

my solution to discussion problem Nov 10 2017, ME 240 Dynamics, Fall 2017

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My solution is below

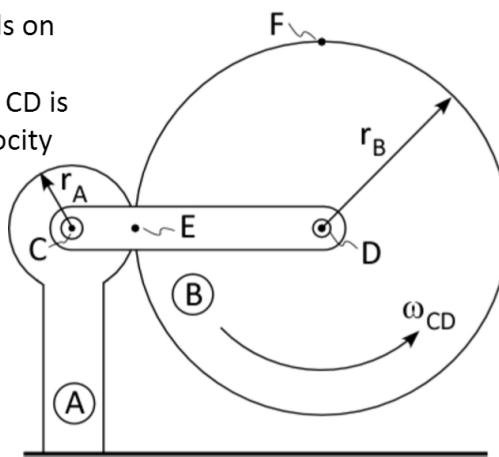
0.1 Problem 1

General Motion: Velocity

Example: The cylinder B rolls on the fixed cylinder A without slipping. The connecting bar CD is rotating with an angular velocity of $\omega_{CD} = 5 \text{ rad/s}$.

Determine:

- 1) the angular velocity of cylinder B
- 2) the velocity of point F



0.1.1 Part 1

Notice that the point E is not on the bar CD . It is the point where the disks meet at this instance shown.

$$\begin{aligned} \vec{V}_D &= \vec{V}_C + \vec{\omega}_{CD} \times \vec{r}_{D/C} \\ &= 0 + \omega_{CD} \hat{k} \times (r_A + r_B) \hat{i} \\ &= \omega_{CD} (r_A + r_B) \hat{j} \end{aligned} \quad (1)$$

But we also see that \vec{V}_D can be written as

$$\begin{aligned} \vec{V}_D &= \vec{V}_E + \vec{\omega}_{disk} \times \vec{r}_{D/E} \\ &= 0 + \omega_{disk} \hat{k} \times r_B \hat{i} \\ &= \omega_{disk} r_B \hat{j} \end{aligned} \quad (2)$$

Where in the above we used the fact that $\vec{V}_E = \vec{V}_C = 0$ at the instance shown. Equating (1) and (2)

$$\begin{aligned} \omega_{CD} (r_A + r_B) &= \omega_{disk} r_B \\ \omega_{disk} &= \omega_{CD} \frac{r_A + r_B}{r_B} \end{aligned} \quad (3)$$

0.1.2 Part 2

$$\begin{aligned}\vec{V}_F &= V_D + \vec{\omega}_{disk} \times \vec{r}_{F/D} \\ &= \omega_{CD} (r_A + r_B) \hat{j} + \omega_{disk} \hat{k} \times r_B \hat{j}\end{aligned}$$

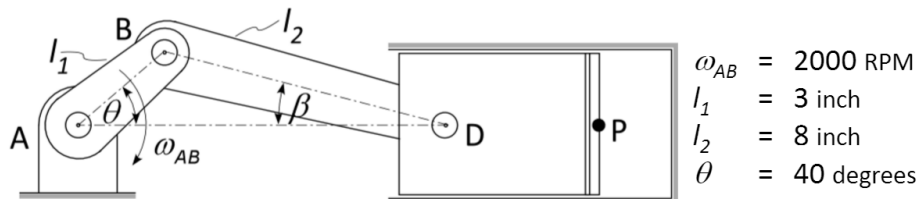
Hence

$$\begin{aligned}\vec{V}_F &= -\omega_{disk} r_B \hat{i} + \omega_{CD} (r_A + r_B) \hat{j} \\ &= -\omega_{CD} \frac{r_A + r_B}{r_B} r_B \hat{i} + \omega_{CD} (r_A + r_B) \hat{j} \\ &= \omega_{CD} (r_A + r_B) \hat{i} + \omega_{CD} (r_A + r_B) \hat{j}\end{aligned}$$

0.2 Problem 2

General Motion - Velocity

Example: In the piston system shown, the crank AB has a constant clockwise angular velocity of 2000 RPM.



Determine the velocity of point P on the piston for the configuration parameters given above

$$\begin{aligned}\vec{V}_D &= \vec{V}_B + \vec{V}_{D/B} \\ &= \vec{V}_B + \vec{\omega}_{BD} \times \vec{r}_{D/B}\end{aligned} \tag{1}$$

But

$$\begin{aligned}\vec{V}_B &= \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} \\ &= 0 - \omega_{AB} \hat{k} \times (L_1 \cos \theta \hat{i} + L_1 \sin \theta \hat{j}) \\ &= -\omega_{AB} L_1 \cos \theta \hat{j} + \omega_{AB} L_1 \sin \theta \hat{i}\end{aligned} \tag{2}$$

Where

$$\omega_{AB} = 2000 \left(\frac{2\pi}{60} \right) = \frac{200}{3} \pi = 209.4395 \text{ rad/sec}$$

The angle β can be found as follows

$$\begin{aligned}\frac{\sin \theta}{L_2} &= \frac{\sin \beta}{L_1} \\ \sin \beta &= \frac{L_1}{L_2} \sin \theta = \frac{3}{8} \sin \left(40 \left(\frac{\pi}{180} \right) \right) = 0.241 \text{ radians} \\ &= 13.808^\circ\end{aligned}$$

Now we know everything to evaluate (1). Therefore

$$\begin{aligned}\vec{V}_D &= \vec{V}_B + \vec{\omega}_{BD} \times \vec{r}_{D/B} \\ &= (-\omega_{AB} L_1 \cos \theta \hat{j} + \omega_{AB} L_1 \sin \theta \hat{i}) + \omega_{BD} \hat{k} \times (L_2 \cos \beta \hat{i} - L_2 \sin \beta \hat{j}) \\ &= (-\omega_{AB} L_1 \cos \theta \hat{j} + \omega_{AB} L_1 \sin \theta \hat{i}) + \omega_{BD} L_2 \cos \beta \hat{j} + \omega_{BD} L_2 \sin \beta \hat{i} \\ &= \hat{i} (\omega_{AB} L_1 \sin \theta + \omega_{BD} L_2 \sin \beta) + \hat{j} (-\omega_{AB} L_1 \cos \theta + \omega_{BD} L_2 \cos \beta)\end{aligned} \tag{3}$$

But the y component of $\vec{V}_D = 0$ since D can only move in x direction. Therefore from

the above

$$\begin{aligned}
 -\omega_{AB}L_1 \cos \theta + \omega_{BD}L_2 \cos \beta &= 0 \\
 \omega_{BD} &= \omega_{AB} \frac{L_1 \cos \theta}{L_2 \cos \beta}
 \end{aligned} \tag{4}$$

Substituting (4) into the x component of (3) gives the answer we want

$$\begin{aligned}
 \bar{V}_D &= \hat{i} \left(\omega_{AB}L_1 \sin \theta + \left(\frac{L_1 \cos \theta}{L_2 \cos \beta} \omega_{AB} \right) L_2 \sin \beta \right) \\
 &= \hat{i} \left(L_1 \sin \theta + \left(\frac{L_1 \cos \theta}{L_2 \cos \beta} \right) L_2 \sin \beta \right) \omega_{AB} \\
 &= \hat{i} \left(3 \sin \left(40 \left(\frac{\pi}{180} \right) \right) + \frac{3 \cos \left(40 \left(\frac{\pi}{180} \right) \right) \sin \left(13.808 \left(\frac{\pi}{180} \right) \right)}{\cos \left(13.808 \left(\frac{\pi}{180} \right) \right)} \right) 209.4395 \\
 &= 522.170 \hat{i} \text{ inch/sec} \\
 &= 43.514 \hat{i} \text{ ft/sec}
 \end{aligned}$$

$$\text{And } \omega_{BD} = \omega_{AB} \frac{L_1 \cos \theta}{L_2 \cos \beta} = 209.4395 \frac{3 \cos \left(40 \left(\frac{\pi}{180} \right) \right)}{8 \cos \left(13.808 \left(\frac{\pi}{180} \right) \right)} = 61.955 \text{ rad/sec}$$