HW 12,ME 240 Dynamics, Fall 2017

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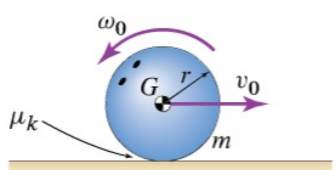
0.1 Problem 1

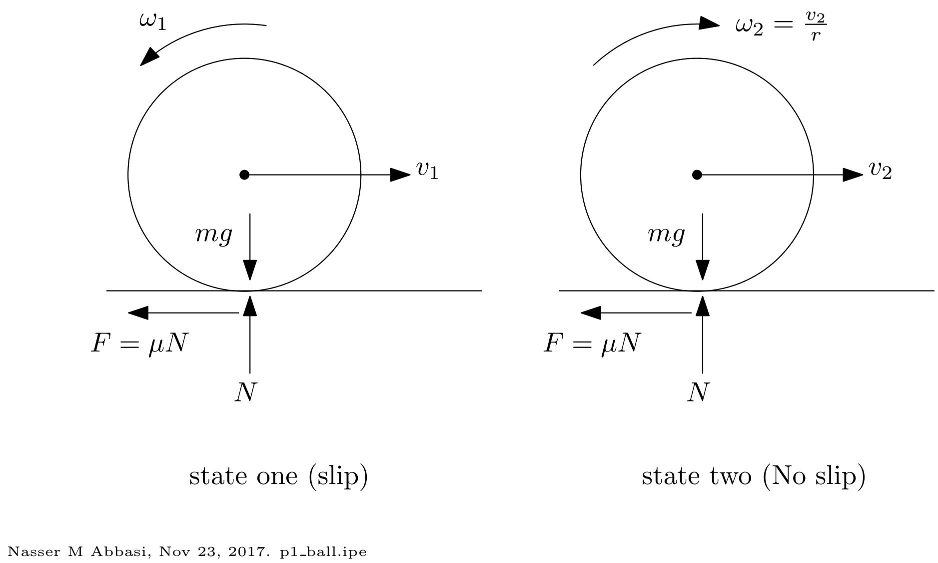
A bowling ball is thrown onto a lane with a backspin ω_0 and forward velocity v_0. The mass of the ball is m, its radius is r, its radius of gyration is k_G, and the coefficient of kinetic friction between the ball and the lane is μ_k. Assume the mass center G is at the geometric center. For a15 lb ball with $r = 4.25$ in., $k_G = 2.4$ in., $\omega_0 = 9$ rad/s, and $v_0 = 17$ mph, determine the time it takes for the ball to start rolling without slip and its speed when it does so. In addition, determine the distance it travels before it starts rolling without slip. Use $\mu_k = 0.11$.



t = ______ **s**

d = ______ **ft**

A ball will roll with slip when the linear velocity v of its center of mass is different from $r\omega$ where r is the radius and ω is the spin angular velocity. Therefore, to find when the ball will roll without slipping, we need to find when $v = r\omega$. Let the initial state be such that $v_1 = v_0$ (given) and $\omega_1 = \omega_0$ (given). So we need to find the time t to get to new state, such that $v_2 = r\omega_2$



Using linear momentum

$$mv_1 + \int_0^{t_{final}} F_{friction} dt = mv_2$$

But $F_{friction} = -\mu N = -\mu mg$ and the above becomes

$$mv_1 - \mu mgt = mv_2 \tag{1}$$

Using the angular momentum gives

$$I\omega_1 + \int_0^{t_{final}} F_{friction} r dt = I\omega_2$$

$$mr_G^2\omega_1 - \mu mgrt = -mr_G^2\left(\frac{v_2}{r}\right) \tag{2}$$

Where in (2), r_G is radius of gyration, and we replaced ω_2 by $\frac{v_2}{r}$. Notice the sign in RHS of (2) is negative, since we assume v_2 is moving to the right, so in state 2, the ball will be spinning clock wise, which is negative,. Now we have two equations (1,2) with two unknowns t, which is the time to get to the state such that center of mass moves with same speed as $r\omega$ (i.e. no slip) and the second unknown is v_2 which is the speed at which the ball will be rolling at that time. We now solve (1,2) for t, v_2

(1) becomes

$$\left(\frac{15}{32.2}\right)\left(17\left(\frac{5280}{3600}\right)\right) - (0.11)\left(\frac{15}{32.2}\right)(32.2)\,t = \left(\frac{15}{32.2}\right)v_2 \tag{1A}$$

$$\left(\frac{15}{32.2}\right)\left(\frac{2.4}{12}\right)^2(9) - (0.11)\left(\frac{15}{32.2}\right)(32.2)\left(\frac{4.25}{12}\right)t = -\left(\frac{15}{32.2}\right)\left(\frac{2.4}{12}\right)^2\left(\frac{v_2}{\left(\frac{4.25}{12}\right)}\right) \tag{2A}$$

Or

$$11.615 - 1.65t = 0.466v_2 \tag{1A}$$
$$0.168 - 0.584\,t = -0.0526v_2 \tag{2A}$$

Solution is:

$$t = 1.9196 \text{ sec}$$
$$v_2 = 18.134 \text{ ft/sec}$$

Now that we know the time and the final velocity, we can find the acceleration of the ball

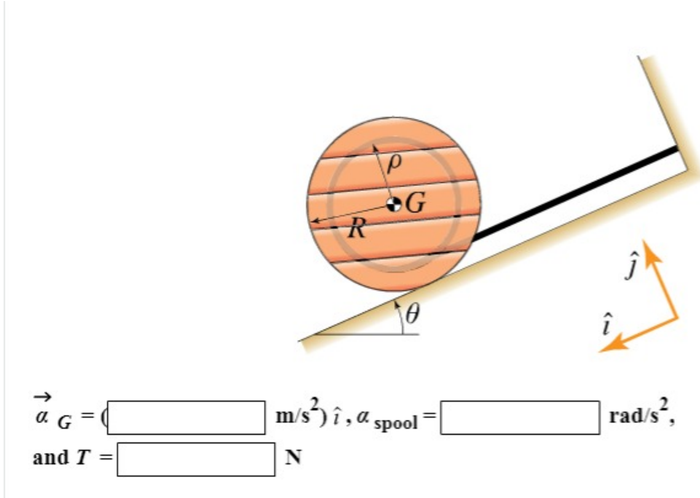
$$\begin{aligned} v_2 &= v_1 + at \\ a &= \frac{v_2 - v_1}{t} \\ &= \frac{18.134 - 17\left(\frac{5280}{3600}\right)}{1.9196} \\ &= -3.542 \text{ ft/s}^2 \end{aligned}$$

Hence the distance travelled is

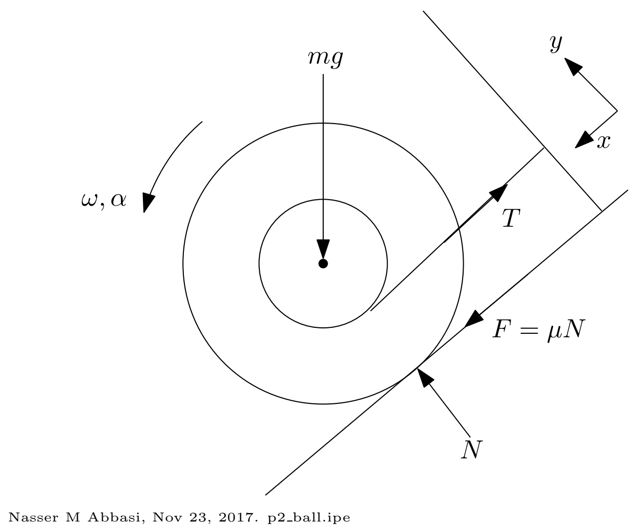
$$\begin{aligned} s &= v_0 t + \frac{1}{2}at^2 \\ &= 17\left(\frac{5280}{3600}\right)(1.9196) + \frac{1}{2}(-3.542)(1.9196)^2 \\ &= 41.3361 \text{ ft} \end{aligned}$$

0.2 Problem 2

A spool of mass $m = 213$ kg, inner and outer radii $\rho = 1.74$ m and $R = 2.24$ m, respectively, and radius of gyration $k_G = 2$ m, is being lowered down an incline with $\theta = 27°$. If the static and kinetic friction coefficients between the incline and the spool are $\mu_s = 0.45$ and $\mu_k = 0.31$, respectively, determine the acceleration of G, the angular acceleration of the spool, and the tension in the cable.



Using the following FBD



Notice that the Friction force F is pointing downwards since the spool is spinning counter clockwise. Resolving forces along x gives

$$F - T + mg\sin\theta = m\ddot{x} \tag{1}$$

Taking moment about CG, using clockwise as positive now, since we changed x positive direction from normal

$$FR - T\rho = I_{cg}\alpha \tag{2}$$

Where α is angular acceleration of spool. But $\ddot{x} = -\rho\alpha$ then (1) becomes

$$F - T + mg\sin\theta = -m\rho\alpha \tag{3}$$

But

$$\begin{aligned} F &= \mu_k N \\ &= \mu_k mg\cos\theta \end{aligned}$$

Therefore (2) and (3) become

$$\mu_k mg\cos\theta R - T\rho = I_{cg}\alpha \tag{2A}$$

$$\mu_k mg\cos\theta - T + mg\sin\theta = -m\rho\alpha \tag{3A}$$

In (2A) and (3A) there are 2 unknowns, α and T. Plugging numerical values gives

$$(0.31)(213)(9.81)\cos\left(27\left(\frac{\pi}{180}\right)\right)(2.24) - T(1.74) = (213)(2)^2\alpha$$

$$(0.31)(213)(9.81)\cos\left(27\left(\frac{\pi}{180}\right)\right) - T + (213)(9.81)\sin\left(27\left(\frac{\pi}{180}\right)\right) = -(213)(1.74)\alpha$$

Or

$$1292.823 - 1.74T = 852.0\alpha \tag{2A}$$

$$1525.78 - 1.0T = -370.62\alpha \tag{3A}$$

Solution is:

$$T = 1188.547 \text{ N}$$

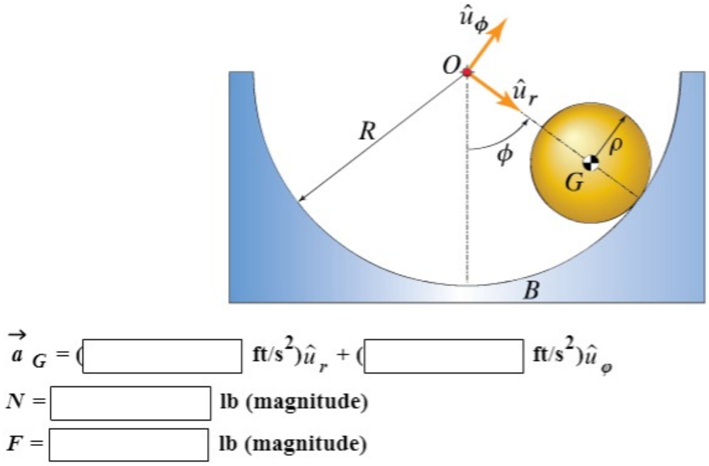
$$\alpha = -0.9099 \text{ rad/s}^2$$

Now since $\ddot{x} = -\rho\alpha$ then

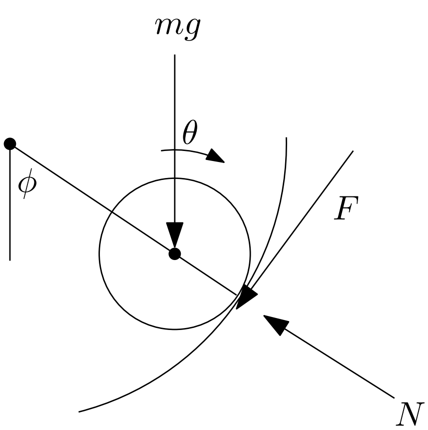
$$\begin{aligned} \ddot{x} &= -(1.74)(-0.9099) \\ &= 1.583 \text{ m/s}^2 \end{aligned}$$

0.3 Problem 3

The uniform ball of radius ρ and mass m is gently placed in the bowl B with inner radius R and is released. The angle φ measures the position of the center of the ball at G with respect to a vertical line, and the angle θ measures the rotation of the ball with respect to a vertical line. Assume that the system lies in the vertical plane. Assuming that the ball rolls without slip that it weighs 2.9 lb, is at the position $\varphi = 40°$, and is moving clockwise at 9.1 ft/s, determine the acceleration of the center of the ball aG and the normal and friction force between the ball and the bowl. Use $R = 4.2$ ft and $\rho = 1.2$ ft. *Hint:* In working the following problem, we recommend using the $r\varphi$ coordinate system shown.



The forces in play are



Resolving forces along $\hat{u}_\phi$

$$-F - mg\sin\phi = m\left(R-\rho\right)\ddot{\phi} \tag{1}$$

Taking moment around C.G. of ball

$$-F\rho = I_{cg}\ddot{\theta} \tag{2}$$

The above are 2 equations in 3 unknowns $(F, \ddot{\theta}, \ddot{\phi})$. So we need one more equation. Resolving along $\hat{u}_r$ will not give us an equation in any of these unknowns so it will not be useful for this. Here we must notice that acceleration of point D, where the ball touches the bottom of the bowl will be zero. This is because the ball rolls without slip. We can use this to come up with the third equation. The acceleration of this point in the $\hat{u}_\phi$ direction is zero, and given by

$$a_{D,\phi} = \left(R - \rho\right)\ddot{\phi} + \rho\ddot{\theta} = 0 \tag{3}$$

Now we have three equations with three unknowns. Plug-in numerical values, using $I_{cg} = \frac{2}{5}m\rho^2$

$$-F - (2.9)\sin\left(40\left(\frac{\pi}{180}\right)\right) = \left(\frac{2.9}{32.2}\right)(4.2 - 1.2)\,\ddot{\phi} \tag{1A}$$

$$-F\,(1.2) = \left(\frac{2}{5}\left(\frac{2.9}{32.2}\right)\left(1.2^2\right)\right)\ddot{\theta} \tag{2A}$$

$$0 = (4.2 - 1.2)\,\ddot{\phi} + (1.2)\,\ddot{\theta} \tag{3A}$$

Or

$$-F - 1.864 \ = 0.27\ddot{\phi} \tag{1A}$$

$$-1.2F = 0.0519\ddot{\theta} \tag{2A}$$

$$0 = 1.2\ddot{\theta} + 3\ddot{\phi} \tag{3A}$$

Solving gives

$$\begin{aligned} F &= -0.5326 \text{ N} \\ \ddot{\theta} &= 12.32 \text{ rad/s}^2 \\ \ddot{\phi} &= -4.928 \text{ rad/s}^2 \end{aligned}$$

To find N, we resolve forces along $\hat{u}_r$

$$-N + mg\cos\phi = -m\left(R - \rho\right)\dot{\theta}^2$$

But $\dot{\theta} = \frac{v}{(R-\rho)}$, where $v = 9$ ft/sec in this problem. Hence the above becomes

$$\begin{aligned} -N + mg\cos\phi &= -m\left(\frac{v^2}{R - \rho}\right) \\ N &= mg\cos\phi + m\left(\frac{v^2}{R - \rho}\right) \\ &= (2.9)\cos\left(40\frac{\pi}{180}\right) + \frac{2.9}{32.2}\left(\frac{(9.1)^2}{(4.2 - 1.2)}\right) \\ &= 4.708 \text{ N} \end{aligned}$$

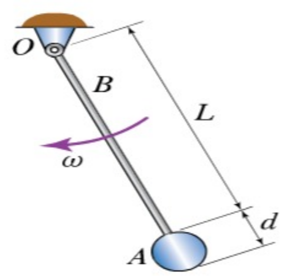
Now to find $\vec{a}_G$. Since

$$\vec{a}_G = \left(R - \rho\right)\ddot{\phi}\hat{u}_\phi - \frac{v^2}{R - \rho}\hat{u}_r$$

Then

$$\begin{aligned} \vec{a}_G &= -(4.1 - 1.2)\,4.928\hat{u}_\phi - \frac{(9.1)^2}{(4.2 - 1.2)}\hat{u}_r \\ &= -14.291\hat{u}_\phi - 27.603\,\hat{u}_r \end{aligned}$$

0.4 Problem 4



A pendulum consists of a uniform disk A of diameter d = 0.16 m and mass m_A = 0.37 kg attached at the end of a uniform bar B of length L = 0.75 m and mass m_B = 0.7 kg. At the instant shown, the pendulum is swinging with an angular velocity ω = 0.23 rad/s clockwise. Determine the kinetic energy of the pendulum at this instant, using $T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega_B^2$.

T = [ ] J

Let r be radius of disk. Then, about joint O at top,

$$
\begin{aligned}
I_{disk} &= m_{disk}\frac{r^2}{2} + m_{disk}(L+r)^2 \\
&= (0.37)\frac{(0.08)^2}{2} + 0.37\,(0.75+0.08)^2 \\
&= 0.256\,077
\end{aligned}
$$

And

$$
\begin{aligned}
I_{bar} &= m_{bar}\frac{L^2}{3} \\
&= (0.7)\frac{(0.75)^2}{3} \\
&= 0.131
\end{aligned}
$$

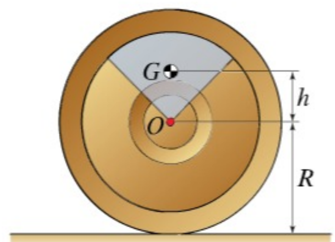
Hence overall

$$
\begin{aligned}
I_o &= I_{disk} + I_{bar} \\
&= 0.256 + 0.131 \\
&= 0.387
\end{aligned}
$$

Therefore

$$
\begin{aligned}
KE &= \frac{1}{2} I_o \omega^2 \\
&= \frac{1}{2} (0.387)(0.23)^2 \\
&= 0.01024 \mathrm{J}
\end{aligned}
$$

0.5 Problem 5



An eccentric wheel with weight $W = 260$ lb, mass center G, and radius of gyration $k_G = 1.32$ ft is initially at rest in the position shown. Letting $R = 1.76$ ft and $h = 0.6$ ft, and assuming that the wheel is gently nudged to the right and rolls without slip, determine the speed of O when G is closest to the ground.

v_O = ____ ft/s

Since wheel rolls without spin, then friction on the ground against the wheel does no work. Therefore we can use work-energy to find v_{final} since we do not need to find friction force and this gives us one equation with one unknown to solve for.

$$
\begin{aligned}
T_1 + U_1 &= T_2 + U_2 \\
0 + mgh &= \frac{1}{2} m v_{cg}^2 + \frac{1}{2} I_{cg} \omega^2 - mgh \qquad (1)
\end{aligned}
$$

Where in the above, the datum is taken as horizontal line passing through the middle of the wheel. But

$$
I_{cg} = m r_G^2
$$

Where r_G is radius of gyration. And

$$
v_{cg} = v_o \frac{(R-h)}{R}
$$

And $\omega = \frac{v_o}{R}$ since rolls with no slip. Now we have all the terms needed to evaluate (1) and solve for v_o. Here

$$
m = \frac{260}{32.2} = 8.075 \text{ slug}
$$

Hence (1)

$$mgh = \frac{1}{2}m\left(v_o\frac{(R-h)}{R}\right)^2 + \frac{1}{2}mr_G^2\left(\frac{v_o}{R}\right)^2 - mgh$$

$$260\,(0.6) = \frac{1}{2}\left(\frac{260}{32.2}\right)\left(v_o\frac{(1.76-0.6)}{1.76}\right)^2 + \frac{1}{2}\left(\frac{260}{32.2}\right)(1.32)^2\left(\frac{v_o}{1.76}\right)^2 - 260\,(0.6)$$

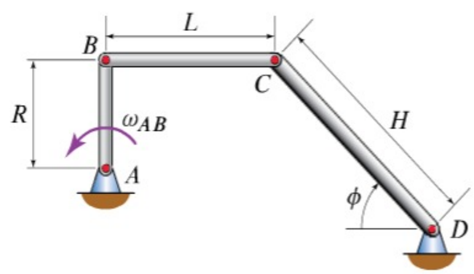
$$156 = 4.025v_o^2 - 156$$

Therefore

$$v_o = 8.805 \text{ ft/s}$$

Where the positive root is used since it is moving to the right.

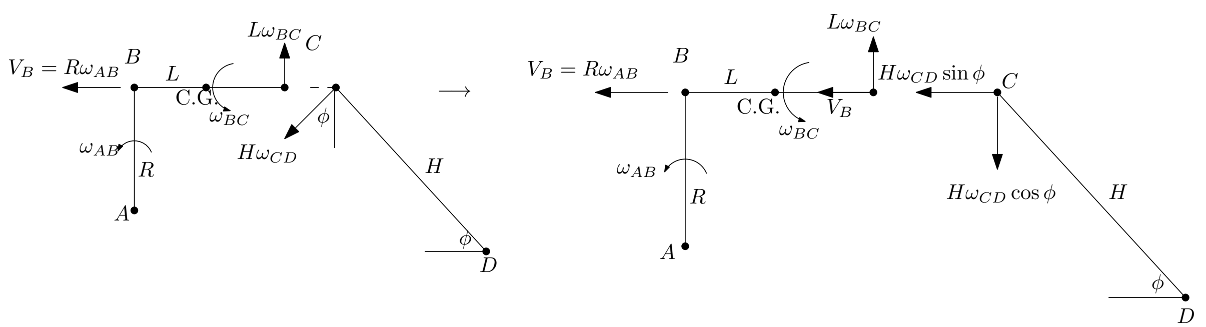
0.6 Problem 6



The weights of the uniform thin pin−connected bars AB, BC, and CD are W_{AB} = 3 lb, W_{BC} = 6.5 lb, and W_{CD} = 11 lb, respectively. Letting φ = 49°, R = 4 ft, L = 5.5 ft, and H = 6.5 ft, and knowing that bar AB rotates at an angular velocity ω_{AB} = 3 rad/s, compute the kinetic energy T of the system at the instant shown.

T = ______ ft · lb

The velocities at each point are given by



$$
\begin{aligned}
V_B &= R\omega_{AB} \\
&= 4\,(3) \\
&= 12 \text{ ft/s}
\end{aligned}
$$

Looking at point C, we obtain two equations

$$
\begin{aligned}
L\omega_{BC} &= -H\omega_{CD}\cos\phi \\
-V_B &= -H\omega_{CD}\sin\phi
\end{aligned}
$$

Or

$$
\begin{aligned}
(5.5)\,\omega_{BC} &= -\,(6.5)\,\omega_{CD}\cos\left(49\left(\frac{\pi}{180}\right)\right) \\
-12 &= -\,(6.5)\,\omega_{CD}\sin\left(49\left(\frac{\pi}{180}\right)\right)
\end{aligned}
$$

Solving gives

$$
\begin{aligned}
\omega_{BC} &= -1.897 \ \text{ rad/sec} \\
\omega_{CD} &= 2.446 \ \text{ rad/sec}
\end{aligned}
$$

We now need to find velocity of center of mass of bar BC. We see from diagram that it is given by

$$
\begin{aligned}
\vec{v}_{CG} &= -V_B\hat{\imath} - \frac{L}{2}\omega_{BC}\hat{\jmath} \\
&= -12\hat{\imath} - \frac{5.5}{2}\,(-1.897)\,\hat{\jmath} \\
&= -12\hat{\imath} + 5.217\hat{\jmath}
\end{aligned}
$$

Hence

$$
\begin{aligned}
\left|\vec{v}_{CG}\right| &= \sqrt{12^2 + 5.217^2} \\
&= 13.085 \text{ ft/sec}
\end{aligned}
$$

Now we have all the velocities needed. The K.E. of bar AB is

$$
\begin{aligned}
T_{AB} &= \frac{1}{2}I_{AB}\frac{1}{2}\omega_{AB}^2 \\
&= \frac{1}{2}\left(\frac{1}{3}m_{AB}R^2\right)\omega_{AB}^2 \\
&= \frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{32.2}\right)(4)^2\right)(3)^2 \\
&= 2.236
\end{aligned}
$$

For bar BC it has both translation and rotation KE

$$
\begin{aligned}
T_{BC} &= \frac{1}{2} I_{BC} \frac{1}{2} \omega_{BC}^2 + \frac{1}{2} m_{BC} v_{CG}^2 \\
&= \frac{1}{2} \left(\frac{1}{12} m_{BC} L^2 \right) \omega_{BC}^2 + \frac{1}{2} m_{BC} v_{CG}^2 \\
&= \frac{1}{2} \left(\frac{1}{12} \left(\frac{6.5}{32.2} \right) (5.5)^2 \right) (-1.897)^2 + \frac{1}{2} \left(\frac{6.5}{32.2} \right) (13.085)^2 \\
&= 18.197
\end{aligned}
$$

And for bar CD it has only rotation KE

$$
\begin{aligned}
T_{CD} &= \frac{1}{2} I_{CD} \frac{1}{2} \omega_{CD}^2 \\
&= \frac{1}{2} \left(\frac{1}{3} m_{CD} H^2 \right) \omega_{CD}^2 \\
&= \frac{1}{2} \left(\frac{1}{3} \left(\frac{11}{32.2} \right) (6.5)^2 \right) (2.446)^2 \\
&= 14.392
\end{aligned}
$$

Therefore the total KE is

$$
\begin{aligned}
KE &= T_{AB} + T_{BC} + T_{CD} \\
&= 2.236 + 18.197 + 14.392 \\
&= 34.825 \text{ J}
\end{aligned}
$$