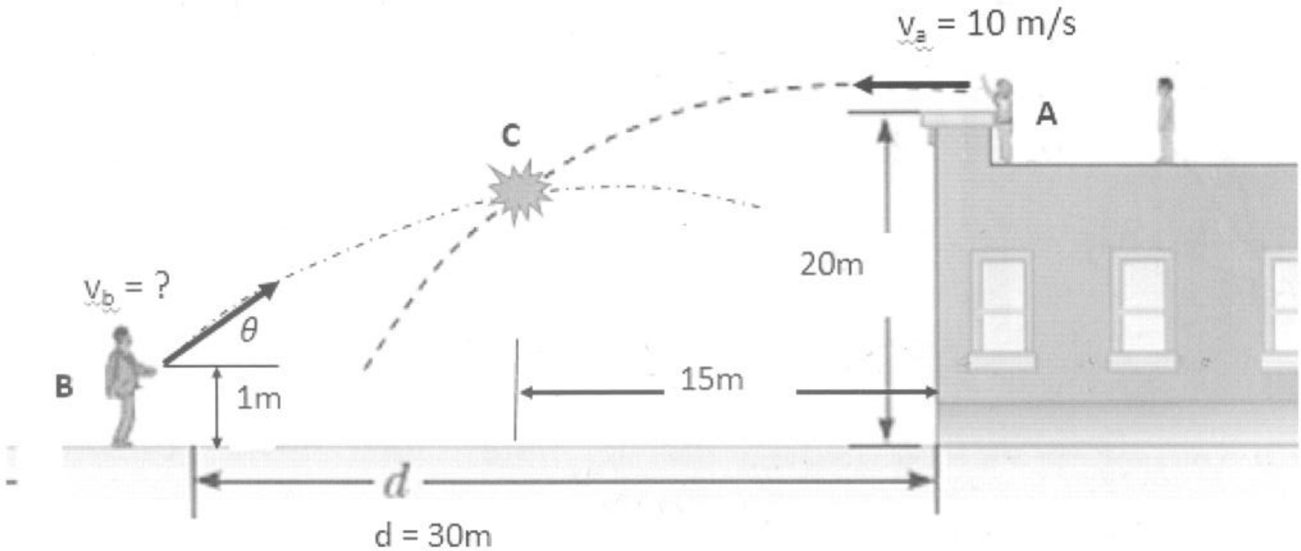
Question 2 (25 points)

A ball is thrown horizontally from the top of a 20m tall building with a velocity of 10m/s. A short time later, a person on the ground (point B) throws another ball such that the two balls collide at point C. Given that the person on the ground throws from a height of 1m and an angle $\theta = 35°$,

8 pts a) Find the height at which the balls collide
12pts b) Find the velocity of the ball thrown from point B
5pts c) Find the time delay between when ball A and B are thrown



A) $X_a = X_{ao} + V_{oax} t$

$15_m = 30_m - 10^m/_s \, t$

(4 pts) $t = 1.5$ sec @ impact location

$Y = Y_{ao} + \cancel{V_{oay}} t - \frac{g}{2} t^2$

V_{oA} = horizontal → 0

$Y_a = 20_m + 0 - \frac{9.81 \, m/s^2}{2} (1.5 \text{sec})^2$

(4pts) $\boxed{Y_a = 8.96 \text{ m}}$ height @ impact point C

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(Additional workspace for Question 2)

B. velocity of ball b.

(2pts) $X_b = \cancel{X_{bo}} + V_{box}\, t_2$ ($X_{bo} = 0$)

$X_b = V_{bo} \cos\theta \; t_2$

(2pts) $t_2 = \dfrac{X_b}{V_{bo} \cos\theta} = \dfrac{15\,m}{V_{bo} \cos 35^\circ}$

(2pts) $Y_b = Y_{bo} + V_{boy} t_2 - \dfrac{g}{2} t_2^2$

$8.96\,m = 1m + V_{bo} \sin\theta \; t_2 - \dfrac{9.81\,m/s^2}{2} t_2^2$

(2pts) Sub in for t_2 & solve for V_{bo}

$(8.96m - 1m) = \cancel{V_{bo}} \sin(35) \dfrac{15m}{\cancel{V_{bo}} \cos(35)} - \dfrac{9.81\,m/s^2}{2} \left(\dfrac{15}{V_{bo} \cos(35)}\right)^2$

(2pts) $\boxed{V_{bo} = 25.43\ m/s}$

C. time delay

$t_1 = t_2 + \Delta t$

(2pts) $\Delta t = t_1 - t_2$

(2pts) $t_2 = \dfrac{15m}{(25.43\,m/s) \cos 35} = .72\ sec$

(1pt) $\boxed{\Delta t = 1.5 - .72 = .78\ sec}$ ball b is thrown .78 sec after ball A.

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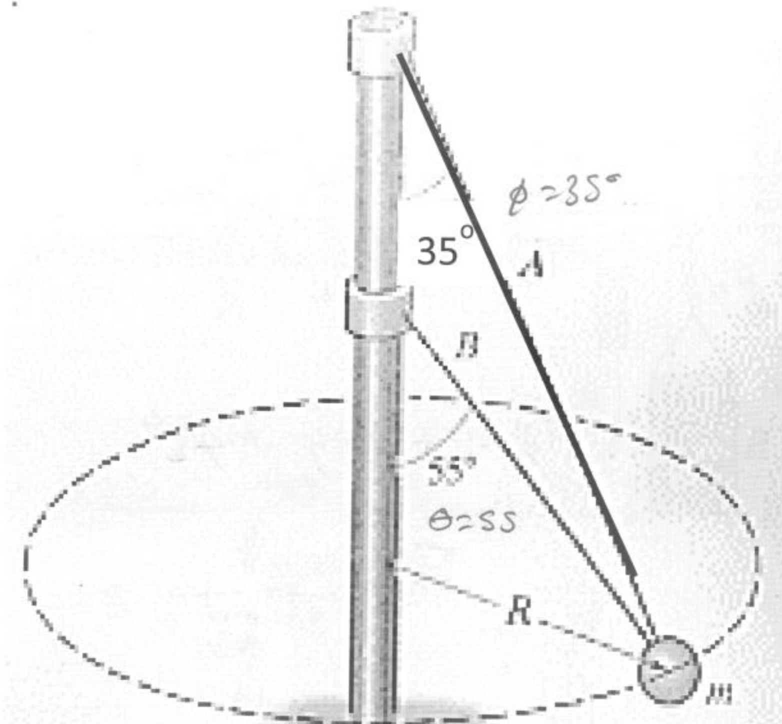
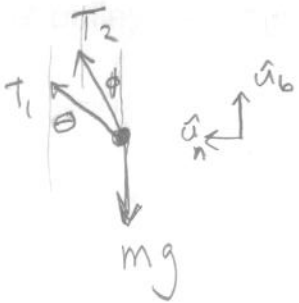
Question 3 (25 points)

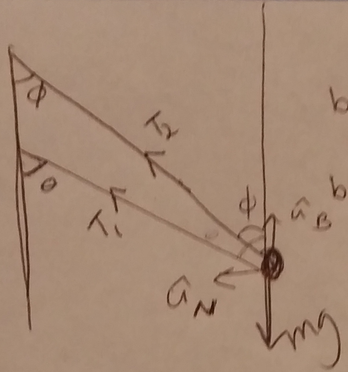
A 10kg mass, m, rotates around a vertical pole in a horizontal circular path with radius R=1m. The angles of the ropes with respect to the vertical direction are 35° and 55°.

Find the range of the velocity of the mass such that the mass will remain on the circular path described above?



5 pts





ball goes up $t_2 \to 0$

ball goes down $t_1 \to 0$

$$\overset{+}{\leftarrow} \sum \vec{F}_n = T_1 \sin\theta + T_2 \sin\phi = m \cancel{a_n}^{\frac{V^2}{R}}$$

$$+\uparrow \sum \vec{F}_B = -mg + T_1 \cos\theta + T_2 \cos\phi = m \cancel{a_b}^{0}$$

$a_b = 0$ for a circular path
- not moving up or down

For $T_2 \to 0$ ball moves up

F_n) $T_1 \sin\theta = m \frac{V^2}{R}$ $\qquad V^2 = \frac{T_1 R}{m} \sin\theta$

F_B) $-mg + T_1 \cos\theta = 0$ $\qquad T_1 = \frac{mg}{\cos\theta}$

$$V^2 = \frac{\cancel{m} g R}{\cancel{m}} \tan\theta = 1m\,(9.81\,m/s^2)\tan 55$$

$$\boxed{V = 3.74\ m/s}$$

For $T_1 \to 0$ ball moves down

$F_n)$ $T_2 \sin\phi = m \frac{v^2}{R}$ $v^2 = T_2 \sin\phi \cdot \frac{R}{m}$

$F_b)$ $-mg + T_2 \cos\phi = 0$ $T_2 = \frac{mg}{\cos\phi}$

$$v^2 = \frac{\cancel{m} g R}{\cancel{m}} \tan\phi$$

$$v^2 = (9.81 \, m/s^2)(1 m) \tan 35°$$

$$\boxed{v = 2.62 \, m/s}$$

Velocity range for circular path

$$\boxed{2.62 \, m/s \leq v \leq 3.74 \, m/s}$$

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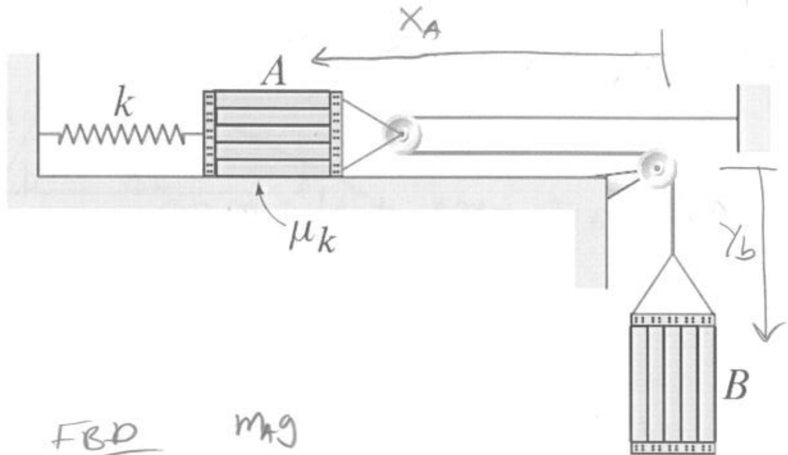
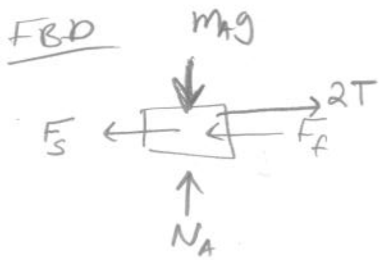
Question 4 (25 points)

Crates A and B of mass 50 kg and 75 kg, respectively, are released from rest. The linear elastic spring has stiffness k=500Nm.

Neglect the mass of the pulleys and cables and neglect friction in the pulley bearings.

If μ_k=0.25 and the spring is initially unstretched, determine the speed of B after A slides 4 m.





Energy method

$$T_1 + V_1 + U_{NC}^{ext} = T_2 + V_2$$

3pts $T_1 = \frac{1}{2} m_A \cancel{V_{A1}^2}^{0} + \frac{1}{2} m_b \cancel{V_{b1}^2}^{0} = \underline{0}$ released from rest

3pts $T_2 = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_b V_{b2}^2$

3pts $V_1 = V_{1s} + V_{1g} = \frac{1}{2} k \cancel{\delta_1^2}^{0} + m_b \cancel{g Y_{b1}}$, let $Y_{b1} = 0$ & $\delta_1 = 0$

$\underline{V_1 = 0}$

3pts $V_2 = \frac{1}{2} k \delta_2^2 - m_b g Y_{b2}$, $\delta_2 = 4m$ $Y_{b2} = ?$

Cable length to relate V_a & V_b

5pts $L = 2X_A + Y_b$

$\dot{L} = 0 = 2V_A + V_b$

$\underline{V_b = -2V_A}$ $\therefore$ $Y_{b_2} = 8m$ @ $X_{A2} = -4m$

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(Additional workspace for Question 4)

external work from Friction

3pts $U_{NC}^{ext} = -F_f \cdot d$ w/ $F_F = \mu_k N$

$+\uparrow \Sigma F_{y_A} = -mg + N = m\cancel{a_y}^{\,0}$

$\underline{N = mg}$

$\underline{U_{nc}^{ext} = -\mu_k mg\,\delta_2}$ w/ $\delta_2 = 4m$

Solve for V_{b2}

$\cancel{T_1}^{\,0} + \cancel{V_1}^{\,0} + U_{NC}^{ext} = T_2 + V_2$

$-\mu_k m_A g \delta_2 = \frac{1}{2} m_a \left(\frac{V_{b2}}{2}\right)^2 + \frac{1}{2} m_b V_{b2}^2 + \frac{1}{2} k \delta_2^2 - m_b g Y_{b2}$

$-\mu_k m_A g \delta_2 = \frac{1}{8}(m_a + 4 m_b) V_{b2}^2 + \frac{1}{2} k \delta_2^2 - m_b g (8m)$

$$V_{b2}^2 = \frac{m_b g (8m) - \mu_k m_A g \delta_2 - \frac{1}{2} k \delta_2^2}{\frac{1}{8}(m_a + 4m_b)}$$

$$V_{b2}^2 = \frac{75kg(9.81)8m - .25(50kg)(9.81 m/s^2)(4m) - \frac{1}{2}(500 N/m)(4m)^2}{\frac{1}{8}(50kg + 4 \cdot 75kg)}$$

5pts $\boxed{V_{b2} = 5.65\ m/s}$ $\downarrow$