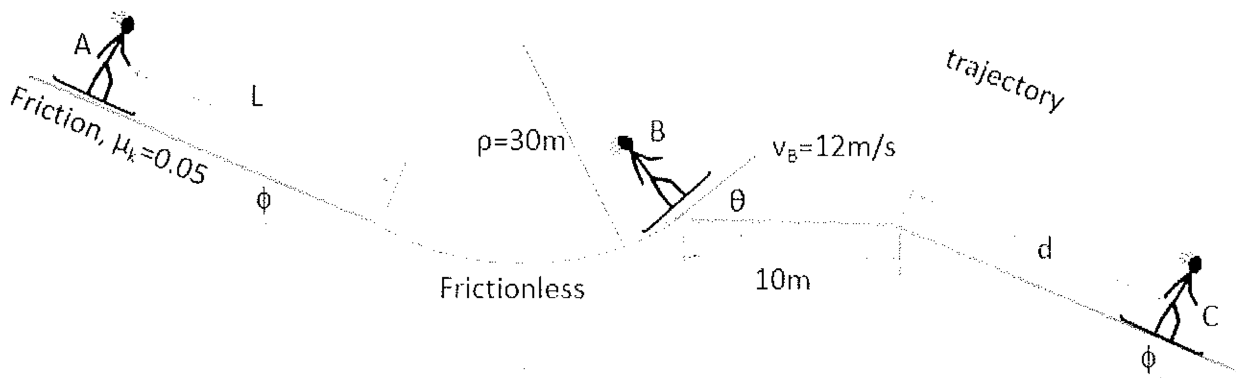
ME 240 / EMA 202, Fall 2017 **Name:** ____________________

Midterm #1, closed book/notes, 60 min.

<u>Question 1 (35 points)</u>

A 70kg snowboarder starts from rest at point A and travels downhill to hit a jump at point B. His velocity at point B is 12m/s. He leaves the jump at an angle θ with respect to the ground, then lands at point C.

Let angles ϕ = **30°** and **θ = 40°**

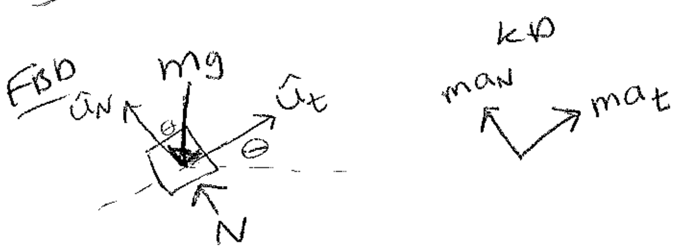


A) Calculate the normal force between the snowboarder and the ground at point B just before he leaves the jump.

B) Calculate the distance d where the snowboarder touches down on the landing ramp.

C) Calculate the distance L to the starting position A, such that the snowboarder reaches 12m/s at point B. Assume the kinetic friction coefficient on the slope is 0.05, and that the curved portion of the ramp is frictionless.

A) Normal Force @ pt. B



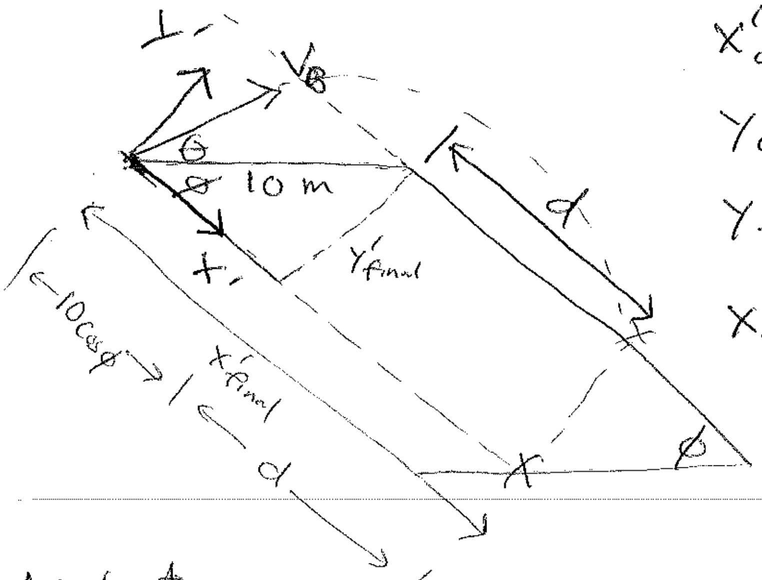
$\nwarrow_+ \Sigma F_N$: $N - mg\cos\theta = ma_N$ w/ $a_N = \frac{v^2}{\rho} = \frac{(12\,m/s)^2}{30\,m}$

$$N = m\frac{v^2}{\rho} + mg\cos\theta$$

$$N = 70\,kg\left(\frac{(12\,m/s)^2}{30m}\right) + 70kg\,(9.81\,m/s^2)\cos 40°$$

$$\boxed{N = 862.0\ N}$$

B) find distance d for landing



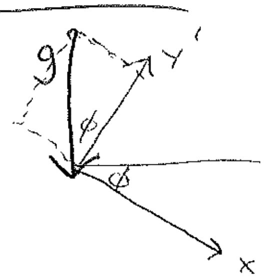
$x'_0 = 0$

$y'_0 = 0$

$y'_{final} = 10m \sin\phi$

$x'_{final} = d + 10\cos\phi$

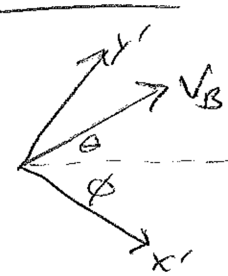
Acceleration



$a_x' = g\sin\phi$

$a_y' = -g\cos\phi$

Initial velocity



$V_{ox'} = V_B\cos(\theta+\phi)$

$V_{oy'} = V_B\sin(\theta+\phi)$

x) $x'_f = x'_0 + V_{ox'}t + \frac{a_{x'}}{2}t^2$

y) $y_{f'} = y'_0 + V_{oy'}t + \frac{a_{y'}}{2}t^2$ → use y to find t

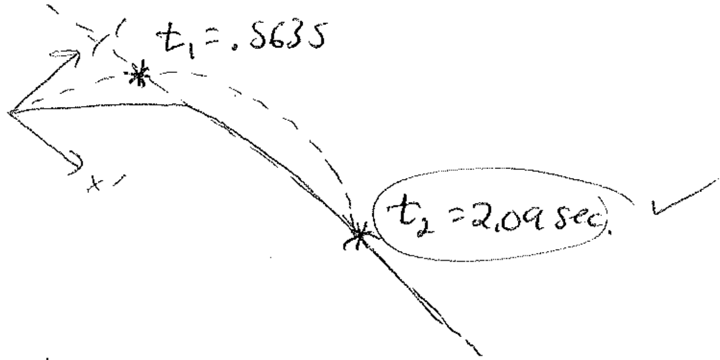
$$10\sin\phi = 0 + V_B\sin(\theta+\phi)t - \frac{g\cos\phi}{2}t^2$$

$$0 = \frac{-9.81\,m/s^2\cos 30^\circ}{2}t^2 + 12\,m/s\,\sin(40^\circ+30^\circ)t - 10\,m/s\,\sin\phi$$

$$0 = -4.248\,t^2 + 11.276\,t - 5$$ → solve w/ quadratic eqn

$t = 2.09$ & $.563$ sec

B)



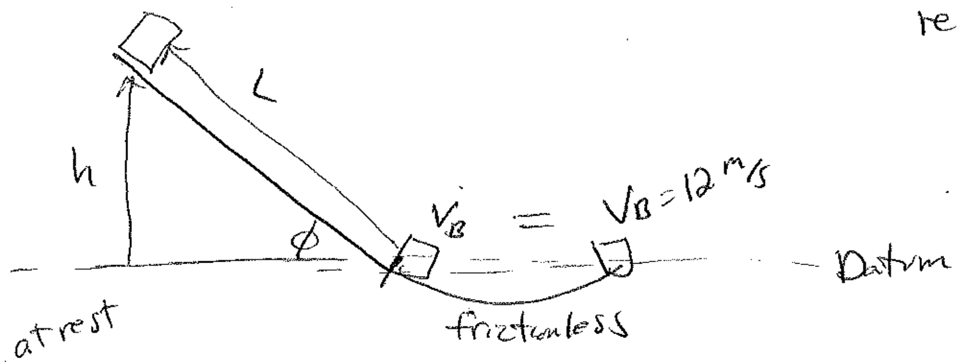
X eqn) $d + 10\cos\phi = 0 + V_B\cos(\theta+\phi)t + \frac{g\sin\phi}{2}t^2$

$d = -10\,m/s\cos(30) + 12\,m/s\cos(70°)(2.09\,s) + \frac{9.81\,m/s^2\sin 30}{2}(2.09)^2$

$\boxed{d = 10.63\text{ m}}$

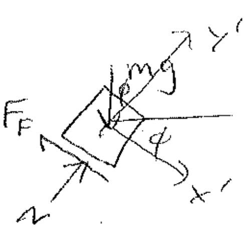
c) How far up ramp, L to start?

released from rest $V_1 = 0\,m/s$



$$\begin{cases} V_1 = mgh \text{ w/ } h = L\sin\phi \\ T_2 = \frac{1}{2}mV_B^2 \\ U_{12_{NC}} = -F_F\,L = -F_F L \end{cases}$$

$$\underbrace{T_1}_{0,\text{ at rest}} + V_1 + U_{12_{NC}} = T_2 + \underbrace{V_2}_{0,\text{ Datum}}$$



$+\nearrow \sum F_{y'}$: $N - mg\cos\phi = ma_{y'}^{\,0}$

$N = mg\cos\phi$

Friction $F_F = \mu_k N = \underline{\mu_k mg\cos\phi}$

W-E

$\cancel{m}g(L\sin\phi) + -\mu_k\cancel{m}g\cos\phi\,L = \frac{1}{2}\cancel{m}V_B^2$

$$\boxed{L = \frac{1}{2}V_B^2\left(\frac{1}{g\sin\phi - \mu_k g\cos\phi}\right)} = \boxed{16.06\text{ m}}$$

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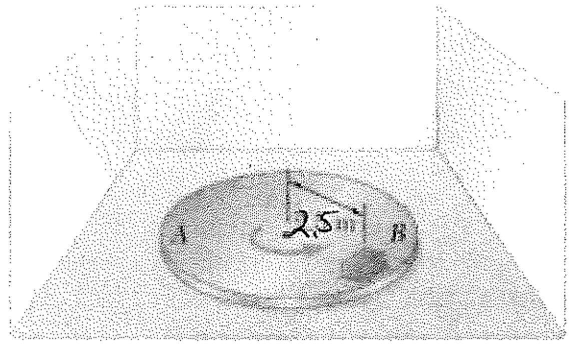
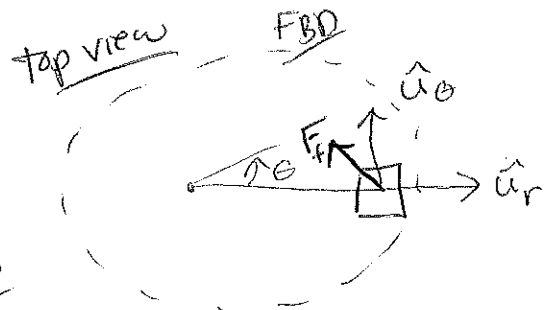
Question 2 (30 points)

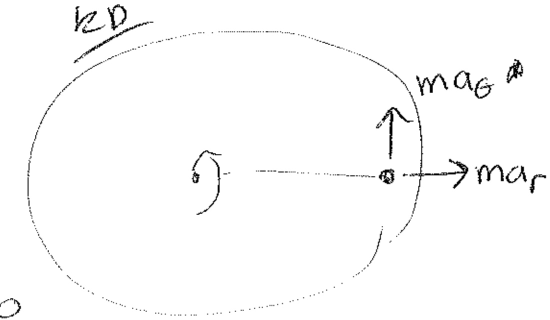
It is observed that a 50kg block B begins to slide on the turntable 10 seconds after the turntable begins to spin.

Knowing that the turntable has a constant angular acceleration of 0.1 rad/s^2, determine the coefficient of static friction between the block and the turntable.



$r = 2.5$ m





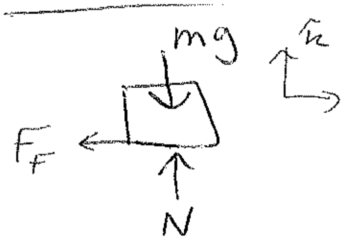
polar

$$\vec{a} = (\cancel{\ddot{r}}^{\,0} - r\dot{\theta}^2)\hat{u}_r + (r\ddot{\theta} + \cancel{2\dot{r}\dot{\theta}}^{\,0})\hat{u}_\theta$$

Before sliding $\dot{r} = 0$ & $\ddot{r} = 0$

$$\vec{a} = -r\dot{\theta}^2\hat{u}_r + r\ddot{\theta}\hat{u}_\theta \qquad w/ \quad \ddot{\theta} = 0.1 \text{ r/s}^2$$

Side view



$$+\uparrow\Sigma F_z: \; N - mg = m\cancel{a_z}^{\,0} \quad \text{flat disc}$$

$$N = mg$$

@ impending slip $\quad F_F = +\mu_s N = +\underline{\mu_s mg}$

$\vec{F} = m\vec{a}$ — friction for acts in the direction of total acceleration

$$\mu_s \cancel{m} g = m|\vec{a}| = \cancel{m}\sqrt{(-r\dot{\theta}^2)^2 + (r\ddot{\theta})^2}$$

$$\mu_s = \frac{1}{g}\sqrt{(-r\dot{\theta}^2)^2 + (r\ddot{\theta})^2}$$

w/ $r = 2.5\ m$
$g = 9.81\ m/s^2$
$\ddot{\theta} = .1\ rad/s^2$

$\dot{\theta} = \ddot{\theta}t = (.1\ r/s^2)(10\ sec)$
$\dot{\theta} = 1\ r/s$

$$\boxed{\mu_s = .256}$$

or in N-T coords

$$\vec{a} = \dot{v}\,\hat{u}_t + \frac{v^2}{\rho}\hat{u}_N$$

$$|\vec{a}| = \sqrt{\dot{v}^2 + \left(\frac{v^2}{\rho}\right)^2}$$

w/
$\dot{v} = r\ddot{\theta} = 2.5m\,(.1\ r/s^2)$
$\dot{v} = .25\ m/s^2$

$v = \dot{v}\,(10\ sec) = 2.5\ m/s$
$\rho = 2.5\ m$

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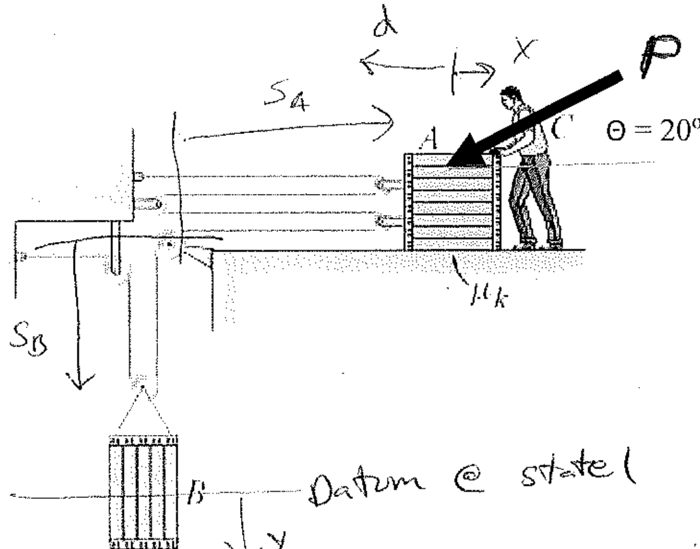
<u>Question 3 (35 points)</u>

The two crates A and B of mass m_A=100 kg and m_B=70 kg, respectively, are connected by a system of pulleys. The system is initially at rest, when a man starts pushing on crate A with a constant 350N force. The man's pushing force is at an angle of 20° with respect to the horizontal. Neglect the mass of the cables and friction in the pulleys.

If μ_k = 0.3, determine the distance the man pushes crate A to achieve a speed of 3m/s at crate A.



W-E $\quad V_1 = 0 \quad , \quad V_{2A} = 3\ m/s \quad , \quad P = 350N$

rope eqn

$L = 4S_A + 2S_B$

$\dot{L} = 0 = 4V_A + 2V_B$

$\boxed{2V_A = V_B}$ & $y = 2d$

$$\underbrace{\cancel{T_1}}_{@\text{ rest}} + \underbrace{\cancel{V_1}}_{\text{Datum}} + \underbrace{U_{1\text{-}2}}_{ext} + \underbrace{\cancel{U_{1\text{-}2}}}_{\text{INT}\text{, 0, for cables}} = T_2 + V_2$$

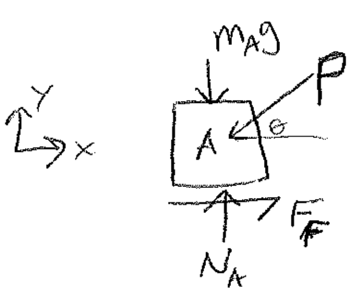
$$T_2 = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B \underbrace{V_{B2}^2}_{2V_A^2} = \frac{1}{2}(m_A + 4m_B)V_A^2$$

$$V_2 = -m_B g Y_2$$

F_x component

$$\underset{ext}{U_{1\text{-}2}} = \vec{F}_x \cdot (-d\,\hat{\imath})$$

$$\rightarrow \Sigma F_{x_A}: \quad F_f - P\cos\theta = m a_{x_A}$$



$$\uparrow \Sigma F_{Y_A}: \quad N_A - m_A g - P\sin\theta = m \cancel{a_{Y_A}}^{\,0}$$

$$N_A = m_A g + P\sin\theta$$

$$F_f = \mu_k N_A = \mu_k m_A g + \mu_k P\sin\theta$$

From ΣF_{x_A}:

external force $\vec{F_x} = F_F - P\cos\theta$

$$\vec{F_x} = (\mu_k m_A g + \mu_k P\sin\theta - P\cos\theta)\hat{\imath}$$

$$U_{1\text{-}2_{ext}} = \vec{F_x}\cdot(-d\hat{\imath}) = (-\mu_k m_A g - \mu_k P\sin\theta + P\cos\theta)\,d$$

Back to W-E $\quad U_{12_{ext}} = T_2 + V_2$

$$(-\mu_k m_A g - \mu_k P\sin\theta + P\cos\theta)\,d = \tfrac{1}{2}(m_A + 4m_B)V_{A2}^2 - m_B g\, \underbrace{y_2}_{2d}$$

$$d\,(-\mu_k m_A g - \mu_k P\sin\theta + P\cos\theta + m_B g 2) = \tfrac{1}{2}(m_A + 4m_B)V_{A2}^2$$

$$d = \frac{\frac{1}{2}(\overset{100\,kg}{m_A} + 4\overset{70\,kg}{m_B})(3\,m/s)^2}{(-.3\cdot 100kg\cdot 9.81\,m/s^2 - .3\cdot 350N\sin 20^\circ + 350N\cos 20^\circ + 70kg\, 9.81\,m/s^2\cdot 2)}$$

$$\boxed{d = 1.246\ m}$$