EMA 202, Spring 2015 **Name: ______________________________**
Midterm #1, closed book/notes, 90 min.

Instructions

- Write your name on every sheet.
- Show all of your work.
- To maximize opportunities for partial credit, solve problems symbolically to the extent possible and substitute numerical values at the end.
- Include free body diagrams for all equilibrium equations.

- This is a closed book examination.
- The only notes allowed are the equations provided with this exam.
- Calculators are allowed.
- The use of cell phones is prohibited.

- The instructors and the University of Wisconsin expect the highest standards of honesty and integrity in the academic performance of its students. It is important that the work submitted on this examination is yours and yours alone.
- Receiving or giving aid in an examination or using notes on a closed note exam will be considered cheating and will result in a grade of F and the case being reported to the Dean of Students Office.

Circle Your Discussion Section:

301 (Tu 1:20, Harsha)

302 (Tu 2:25, Harsha)

303 (W 3:30, David)

304 (Th 8:50, David)

305 (Th 12:05, David)

306 (Th 1:20, David)

Grading:

Q1	/10
Q2	/30
Q3	/30
Q4	/30
Total	/100

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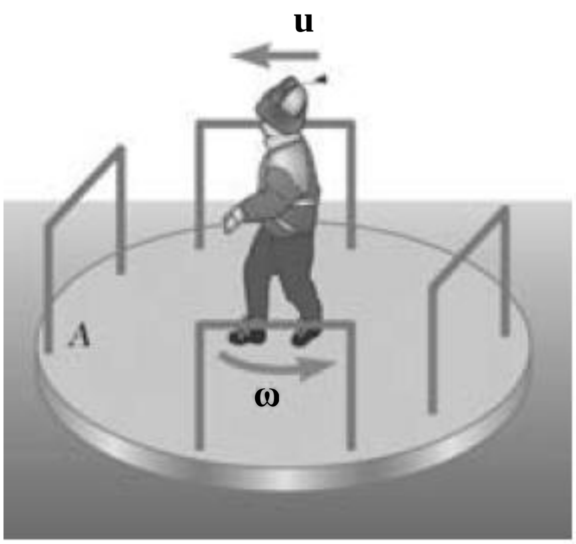
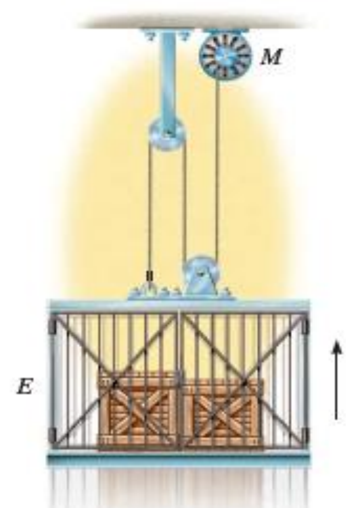
Question 1 (10 points)

1A (4 points) A child walks across a merry-go-round with constant speed *u* relative to the platform. The merry-go-round is rotating about its center at a constant angular velocity, ω, in the direction shown. When the child is at the center of the platform, write an expression for the acceleration vector and note all of the terms that are equal to zero. Additionally, draw the acceleration vector (with coordinate system) at the instant shown on the figure below.



1B (6 points)

A 500kg elevator starts from rest and travels upward with a constant acceleration of $2 m/s^2$. Determine the power output of motor M at t = 3 seconds.



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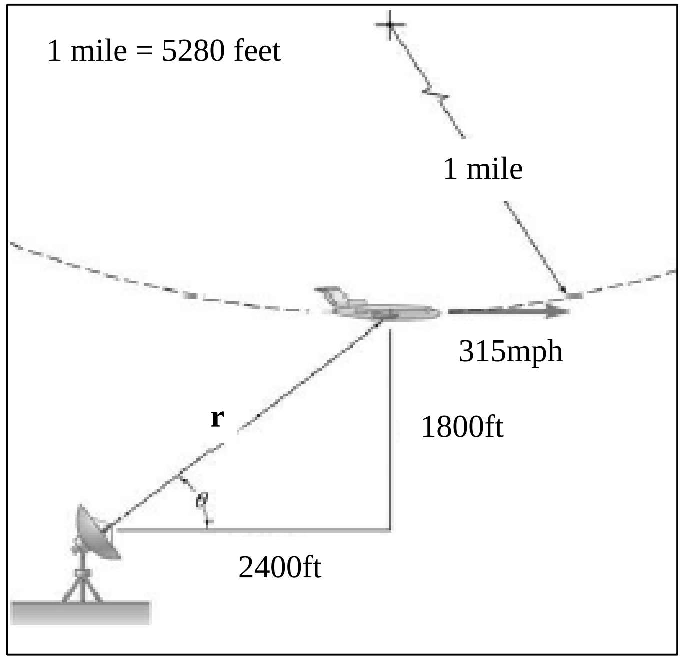
<u>Question 2 (30 points)</u>

At the bottom of a loop in the vertical plane, an airplane has a horizontal velocity of 315mph and is accelerating at a rate of 10 ft/s^2. The radius of curvature of the loop is 1 mile. The plane is being tracked by radar at O. What are the values for $\dot{r},\ \ddot{r}, \dot{\theta}, and\ \ddot{\theta}$ at this instant?



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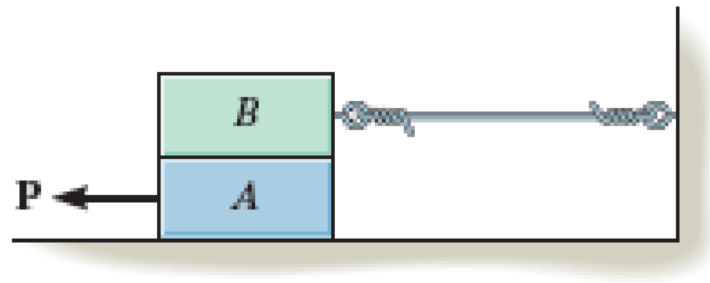
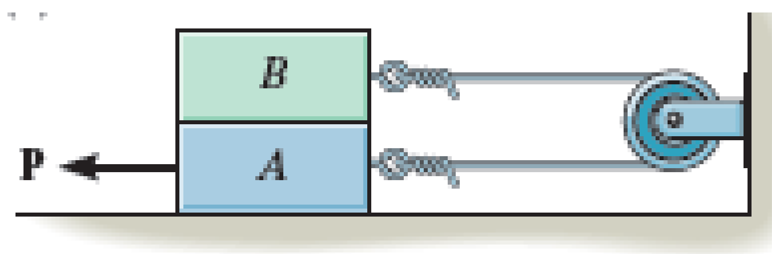
<u>Question 3 (30 points)</u>

Each of the two blocks has mass *m*. The coefficient of kinetic friction on all surfaces of contact is μ. If a horizontal force P moves the bottom block, determine the acceleration of the bottom block in cases (a) and (b).

Additionally, solve for the force P, in which case (a) and (b) have the same acceleration.



(a)



(b)

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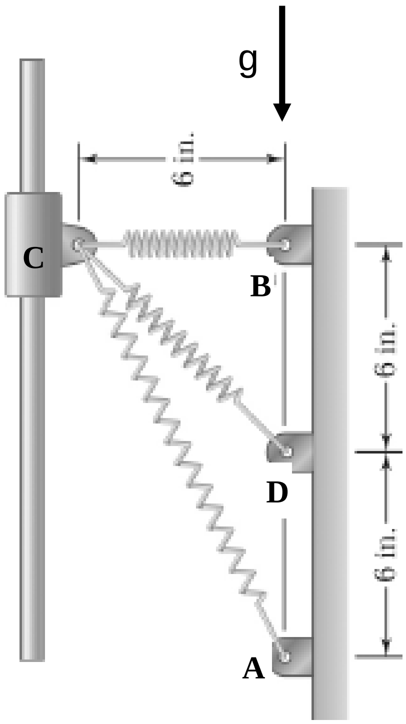
Question 4 (30 points)



A 3-lb collar C may slide without friction along the vertical rod. It is attached to 3 springs, each with spring constant k = 2 lb/in and an undeformed length of 6 inches. Given that the collar is released from rest, determine the speed of the collar after it has traveled 6 inches down.

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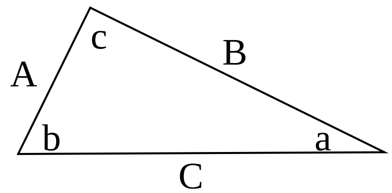
Geometry:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

$$C^2 = A^2 + B^2 - 2AB\cos c$$

$$\cos^2\theta + \sin^2\theta = 1$$



Particle Rectilinear Motion:

$$a = \frac{dv}{dt} \qquad v = \frac{ds}{dt} \qquad ads = vdv$$

For the special case of constant acceleration (a_c), and assuming initial conditions are specified at t = 0:

$$v(t) = v_o + a_c t \qquad s(t) = s_o + v_o t + \frac{1}{2}a_c t^2 \qquad v^2 = v_o^2 + 2a_c(s - s_o)$$

Particle Curvilinear Motion:

Cartesian Results: $\vec{\mathbf{v}} = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}} + \dot{z}\hat{\mathbf{k}}$ $\qquad \vec{\mathbf{a}} = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}}$

Normal/Tangential Results:

$$\vec{\mathbf{v}} = \dot{s}\hat{\mathbf{u}}_t$$

$$\vec{\mathbf{a}} = \dot{v}\hat{\mathbf{u}}_t + \frac{v^2}{\rho}\hat{\mathbf{u}}_n \qquad \text{where} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|\,d^2y/dx^2\,|}$$

Polar Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\theta}\,\hat{\mathbf{u}}_\theta$$

$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{u}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\mathbf{u}}_\theta$$

Spherical Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\phi}\,\hat{\mathbf{u}}_\phi + r\dot{\theta}\sin\phi\,\hat{\mathbf{u}}_\theta$$

$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\sin^2\phi)\,\hat{\mathbf{u}}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi} - r\dot{\theta}^2\sin\phi\cos\phi)\,\hat{\mathbf{u}}_\phi$$

$$+ (r\ddot{\theta}\sin\phi + 2\dot{r}\dot{\theta}\sin\phi + 2r\dot{\phi}\dot{\theta}\cos\phi)\hat{\mathbf{u}}_\theta$$

General Time Derivatives of a Vector:

$$\dot{\hat{\mathbf{u}}}(t) = \vec{\boldsymbol{\omega}}_u \times \hat{\mathbf{u}}$$

$$\dot{\vec{\mathbf{A}}}(t) = \dot{A}\hat{\mathbf{u}}_A + \vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}} \qquad \ddot{\vec{\mathbf{A}}}(t) = \ddot{A}\hat{\mathbf{u}}_A + 2\vec{\boldsymbol{\omega}}_A \times \dot{A}\hat{\mathbf{u}}_A + \dot{\vec{\boldsymbol{\omega}}}_A \times \vec{\mathbf{A}} + \vec{\boldsymbol{\omega}}_A \times (\vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}})$$

Principle of Work and Energy: $\Sigma U_{1\to 2} = T_2 - T_1$ where $U_{1\to 2} = \int_1^2 \vec{\mathrm{F}} \cdot d\vec{\mathrm{r}}$ and $T = \frac{1}{2}mv^2$

Conservation of Energy (assumes only conservative forces): $T_1 + V_1 = T_2 + V_2$

(For rigid bodies, these results are unchanged except for the fact that $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$)

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V is potential energy and takes different forms depending on the source. For terrestrial gravity, $V = mgy$, (mgy_G for a rigid body) while for a linear elastic spring, $V = \frac{1}{2}k\delta^2$

Power: $P = \vec{F} \cdot \vec{v}$

Efficiency: $\varepsilon = \dfrac{\text{power output}}{\text{power input}}$

Linear Impulse and Momentum: $\sum \vec{\mathrm{I}}_{1\to2} = \vec{\mathrm{p}}_2 - \vec{\mathrm{p}}_1$ where $\vec{\mathrm{I}}_{1\to2} = \int_1^2 \vec{\mathrm{F}}dt$ and $\vec{\mathrm{p}} = m\vec{\mathrm{v}}$
(For rigid bodies, the linear impulse and momentum expressions are identical but with $\vec{\mathrm{v}} = \vec{\mathrm{v}}_G$)

Coefficient of restitution: $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum: $\sum \vec{\mathrm{I}}_{M\,1\to2} = \vec{\mathrm{H}}_{O2} - \vec{\mathrm{H}}_{O1}$ where
$\vec{\mathrm{I}}_{M\,1\to2} = \int_1^2 \vec{\mathrm{M}}_O dt$ and $\vec{\mathrm{H}} = \vec{r} \times m\vec{\mathrm{v}}$

(For rigid bodies, this principle holds as long as O is either the center of gravity G or a point of fixed rotation. If the center of gravity, $H_G = I_G\omega$; if a fixed axis, $H_O = I_O\omega$.)

Relative General Plane Motion:

Translating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \vec{\mathbf{v}}_{B/A} = \vec{\mathbf{v}}_A + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \vec{\mathbf{a}}_{B/A} = \vec{\mathbf{a}}_A + \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}_{B/A} - \omega^2\vec{\mathbf{r}}_{B/A}$$

Rotating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \left(\vec{\mathbf{v}}_{B/A}\right)_{xy} + \vec{\Omega} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \left(\vec{\mathbf{a}}_{B/A}\right)_{xy} + \dot{\vec{\Omega}} \times \vec{\mathbf{r}}_{B/A} + \Omega \times \left(\Omega \times \vec{\mathbf{r}}_{B/A}\right) + 2\Omega \times \left(\vec{\mathbf{v}}_{B/A}\right)_{xy}$$

Equations of Motion: $\sum F_x = m(a_G)_x$; $\sum F_y = m(a_G)_y$
or $\sum F_t = m(a_G)_t$; $\sum F_n = m(a_G)_n$

$\sum M_G = I_G\alpha$ *or* $\sum M_O = I_O\alpha$ *or* ... if you must ...
$\sum M_P = -\bar{y}m(a_G)_x + \bar{x}m(a_G)_y + I_G\alpha$

Mass Moment of Inertia: Definition, and in terms of radius of gyration, k: $I = \int r^2 dm = k^2 m$

Parallel axis theorem: $I = I_G + md^2$