HW 8,ME 240 Dynamics, Fall 2017

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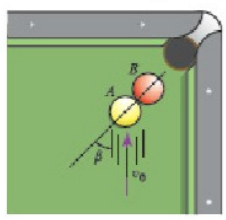
0.1 Problem 1

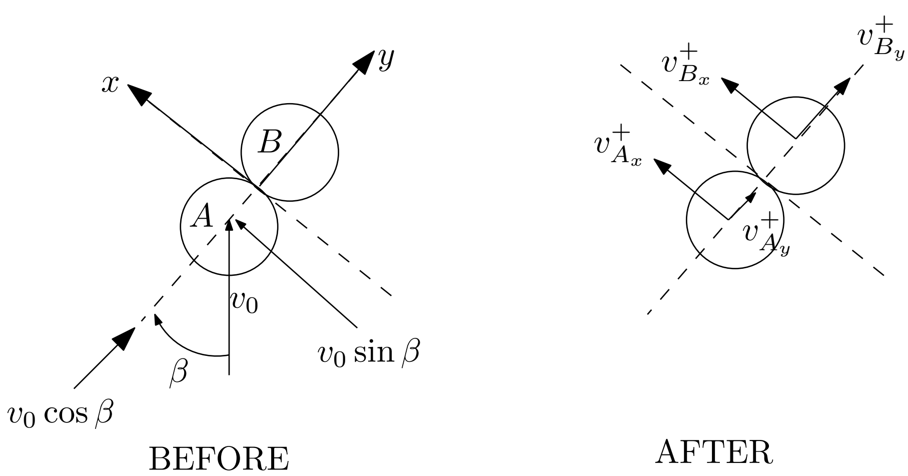
Ball B is stationary when it is hit by an identical ball A as shown, with $\beta = 45°$. The preimpact speed of ball A is $v_0 = 9$ m/s.

Determine the postimpact velocity of ball B if the COR of the collision $e = 1$.



$\vec{v}_B^{\,+} =$ [ ] $\hat{\imath}$ m/s. $\hat{\jmath}$ − 45°

The before and after impact diagram is



Along the y direction

$$m_A v_0 \cos\beta = m_A v_{A_y}^{+} + m_B v_{B_y}^{+}$$

$$-e = -1 = \frac{v_{A_y}^{+} - v_{B_y}^{+}}{v_{A_y}^{-} - v_{B_y}^{-}} = \frac{v_{A_y}^{+} - v_{B_y}^{+}}{v_0 \cos\beta}$$

These are 2 equations with 2 unknowns $v_{A_y}^{+}, v_{B_y}^{+}$. From the second equation

$$-v_0 \cos\beta = v_{A_y}^{+} - v_{B_y}^{+} \tag{1}$$

Substituting this in the first equation (and canceling the mass since they are the same), gives

$$-v_{A_y}^+ + v_{B_y}^+ = v_{A_y}^+ + v_{B_y}^+$$
$$v_{A_y}^+ = 0$$

Therefore from (1)

$$\begin{aligned} v_{B_y}^+ &= v_0 \cos\beta \\ &= 9\cos\left(45\left(\frac{\pi}{180}\right)\right) \\ &= 6.364 \text{ m/s} \end{aligned}$$

Along the x direction, since this is perpendicular to the line of impact then we know that

$$\begin{aligned} v_{A_x}^+ &= v_{A_x}^- = v_0 \sin\beta = 9\sin\left(45\left(\frac{\pi}{180}\right)\right) = 6.364 \text{ m/s} \\ v_{B_x}^+ &= v_{B_x}^- = 0 \end{aligned}$$

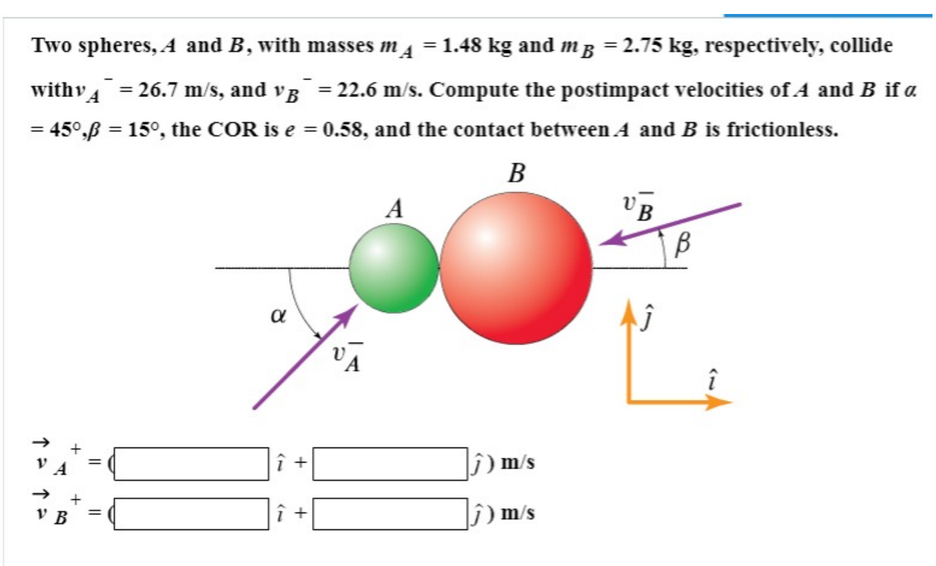
Hence velocity of B is

$$\bar{v}_B = 0\hat{\imath} + 6.364\hat{\jmath}$$

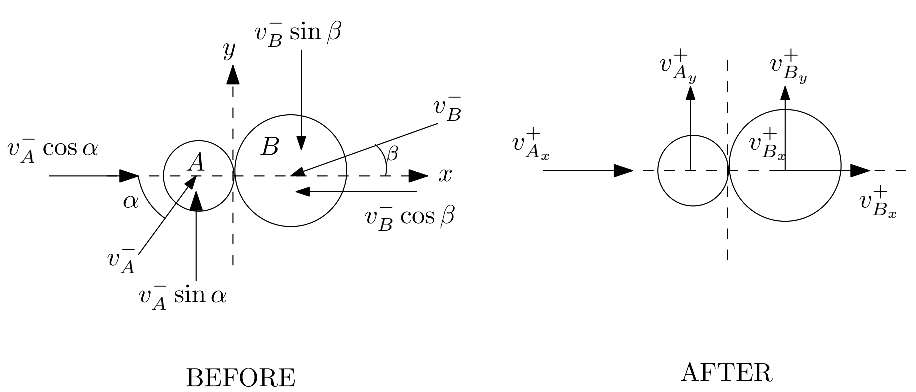
And velocity of A is

$$\bar{v}_A = 6.364\hat{\imath} + 0\hat{\jmath}$$

0.2 Problem 2



The before and after impact diagram is



Along the x axis, the conservation of linear momentum gives

$$\begin{aligned}
m_A v_A^- \cos\alpha - m_B v_B^- \cos\beta &= m_A v_{A_x}^+ + m_B v_{B_x}^+ \\
(1.48)(26.7)\cos\left(45\left(\frac{\pi}{180}\right)\right) - (2.75)(22.6)\cos\left(15\left(\frac{\pi}{180}\right)\right) &= (1.48)\, v_{A_x}^+ + (2.75)\, v_{B_x}^+ \\
-32.09 &= (1.48)\, v_{A_x}^+ + (2.75)\, v_{B_x}^+ \qquad (1)
\end{aligned}$$

And

$$\begin{aligned}
-e &= \frac{v_{A_x}^+ - v_{B_x}^+}{v_{A_x}^- - v_{B_x}^-} \\
-0.58 &= \frac{v_{A_x}^+ - v_{B_x}^+}{v_A^- \cos\alpha + v_B^- \cos\beta} \\
-0.58 &= \frac{v_{A_x}^+ - v_{B_x}^+}{(26.7)\cos\left(45\left(\frac{\pi}{180}\right)\right) + (22.6)\cos\left(15\left(\frac{\pi}{180}\right)\right)} \\
-0.58 &= \frac{v_{A_x}^+ - v_{B_x}^+}{40.71} \\
-23.612 &= v_{A_x}^+ - v_{B_x}^+ \qquad (2)
\end{aligned}$$

Now $v_{A_x}^+, v_{B_x}^+$ is solved for using (1),(2). From (2) $v_{A_x}^+ = -23.612 + v_{B_x}^+$, substituting this in (1) gives

$$\begin{aligned}
-32.09 &= (1.48)\left(-23.612 + v_{B_x}^+\right) + (2.75)\, v_{B_x}^+ \\
-32.09 &= -34.945 + 4.23 v_{B_x}^+ \\
v_{B_x}^+ &= \frac{-32.09 + 34.945}{4.23} \\
&= 0.675 \text{ m/s}
\end{aligned}$$

From (2)

$$\begin{aligned}
v_{A_x}^+ &= -23.612 + 0.675 \\
&= -22.937 \text{ m/s}
\end{aligned}$$

Now we do the same for the y direction. But along this direction we know that

$$\begin{aligned}
v_{A_y}^+ &= v_{A_y}^- \\
&= v_A^- \sin\alpha \\
&= (26.7)\sin\left(45\left(\frac{\pi}{180}\right)\right) \\
&= 18.88 \text{ m/s}
\end{aligned}$$

And

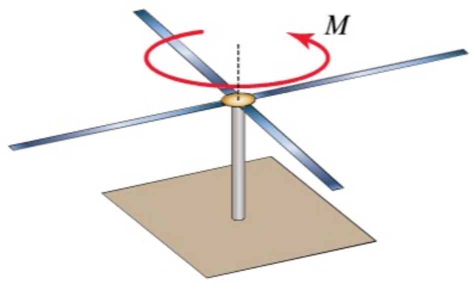
$$\begin{aligned}
v_{B_y}^+ &= v_{B_y}^- \\
&= -v_B^- \sin\beta \\
&= (-22.6)\sin\left(15\left(\frac{\pi}{180}\right)\right) \\
&= -5.849 \text{ m/s}
\end{aligned}$$

Therefore, after impact

$$\begin{aligned}
\bar{v}_A &= -22.938\,\hat{\imath} + 18.879\,\hat{\jmath} \\
\bar{v}_B &= 0.675\hat{\imath} - 5.849\,\hat{\jmath}
\end{aligned}$$

0.3 Problem 3

A rotor consists of four horizontal blades each of length $L = 4.5$ m and mass $m = 89$ kg cantilevered off of a vertical shaft. Assume that each blade can be modeled as having its mass concentrated at its midpoint. The rotor is initially at rest when it is subjected to a moment $M = \beta t$, with $\beta = 63$ N · m/s. Determine the angular speed of the rotor after 10 s.



The angular speed is [ ] **rad/s.**

Using

$$\tau = 4I\ddot{\theta}$$

Where τ is applied torque and I is mass moment of inertia around the spin axis of one blade (we have 4). But $I = m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4}$, since blade is modeled as point mass. Therefore

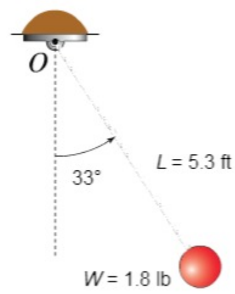
$$\ddot{\theta} = \frac{\tau}{\frac{4mL^2}{4}} = \frac{\beta t}{mL^2}$$

But $\ddot{\theta} = \frac{d}{dt}\dot{\theta}$, then the above becomes

$$\begin{aligned}
\frac{d}{dt}\dot{\theta} &= \frac{\beta t}{mL^2} \\
d\dot{\theta} &= \frac{\beta t}{mL^2} dt \\
\int_0^{\dot{\theta}_f} d\dot{\theta} &= \frac{\beta t}{mL^2} \int_0^{10} t dt \\
\dot{\theta}_f &= \frac{\beta}{mL^2}\left(\frac{t^2}{2}\right)_0^{10} \\
&= \frac{\beta}{2mL^2} 100 \\
&= \frac{(63)}{2(89)(4.5)^2} 100 \\
&= 1.748 \text{ rad/sec}
\end{aligned}$$

0.4 Problem 4

The simple pendulum in the figure is released from rest as shown. Knowing that the bob's weight is $W = 1.8$ lb, determine its angular momentum computed with respect to O as a function of the angle θ.



The angular momentum with respect to O is

$\pm($ [ ] $\text{lb}\cdot\text{ft}\cdot\text{s})\sqrt{\cos\theta -}$ [ ] $\hat{k}$ $\hat{u}_\theta$ $\hat{u}_r$ @ $-57°$.

The angular momentum $\bar{h}$ is the moment of the linear momentum. The linear momentum is $m\bar{v}$. Using radial and tangential coordinates, then

$$m\bar{v} = m\left(L\dot{\theta}\hat{u}_\theta + 0\hat{u}_r\right)$$

Therefore

$$\begin{aligned}\bar{h} &= \bar{r} \times m\bar{v} \\ &= L\hat{u}_r \times mL\dot{\theta}\hat{u}_\theta \\ &= \begin{vmatrix} \hat{u}_r & \hat{u}_\theta & \hat{k} \\ L & 0 & 0 \\ 0 & mL\dot{\theta} & 0 \end{vmatrix} \\ &= \hat{k}mL^2\dot{\theta} \end{aligned} \tag{1}$$

The above is what we want. But we need to find $\dot{\theta}$. Taking time derivative of $\bar{h}$ gives

$$\frac{d}{dt}\bar{h} = \hat{k}mL^2\ddot{\theta}$$

But $\frac{d}{dt}\bar{h}$ is the torque τ, which we can see to be

$$\tau = -mgL\sin\theta$$

The minus sign, since clockwise. Using the above 2 equations, then we write

$$\begin{aligned} -mgL\sin\theta &= mL^2\ddot{\theta} \\ \ddot{\theta} &= -\frac{g}{L}\sin\theta \end{aligned} \tag{2}$$

To integrate this, we need a trick. Since

$$\begin{aligned}\ddot{\theta} &= \frac{d}{dt}\dot{\theta} \\ &= \left(\frac{d}{d\theta}\frac{d\theta}{dt}\right)\dot{\theta} \\ &= \left(\frac{d}{d\theta}\dot{\theta}\right)\dot{\theta} \\ &= \dot{\theta}\frac{d\dot{\theta}}{d\theta}\end{aligned}$$

Then (2) becomes

$$\dot{\theta}\frac{d\dot{\theta}}{d\theta} = -\frac{g}{L}\sin\theta$$

Now it is separable.

$$
\begin{aligned}
\dot{\theta} d\dot{\theta} &= -\frac{g}{L} \sin\theta d\theta \\
\int_0^{\dot{\theta}} \dot{\theta} d\dot{\theta} &= -\frac{g}{L} \int_{33^0}^{\theta} \sin\theta d\theta \\
\frac{\dot{\theta}^2}{2} &= -\frac{g}{L} (-\cos\theta)_{33^0}^{\theta} \\
\frac{\dot{\theta}^2}{2} &= \frac{g}{L} \left(\cos\theta - \cos 33^0\right) \\
\dot{\theta} &= \pm\sqrt{\frac{2g}{L}\left(\cos\theta - \cos 33^0\right)}
\end{aligned}
$$

All this work was to find $\dot{\theta}$. Now we go back to (1) and find the angular momentum

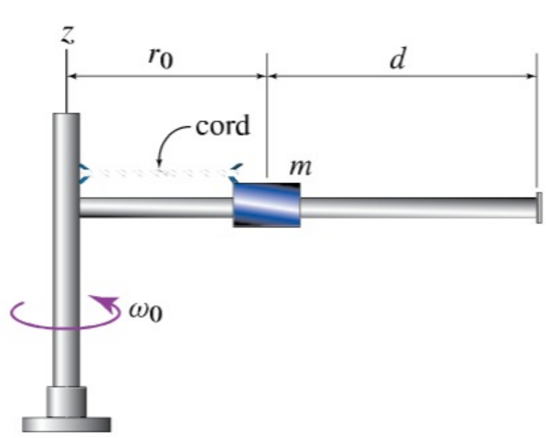
$$
\begin{aligned}
\bar{h} &= \hat{k} m L^2 \dot{\theta} \\
&= \pm\hat{k}\sqrt{\frac{2g}{L}\left(\cos\theta - \cos 33^0\right)} m L^2 \\
&= \pm\hat{k}\sqrt{2gL^3\left(\cos\theta - \cos 33^0\right)} m \\
&= \pm\hat{k}\sqrt{\frac{2L^3}{g}\left(\cos\theta - \cos 33^0\right)} W
\end{aligned}
$$

Substituting numerical values

$$
\begin{aligned}
\bar{h} &= \pm\hat{k} 1.8\sqrt{\frac{2(5.3)^3}{(32.2)}\left(\cos\theta - \cos\left(33\left(\frac{\pi}{180}\right)\right)\right)} \\
&= \pm\hat{k} 1.8\sqrt{9.247(\cos\theta - 0.839)} \\
&= \pm\hat{k} 1.8\sqrt{9.247}\sqrt{(\cos\theta - 0.839)} \\
&= \pm 5.474\sqrt{(\cos\theta - 0.839)}\hat{k}
\end{aligned}
$$

0.5 Problem 5

A collar with mass m = 1.5 kg is mounted on a rotating arm of negligible mass that is initially rotating with an angular velocity ω_0 = 1.6 rad/s. The collar's initial distance from the z axis is r_0 = 0.5 m and d = 1.9 m. At some point, the restraint keeping the collar in place is removed so that the collar is allowed to slide. Assume that the friction between the arm and the collar is negligible. If no external forces and moments are applied to the system, with what speed will the collar impact the end of the arm?



The collar will impact the end of the arm with a speed of ______ m/s.

There is no external torque, hence angular momentum is conserved. Let $\bar{h}_1$ be the angular momentum initially and let $\bar{h}_2$ be angular momentum be at some instance of time later on. Therefore

$$\begin{aligned}
\bar{h}_1 &= \bar{r}_1 \times m\bar{v}_1 \\
&= r_0\hat{u}_r \times m\left(r_0\omega_0\hat{u}_\theta\right) \\
&= \begin{vmatrix} \hat{u}_r & \hat{u}_\theta & \hat{k} \\ r_0 & 0 & 0 \\ 0 & mr_0\omega_0 & 0 \end{vmatrix} \\
&= mr_0^2\omega_0\hat{k}
\end{aligned}$$

And at some later instance

$$\begin{aligned}
\bar{h}_2 &= \bar{r}_2 \times m\bar{v}_2 \\
&= r\hat{u}_r \times m\left(\dot{r}\hat{u}_r + r\omega\hat{u}_\theta\right) \\
&= \begin{vmatrix} \hat{u}_r & \hat{u}_\theta & \hat{k} \\ r & 0 & 0 \\ m\dot{r} & mr\omega & 0 \end{vmatrix} \\
&= mr^2\omega\hat{k}
\end{aligned}$$

Equating the last two results gives

$$\begin{aligned}
mr_0^2\omega_0 &= mr^2\omega \\
\omega &= \left(\frac{r_0}{r}\right)^2\omega_0 \qquad (1)
\end{aligned}$$

Now the equation of motion in radial direction is $F = ma_r$, but $F = 0$, since there is no force on the collar. Therefore

$$\begin{aligned}
ma_r &= 0 \\
m\left(\ddot{r} - r\omega^2\right) &= 0 \\
\ddot{r} &= r\omega^2
\end{aligned}$$

Using (1) in the above

$$\begin{aligned}
\ddot{r} &= r\left[\left(\frac{r_0}{r}\right)^2\right]^2\omega_0^2 \\
\ddot{r} &= \frac{r_0^4}{r^3}\omega_0^2
\end{aligned}$$

But $\ddot{r} = \dot{r}\frac{d\dot{r}}{dr}$, hence the above becomes

$$\dot{r}d\dot{r} = \frac{r_0^4}{r^3}\omega_0^2 dr$$

Now we can integrate

$$\begin{aligned}
\int_0^{\dot{r}} \dot{r}d\dot{r} &= \int_{r_0}^{r} \frac{r_0^4}{r^3}\omega_0^2 dr \\
\frac{\dot{r}^2}{2} &= \frac{1}{2}\omega_0^2 r_0^4\left(\frac{-1}{r^2}\right)_{r_0}^{r} \\
&= \frac{1}{2}\omega_0^2 r_0^4\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right)
\end{aligned}$$

Therefore

$$\dot{r} = \omega_0 r_0^2\sqrt{\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right)}$$

To find $\dot{r}$ when it hits the end, we just need to replace r by $r_0 + d$ in the above

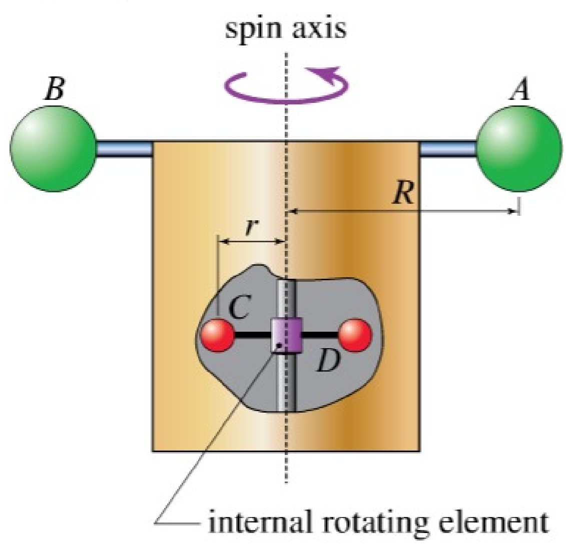
$$\dot{r}_{end} = \omega_0 r_0^2\sqrt{\frac{1}{r_0^2} - \frac{1}{(r_0 + d)^2}}$$

Numerically the above is

$$\dot{r}_{end} = (1.6)\,(0.5)^2 \sqrt{\left(\frac{1}{(0.5)^2} - \frac{1}{(0.5+1.9)^2}\right)}$$
$$= 0.782 \ \ \text{m/s}$$

0.6 Problem 6

The body of the satellite shown has a weight that is negligible with respect to the two spheres A and B that are rigidly attached to it, which weigh 172 lb each. The distance from the spin axis of the satellite to A and B is $R = 3.7$ ft. Inside the satellite there are two spheres C and D weighing 4.3 lb mounted on a motor that allows them to spin about the axis of the cylinder at a distance $r = 0.75$ ft from the spin axis. Suppose that the satellite is released from rest and that the internal motor is made to spin up the internal masses at a constant angular acceleration of 4.7 rad/s^2 for a total of 12 s. Treating the system as isolated, determine the angular speed of the satellite at the end of spin−up.



$\omega_{\text{satellite}} =$ [ ] **rad/s**

Using

$$\bar{h}_1 + \int_0^t \tau dt = \bar{h}_2$$

Where $\bar{h}_1$ is initial angular momentum which is zero, and $\bar{h}_2$ is final angular momentum which is $I\omega_f$ where $I = 2MR^2$ where M is mass of large ball and I is the mass moment of inertial of the large ball about the spin axis.

But torque $\tau = I_2\ddot{\theta}$ where $I_2 = 2\left(mr^2\right)$ where m is mass of each small ball and I_2 is the mass moment of inertial of the small ball about the spin axis. Hence the above becomes

$$\int_0^t \tau dt = \bar{h}_2$$
$$2\left(mr^2\right)\ddot{\theta}\int_0^t dt = \bar{h}_2$$

Since $\ddot{\theta}$ is constant. Hence

$$2\left(mr^2\right)\ddot{\theta}t = 2MR^2\omega_f$$

Solving for final angular velocity

$$\begin{aligned}\omega_f &= \frac{2\left(mr^2\right)\ddot{\theta}t}{2MR^2}\\ &= \frac{2\left(\frac{4.3}{32.2}\right)(0.75)^2(4.7)(12)}{2\left(\frac{172}{32.2}\right)(3.7)^2}\\ &= 0.05793 \text{ rad/sec}\end{aligned}$$