my solution to discussion problem Nov 10 2017,ME 240 Dynamics, Fall 2017

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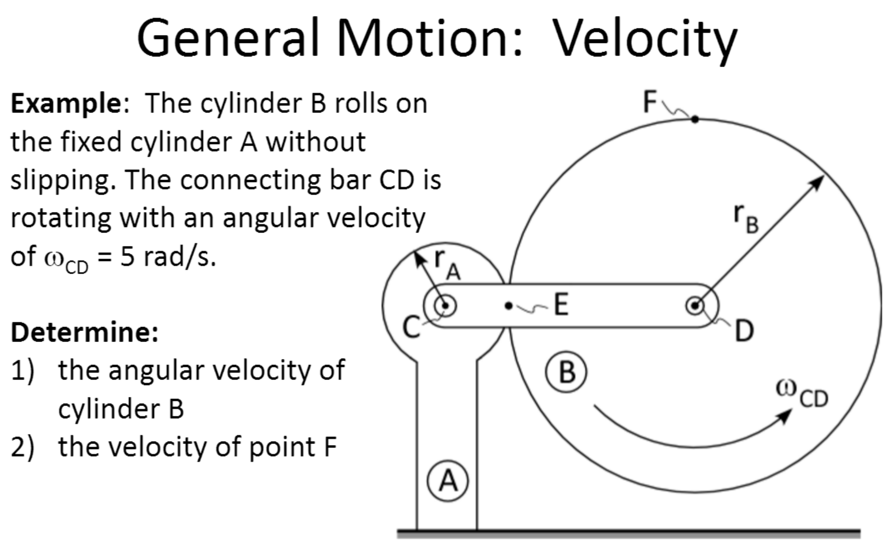
My solution is below

0.1 Problem 1



0.1.1 Part 1

Notice that the point E is not on the bar CD. It is the point where the disks meet at this instance shown.

$$\begin{aligned}\bar{V}_D &= \bar{V}_C + \bar{\omega}_{CD} \times \bar{r}_{D/C} \\ &= 0 + \omega_{CD}\hat{k} \times (r_A + r_B)\,\hat{\imath} \\ &= \omega_{CD}\,(r_A + r_B)\,\hat{\jmath} \end{aligned} \tag{1}$$

But we also see that $\bar{V}_D$ can be written as

$$\begin{aligned}\bar{V}_D &= \bar{V}_E + \bar{\omega}_{disk} \times \bar{r}_{D/E} \\ &= 0 + \omega_{disk}\hat{k} \times r_B\hat{\imath} \\ &= \omega_{disk} r_B \hat{\jmath} \end{aligned} \tag{2}$$

Where in the above we used the fact that $\bar{V}_E = \bar{V}_C = 0$ at the instance shown. Equating (1) and (2)

$$\begin{aligned}\omega_{CD}\,(r_A + r_B) &= \omega_{disk} r_B \\ \omega_{disk} &= \omega_{CD}\frac{r_A + r_B}{r_B} \end{aligned} \tag{3}$$

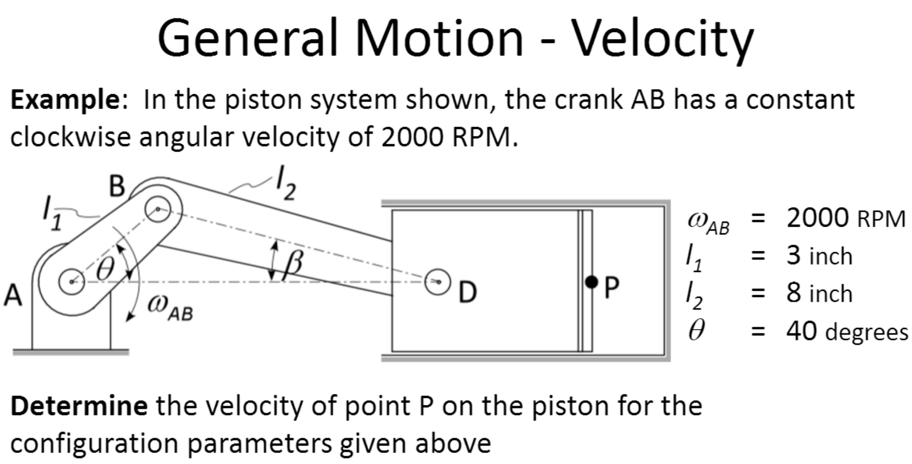
0.1.2 Part 2

$$
\begin{aligned}
\bar{V}_F &= V_D + \bar{\omega}_{disk} \times \bar{r}_{F/D} \\
&= \omega_{CD}\left(r_A + r_B\right)\hat{\jmath} + \omega_{disk}\hat{k} \times r_B\hat{\jmath}
\end{aligned}
$$

Hence

$$
\begin{aligned}
\bar{V}_F &= -\omega_{disk} r_B \hat{\imath} + \omega_{CD}\left(r_A + r_B\right)\hat{\jmath} \\
&= -\omega_{CD}\frac{r_A + r_B}{r_B} r_B \hat{\imath} + \omega_{CD}\left(r_A + r_B\right)\hat{\jmath} \\
&= \omega_{CD}\left(r_A + r_B\right)\hat{\imath} + \omega_{CD}\left(r_A + r_B\right)\hat{\jmath}
\end{aligned}
$$

0.2 Problem 2



$$
\begin{aligned}
\bar{V}_D &= \bar{V}_B + \bar{V}_{D/B} \\
&= \bar{V}_B + \bar{\omega}_{BD} \times \bar{r}_{D/B}
\end{aligned} \tag{1}
$$

But

$$
\begin{aligned}
\bar{V}_B &= \bar{V}_A + \bar{\omega}_{AB} \times \bar{r}_{B/A} \\
&= 0 - \omega_{AB}\hat{k} \times \left(L_1 \cos\theta\hat{\imath} + L_1 \sin\theta\hat{\jmath}\right) \\
&= -\omega_{AB} L_1 \cos\theta\hat{\jmath} + \omega_{AB} L_1 \sin\theta\hat{\imath}
\end{aligned} \tag{2}
$$

Where

$$
\omega_{AB} = 2000\left(\frac{2\pi}{60}\right) = \frac{200}{3}\pi = 209.4395 \text{ rad/sec}
$$

The angle β can be found as follows

$$
\begin{aligned}
\frac{\sin\theta}{L_2} &= \frac{\sin\beta}{L1} \\
\sin\beta &= \frac{L_1}{L_2}\sin\theta = \frac{3}{8}\sin\left(40\left(\frac{\pi}{180}\right)\right) = 0.241 \text{ radians} \\
&= 13.808^0
\end{aligned}
$$

Now we know everything to evaluate (1). Therefore

$$
\begin{aligned}
\bar{V}_D &= \bar{V}_B + \bar{\omega}_{BD} \times r_{D/B} \\
&= \left(-\omega_{AB} L_1 \cos\theta\hat{\jmath} + \omega_{AB} L_1 \sin\theta\hat{\imath}\right) + \omega_{BD}\hat{k} \times \left(L_2 \cos\beta\hat{\imath} - L_2 \sin\beta\hat{\jmath}\right) \\
&= \left(-\omega_{AB} L_1 \cos\theta\hat{\jmath} + \omega_{AB} L_1 \sin\theta\hat{\imath}\right) + \omega_{BD} L_2 \cos\beta\hat{\jmath} + \omega_{BD} L_2 \sin\beta\hat{\imath} \\
&= \hat{\imath}\left(\omega_{AB} L_1 \sin\theta + \omega_{BD} L_2 \sin\beta\right) + \hat{\jmath}\left(-\omega_{AB} L_1 \cos\theta + \omega_{BD} L_2 \cos\beta\right)
\end{aligned} \tag{3}
$$

But the y componenent of $\bar{V}_D = 0$ since D can only move in x direction. Therefore from the above

$$-\omega_{AB}L_1\cos\theta+\omega_{BD}L_2\cos\beta=0$$
$$\omega_{BD}=\omega_{AB}\frac{L_1\cos\theta}{L_2\cos\beta} \tag{4}$$

Substituting (4) into the x component of (3) gives the answer we want

$$\begin{aligned}
\bar{V}_D&=\hat{\imath}\left(\omega_{AB}L_1\sin\theta+\left(\frac{L_1\cos\theta}{L_2\cos\beta}\omega_{AB}\right)L_2\sin\beta\right)\\
&=\hat{\imath}\left(L_1\sin\theta+\left(\frac{L_1\cos\theta}{L_2\cos\beta}\right)L_2\sin\beta\right)\omega_{AB}\\
&=\hat{\imath}\left(3\sin\left(40\left(\frac{\pi}{180}\right)\right)+\frac{3\cos\left(40\left(\frac{\pi}{180}\right)\right)\sin\left(13.808\left(\frac{\pi}{180}\right)\right)}{\cos\left(13.808\left(\frac{\pi}{180}\right)\right)}\right)209.4395\\
&=522.170\hat{\imath}\text{ inch/sec}\\
&=43.514\hat{\imath}\text{ ft/sec}
\end{aligned}$$

And $\omega_{BD}=\omega_{AB}\frac{L_1\cos\theta}{L_2\cos\beta}=209.4395\frac{3\cos\left(40\left(\frac{\pi}{180}\right)\right)}{8\cos\left(13.808\left(\frac{\pi}{180}\right)\right)}=61.955$ rad/sec