HW 6,ME 240 Dynamics, Fall 2017

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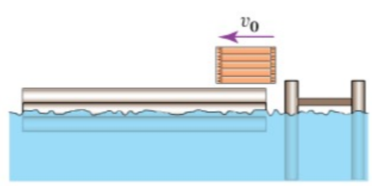
0.1 Problem 1

A 741 lb floating platform is at rest when a 210 lb crate is thrown onto it with a horizontal speed v_0 = 15 ft/s. Once the crate stops sliding relative to the platform, the platform and the crate move with a speed$v$ = 2.652 ft/s. Neglecting the vertical motion of the system as well as any resistance due to the relative motion of the platform with respect to the water, determine the distance that the crate slides relative to the platform if the coefficient of kinetic friction between the platform and the crateis μ_k = 0.28.



d = ft

Let state 1 be when the crate is thrown at the platform. Let the crate by body A and the platform be body B. We will use work-energy to solve this.

$$T_1 + V_1 + U_{12}^{internal} + U_{12}^{external} = T_2 + V_2$$

Where $U_{12}^{internal}$ is work done due to internal forces between the two bodies, which is the friction. We will use notation v_{A_1} to mean velocity of A in state 1 and v_{A_2} to mean velocity of A in state 2 and the same for body B. Therefore the above equation becomes

$$\frac{1}{2}m_A v_{A_1}^2 + \frac{1}{2}m_A v_{B_1}^2 - \int_0^d \mu_k m_A g dx = \frac{1}{2}m_A v_{A_2}^2 + \frac{1}{2}m_A v_{B_2}^2$$

Notice that $V_1 = V_2$ and hence they cancel. Also since in state 2 both body A and B move with same speed v, and also $v_{B_1} = 0$, then the above simplifies to

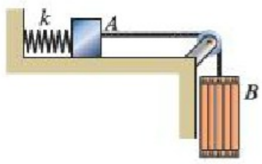
$$\frac{1}{2}m_A v_{A_1}^2 - \mu_k m_A g d = \frac{1}{2}(m_A + m_B) v^2$$

We now solve for d, the distance that body A (the crate) slides. The above is one equation with one unknown.

$$\begin{aligned} d &= \frac{\frac{1}{2}m_A v_{A_1}^2 - \frac{1}{2}(m_A + m_B) v^2}{\mu_k m_A g} \\ &= \frac{1}{2}\frac{(210)(15)^2 - (210 + 741)(2.652)^2}{(0.28)(210)(32.2)} \\ &= 10.712 \text{ ft} \end{aligned}$$

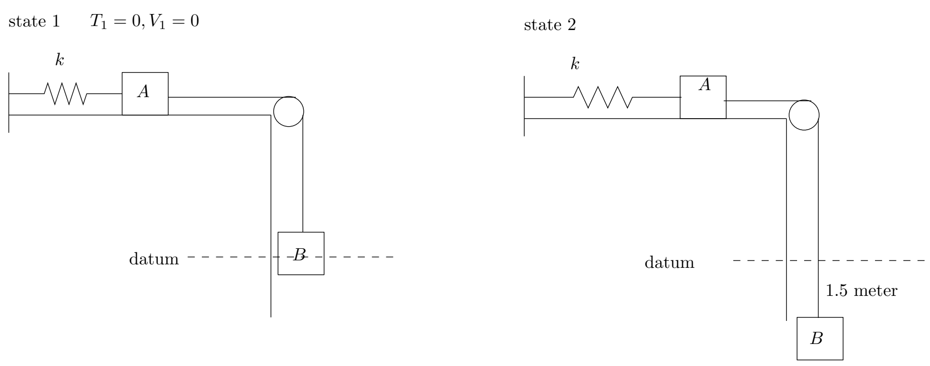
0.2 Problem 2

Blocks A and B are released from rest when the spring is unstretched. Block A has a mass m_A = 4 kg, and the linear spring has stiffness k = 9 N/m. If all sources of friction are negligible, determine the mass of block B such that B has a speed v_B = 1.3 m/s after moving 1.5 m downward, assuming that A never leaves the horizontal surface shown and the cord connecting A and B is inextensible.



m_B = ☐ kg.

Let state 1 be just before release and state 2 after it moves by 1.5 meter



Therefore

$$T_1 + V_1 + U_{12}^{internal} + U_{12}^{external} = T_2 + V_2$$

$T_1 = 0, V_1 = 0$ and $U_{12} = 0$ since there is no friction and no external force. $T_2 = \frac{1}{2}(m_A + m_B)v^2$ since both bodies will have same speed. V_2 comes from spring and gravity. The distance $h = 1.5$ meter. Therefore $V_2 = \frac{1}{2}kh^2 - m_B g h$ since spring extend by same amount m_B and m_A moves. The above becomes

$$0 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kh^2 - m_B g h$$

Where v above is 1.3 since both bodies move with same speed. We want to solve for m_B the only unknown in this equation

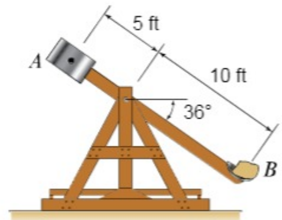
$$\begin{aligned} 0 &= \frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2 + \frac{1}{2}kh^2 - m_B g h \\ &= m_B\left(\frac{1}{2}v^2 - gh\right) + \frac{1}{2}m_A v^2 + \frac{1}{2}kh^2 \\ m_B &= \frac{m_A v^2 + kh^2}{2gh - v^2} \end{aligned}$$

Plug-in numerical values gives

$$\begin{aligned} m_B &= \frac{(4)(1.3)^2 + (9)(1.5)^2}{2(9.81)(1.5) - (1.3)^2} \\ &= 0.974 \text{ kg} \end{aligned}$$

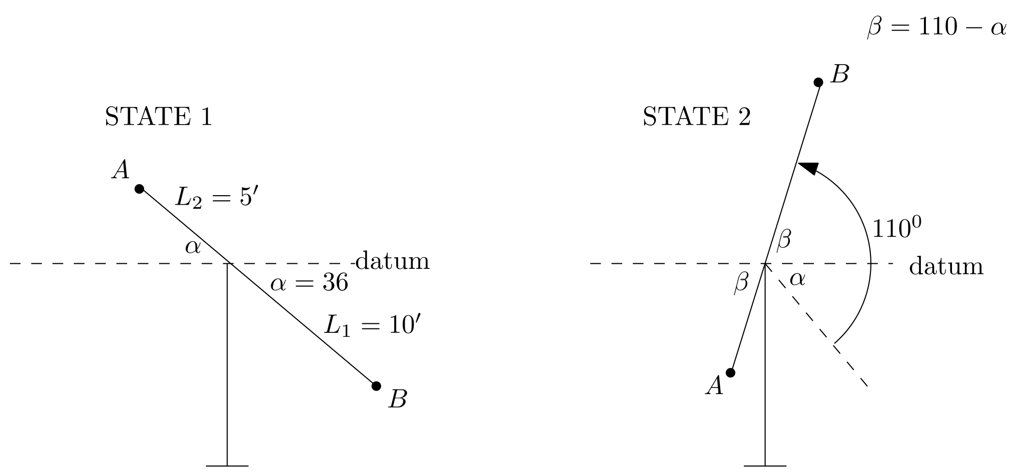
0.3 Problem 3

Consider the simple catapult shown in the figure with an 814 lb counterweight A and a 129 lb projectile B. If the system is released from rest as shown, determine the speed of the projectile after the arm rotates (counterclockwise) through an angle of 110°. Model and B as particles, neglect the mass of the catapult's arm, and assume that friction is negligible. The catapult's frame is fixed with respect to the ground, and the projectile does not separate from the arm during the motion considered.



$v =$ [ ] ft/s

Let state 1 be just before release and state 2 after it rotation.



$T_1 = 0$
$V_1 = -m_B g L_1 \sin\alpha + m_A g L_2 \sin\alpha$

$T_2 = \frac{1}{2} m_B (L_1\omega)^2 + \frac{1}{2} m_A (L_2\omega)^2$
$V_2 = m_B g L_1 \sin\beta - m_A g L_2 \sin\beta$

$$T_1 + V_1 + U_{12}^{internal} + U_{12}^{external} = T_2 + V_2$$

But $U_{12} = 0$ since there is no friction and no external forces. Now $T_1 = 0$ since at rest. And

$$V_1 = -m_B g L_1 \sin\alpha + m_A g L_2 \sin\alpha$$

Where $\alpha = 36^o$. In state 2

$$T_2 = \frac{1}{2} m_B (L_1\omega)^2 + \frac{1}{2} m_A (L_2\omega)^2$$

Where ω is the angular velocity, which we do not know, but will solve for. And

$$V_2 = m_B g L_1 \sin\beta - m_A g L_2 \sin\beta$$

Therefore (1) becomes

$$-m_B g L_1 \sin\alpha + m_A g L_2 \sin\alpha = \frac{1}{2} m_B (L_1\omega)^2 + \frac{1}{2} m_A (L_2\omega)^2 + m_B g L_1 \sin\beta - m_A g L_2 \sin$$

$$-m_B g L_1 \sin\alpha + m_A g L_2 \sin\alpha - m_B g L_1 \sin\beta + m_A g L_2 \sin\beta = \omega^2 \left(\frac{1}{2} m_B L_1^2 + \frac{1}{2} m_A L_2^2\right)$$

$$\omega^2 = \frac{m_A g L_2 \left(\sin\alpha + \sin\beta\right) - m_B g L_1 \left(\sin\alpha + \sin\beta\right)}{\frac{1}{2} m_B L_1^2 + \frac{1}{2} m_A L_2^2}$$

$$\omega = \sqrt{\frac{2\left(m_A g L_2 - m_B g L_1\right)\left(\sin\alpha + \sin\beta\right)}{m_B L_1^2 + m_A L_2^2}}$$

Now we solve for ω and use it to find speed of B from $v_B = L_1\omega$. Since $m_A = 814, m_B =$

$129, L_1 = 10, L_2 = 5, \alpha = 36, \beta = 110 - 36 = 74^o$ then

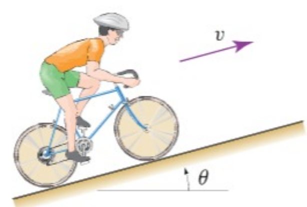
$$\omega = \sqrt{2\frac{((814)(32.2)(5) - (129)(32.2)(10))\left(\sin\left(36\left(\frac{\pi}{180}\right)\right) + \sin\left(74\left(\frac{\pi}{180}\right)\right)\right)}{(129)10^2 + (814)5^2}}$$
$$= 2.888 \text{ rad/sec}$$

Hence

$$\begin{aligned} v_B &= L_1\omega \\ &= 10(2.888) \\ &= 28.88 \text{ ft/sec} \end{aligned}$$

0.4 Problem 4

A cyclist is riding with a speed v = 21 mph over an inclined road with θ = 13°. Neglecting aerodynamic drag, if the cyclist were to keep his output power constant, what speed would he attain if θ were equal to 20°?



mph

Power P is

$$P = Fv$$

Where F is force generated by cyclist. From force balance we see that $F = mg\sin\theta$. Hence for constant power, we want

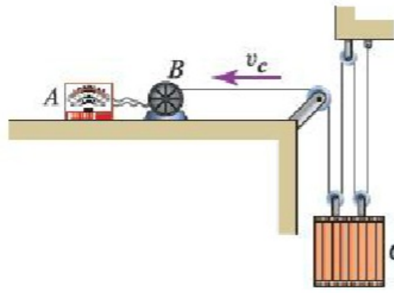
$$\left(mg\sin 13^0\right)v_1 = \left(mg\sin 20^0\right)v_2$$

Solving for v_2

$$\begin{aligned} v_2 &= \frac{\left(mg\sin 13^0\right)v_1}{\left(mg\sin 20^0\right)} \\ &= \frac{\sin\left(13\left(\frac{\pi}{180}\right)\right)}{\sin\left(20\left(\frac{\pi}{180}\right)\right)}21 \\ &= 13.812 \text{ mph} \end{aligned}$$

0.5 Problem 5

The motor B is used to raise and lower the crate C via a pulley system. At the instant shown, the cable is being retracted by the motor with the constant speed $v_c = 4.5$ ft/s. The weight of the crate is $W_C = 460$ lb. If the power meter A shows a power input to the motor of 1.37 hp, determine the overall efficiency of the system.



$\varepsilon =$ ☐

$$\varepsilon = \frac{P_{out}}{P_{in}}$$

Where ε is the efficiency and P_{out} is power out and P_{in} is power in. But $P_{out} = Fv_c$. So we just need to find force in the cable that the motor is pulling with. This force is $\frac{W}{4}$, since there are 4 cables and hence the weight is distributed over them, and then the tension in the one cable attached to the motor is $\frac{W}{4}$. Now we have all the information to find ε

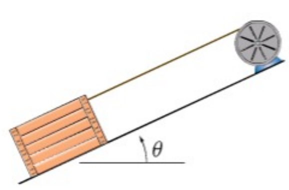
$$\begin{aligned} P_{out} &= (4.5)\left(\frac{460}{4}\right) \\ &= 517.5 \text{ lb-ft/sec} \end{aligned}$$

But $hp = 550$ lb-ft/sec, therefore in hp the above is $\frac{517.5}{550} = 0.941$, hence

$$\varepsilon = \frac{0.941}{1.37} = 0.687$$

0.6 Problem 6

Assuming that the motor shown has an efficiency $\varepsilon = 0.78$, determine the power to be supplied to the motor if it is to pull a 216 lb crate up the incline with a constant speed $v = 6.6$ ft/s. Assume that the kinetic friction coefficient between the slide and the crate is $\mu_k = 0.24$ and that $\theta = 28°$.



The power supplied to the motor is ☐ hp.

$$\varepsilon = \frac{P_{out}}{P_{in}}$$

Mass of crate is $\frac{216}{32.2}$ slug. (note, number given 216 is *weight*) These problem should make it more clear if lb given is meant to be weight or mass.

We need to find P_{in}. We are given ε. We now calculate P_{out} and then will be able to find P_{in}. But $P_{out} = Fv$, where F is force given by motor to pull the crate. From free body diagram, we see that this force is $F = \mu mg\cos\theta + mg\sin\theta$. Hence

$$
\begin{aligned}
P_{out} &= \left(\mu mg\cos\theta + mg\sin\theta\right)v \\
&= mg\left(\mu\cos\theta + \sin\theta\right)v \\
&= \left(\frac{216}{32.2}\right)(32.2)\left(0.24\cos\left(28\left(\frac{\pi}{180}\right)\right) + \sin\left(28\left(\frac{\pi}{180}\right)\right)\right)(6.6) \\
&= 971.374 \text{ lb-ft/sec} \\
&= \frac{971.374}{550} = 1.766 \text{ hp}
\end{aligned}
$$

Hence

$$
\begin{aligned}
P_{in} &= \frac{P_{out}}{\varepsilon} \\
&= \frac{1.766}{0.78} \\
&= 2.264 \text{ hp}
\end{aligned}
$$