EMA 202, Fall 2014 **Name: ______________________________**
Midterm #1, closed book/notes, 90 min.

Instructions

- Write your name on every sheet.
- Show all of your work.
- To maximize opportunities for partial credit, solve problems symbolically to the extent possible and substitute numerical values at the end.
- Include free body diagrams for all equilibrium equations.

- This is a closed book examination.
- The only notes allowed are the equations provided with this exam.
- Calculators are allowed.
- The use of cell phones is prohibited.

- The instructors and the University of Wisconsin expect the highest standards of honesty and integrity in the academic performance of its students. It is important that the work submitted on this examination is yours and yours alone.
- Receiving or giving aid in an examination or using notes on a closed note exam will be considered cheating and will result in a grade of F and the case being reported to the Dean of Students Office.

Circle Your Discussion Section:

301 (Tu 1:20, David)
302 (Tu 2:25, David)
303 (W 3:30, David)
304 (Th 8:50, Matt)
305 (Th 12:05, Matt)
306 (Th 1:20, Matt)

Grading:

Q1	/25
Q2	/25
Q3	/25
Q4	/25
Total	/100

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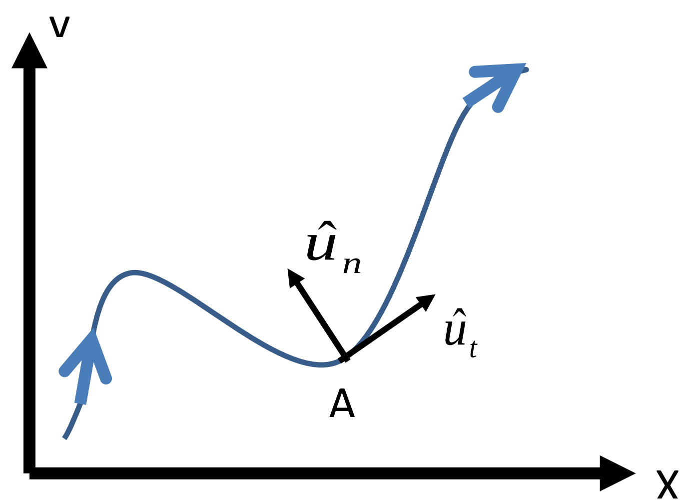
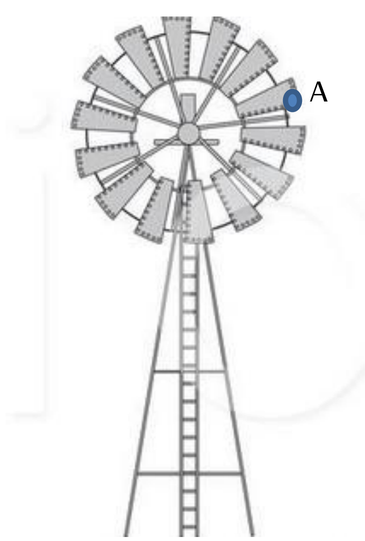
Question 1 (25 points)

Conceptual questions: please include a few sentences or equations to justify your answers

1A (6 points) The trajectory of a particle is shown on the figure below. At point A the speed of the particle is decreasing. Draw the acceleration and velocity vectors at point A (magnitude is not important)



1B (5 points) A windmill sits in an inertial coordinate system. It's blades are rotating at a constant rate ω. Is the acceleration of point A equal to 0?



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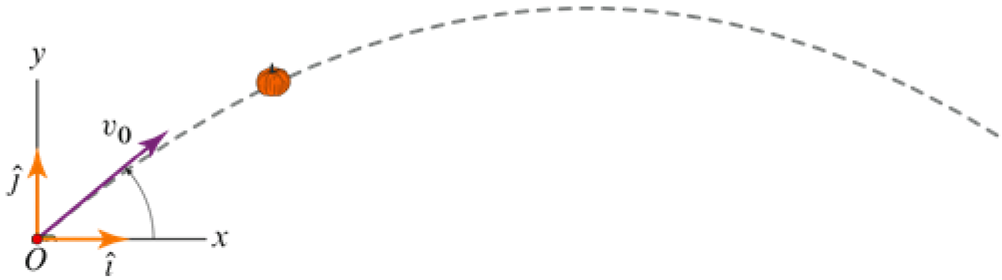
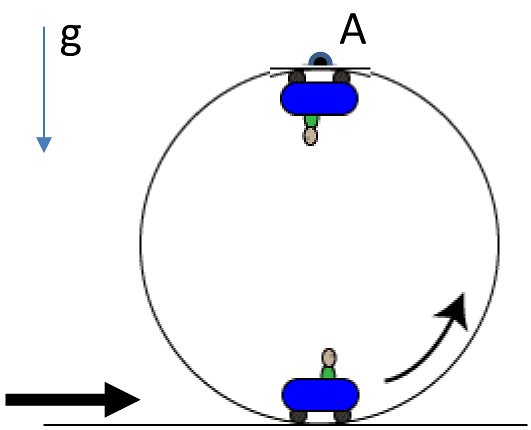
1C (8 points) Determine the equation for the trajectory of a projectile, y(x). Assume x(t=0) = 0 and y(t=0) = 0. (hint: your equation should look like $Y = C_1X+C_2X^2$, Find C1 and C2)



1D (6 points) A toy car is riding a track with a circular loop of radius R. What is the minimum speed that the car must be going at point A to maintain contact with the track? Let m=car mass



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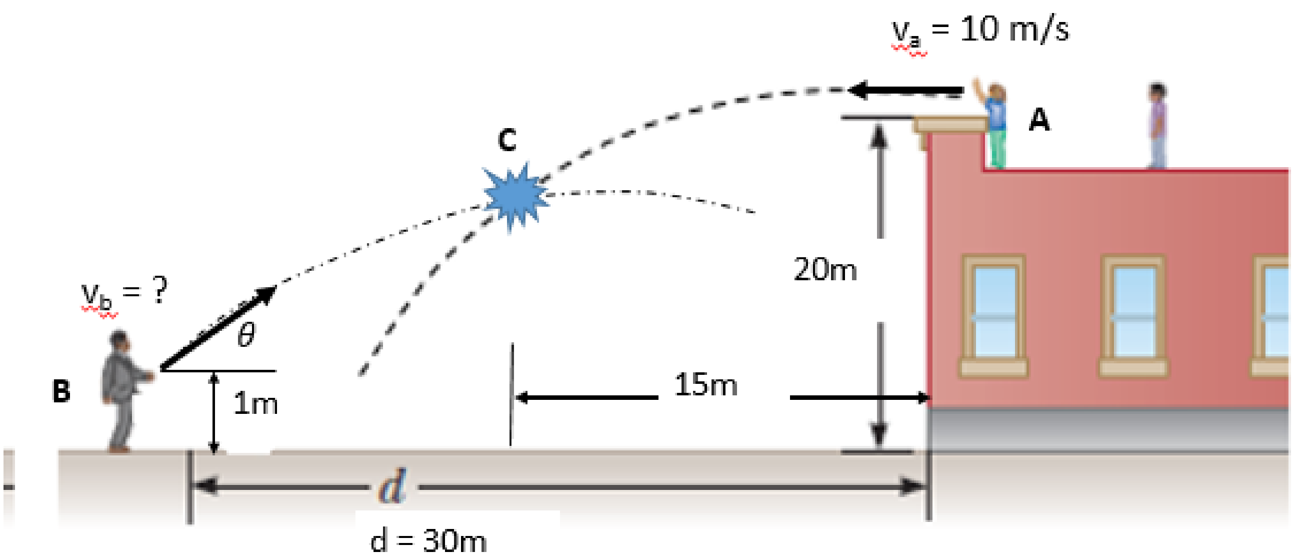
<u>Question 2 (25 points)</u>

A ball is thrown horizontally from the top of a 20m tall building with a velocity of 10m/s. A short time later, a person on the ground (point B) throws another ball such that the two balls collide at point C. Given that the person on the ground throws from a height of 1m and an angle $\theta = 35^o$,

a) Find the height at which the balls collide
b) Find the velocity of the ball thrown from point B
c) Find the time delay between when ball A and B are thrown



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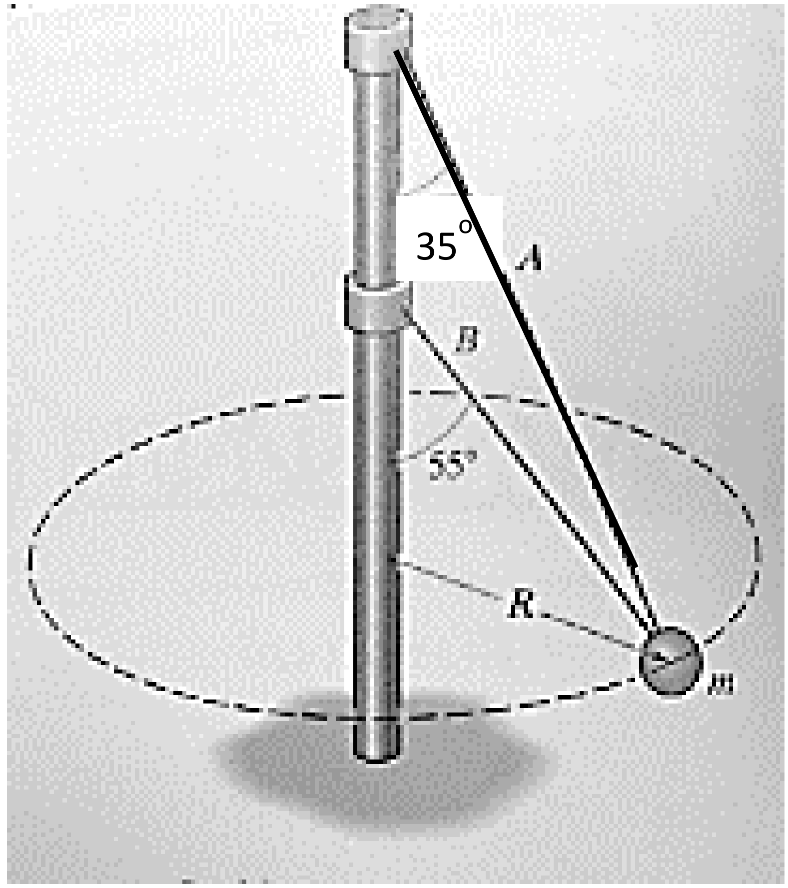
<u>Question 3 (25 points)</u>

A 10kg mass, m, rotates around a vertical pole in a horizontal circular path with radius R=1m. The angles of the ropes with respect to the vertical direction are 35^o and 55^o.

Find the range of the velocity of the mass such that the mass will remain on the circular path described above?



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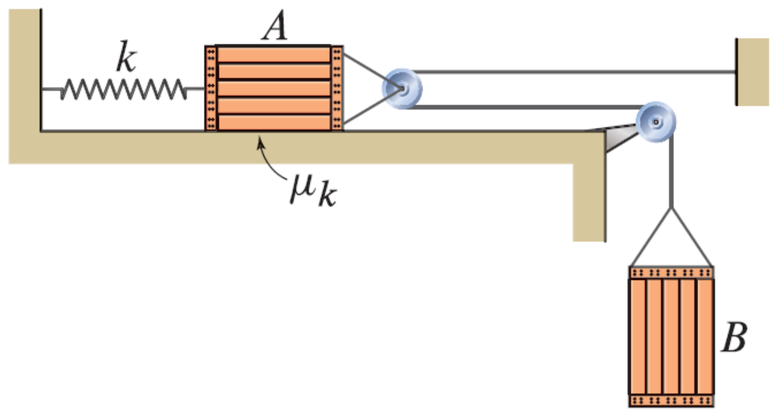
<u>Question 4 (25 points)</u>



Crates A and B of mass 50 kg and 75 kg, respectively, are released from rest. The linear elastic spring has stiffness k=500Nm.

Neglect the mass of the pulleys and cables and neglect friction in the pulley bearings.

If μ_k=0.25 and the spring is initially unstretched, determine the speed of B after A slides 4 m.

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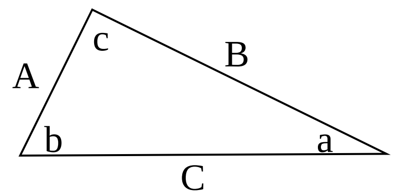
Geometry:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$
$$C^2 = A^2 + B^2 - 2AB\cos c$$
$$\cos^2\theta + \sin^2\theta = 1$$



Particle Rectilinear Motion:

$$a = \frac{dv}{dt} \qquad v = \frac{ds}{dt} \qquad ads = vdv$$

For the special case of constant acceleration (a_c), and assuming initial conditions are specified at $t = 0$:

$$v(t) = v_o + a_c t \qquad s(t) = s_o + v_o t + \frac{1}{2}a_c t^2 \qquad v^2 = v_o^2 + 2a_c(s - s_o)$$

Particle Curvilinear Motion:

Cartesian Results: $\vec{\mathbf{v}} = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}} + \dot{z}\hat{\mathbf{k}}$ $\qquad \vec{\mathbf{a}} = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}}$

Normal/Tangential Results:

$$\vec{\mathbf{v}} = \dot{s}\hat{\mathbf{u}}_t$$

$$\vec{\mathbf{a}} = \dot{v}\hat{\mathbf{u}}_t + \frac{v^2}{\rho}\hat{\mathbf{u}}_n \qquad \text{where} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

Polar Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\theta}\,\hat{\mathbf{u}}_\theta$$
$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{u}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\mathbf{u}}_\theta$$

Spherical Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\phi}\,\hat{\mathbf{u}}_\phi + r\dot{\theta}\sin\phi\,\hat{\mathbf{u}}_\theta$$
$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\sin^2\phi)\,\hat{\mathbf{u}}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi} - r\dot{\theta}^2\sin\phi\cos\phi)\,\hat{\mathbf{u}}_\phi$$
$$+ (r\ddot{\theta}\sin\phi + 2\dot{r}\dot{\theta}\sin\phi + 2r\dot{\phi}\dot{\theta}\cos\phi)\hat{\mathbf{u}}_\theta$$

General Time Derivatives of a Vector:

$$\dot{\hat{\mathbf{u}}}(t) = \vec{\boldsymbol{\omega}}_u \times \hat{\mathbf{u}}$$
$$\dot{\vec{\mathbf{A}}}(t) = \dot{A}\hat{\mathbf{u}}_A + \vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}} \qquad \ddot{\vec{\mathbf{A}}}(t) = \ddot{A}\hat{\mathbf{u}}_A + 2\vec{\boldsymbol{\omega}}_A \times \dot{A}\hat{\mathbf{u}}_A + \dot{\vec{\boldsymbol{\omega}}}_A \times \vec{\mathbf{A}} + \vec{\boldsymbol{\omega}}_A \times (\vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}})$$

Principle of Work and Energy: $\Sigma U_{1\to2} = T_2 - T_1$ $\qquad$ where $U_{1\to2} = \int_1^2 \vec{\mathrm{F}} \cdot d\vec{\mathrm{r}}$ $\qquad$ and $T = \frac{1}{2}mv^2$

Conservation of Energy (assumes only conservative forces): $T_1 + V_1 = T_2 + V_2$

(For rigid bodies, these results are unchanged except for the fact that $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$)

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V is potential energy and takes different forms depending on the source. For terrestrial gravity, $V = mgy$, (mgy_G for a rigid body) while for a linear elastic spring, $V = \frac{1}{2}k\delta^2$

Power: $P = \vec{F} \cdot \vec{v}$

Efficiency: $\varepsilon = \dfrac{\text{power output}}{\text{power input}}$

Linear Impulse and Momentum: $\sum \vec{\mathrm{I}}_{1\to2} = \vec{\mathrm{p}}_2 - \vec{\mathrm{p}}_1$ where $\vec{\mathrm{I}}_{1\to2} = \int_1^2 \vec{\mathrm{F}}dt$ and $\vec{\mathrm{p}} = m\vec{\mathrm{v}}$
(For rigid bodies, the linear impulse and momentum expressions are identical but with $\vec{\mathrm{v}} = \vec{\mathrm{v}}_G$)

Coefficient of restitution: $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum: $\sum \vec{\mathrm{I}}_{M\,1\to2} = \vec{\mathrm{H}}_{O2} - \vec{\mathrm{H}}_{O1}$ where
$\vec{\mathrm{I}}_{M\,1\to2} = \int_1^2 \vec{\mathrm{M}}_O dt$ and $\vec{\mathrm{H}} = \vec{r} \times m\vec{\mathrm{v}}$

(For rigid bodies, this principle holds as long as O is either the center of gravity G or a point of fixed rotation. If the center of gravity, $H_G = I_G\omega$; if a fixed axis, $H_O = I_O\omega$.)

Relative General Plane Motion:

Translating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \vec{\mathbf{v}}_{B/A} = \vec{\mathbf{v}}_A + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \vec{\mathbf{a}}_{B/A} = \vec{\mathbf{a}}_A + \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}_{B/A} - \omega^2 \vec{\mathbf{r}}_{B/A}$$

Rotating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \left(\vec{\mathbf{v}}_{B/A}\right)_{xy} + \vec{\Omega} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \left(\vec{\mathbf{a}}_{B/A}\right)_{xy} + \dot{\vec{\Omega}} \times \vec{\mathbf{r}}_{B/A} + \Omega \times \left(\Omega \times \vec{\mathbf{r}}_{B/A}\right) + 2\Omega \times \left(\vec{\mathbf{v}}_{B/A}\right)_{xy}$$

Equations of Motion: $\sum F_x = m(a_G)_x$; $\sum F_y = m(a_G)_y$
or $\sum F_t = m(a_G)_t$; $\sum F_n = m(a_G)_n$

$\sum M_G = I_G\alpha$ *or* $\sum M_O = I_O\alpha$ *or* ... if you must ...
$\sum M_P = -\bar{y}m(a_G)_x + \bar{x}m(a_G)_y + I_G\alpha$

Mass Moment of Inertia: Definition, and in terms of radius of gyration, k: $I = \int r^2 dm = k^2 m$

Parallel axis theorem: $I = I_G + md^2$