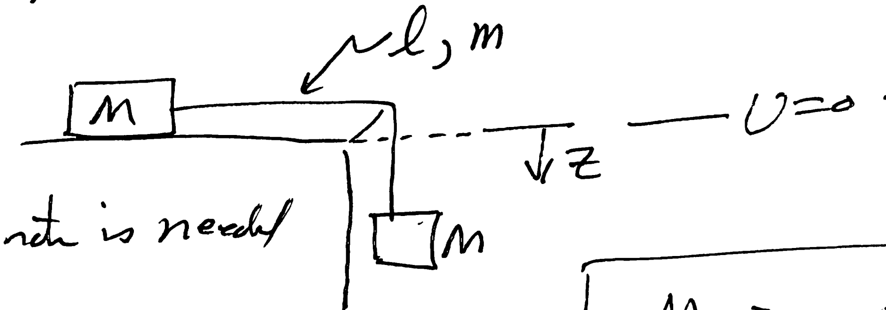
my solution for practice exam 1. Physics 311.
by Nasser M. Abbasi

(1)



(a) one coordinate is needed

(b) $U = -Mg\left(z\right) - \underbrace{m\left(\frac{z}{l}\right)\left(\frac{z}{2}\right)g}_{\text{P.E. for hanging part of wire}} = \boxed{-Mgz - \frac{mz^2}{2l}g}$

$$T = \overbrace{\tfrac{1}{2}M\dot{z}^2 + \tfrac{1}{2}M\dot{z}^2}^{\text{both block move same speed}} + \underbrace{\tfrac{1}{2}m\dot{z}^2}_{\text{K.E. for wire}} = M\dot{z}^2 + \tfrac{1}{2}m\dot{z}^2$$

so $\boxed{L = T - U = \cancel{M}\dot{z}^2\left(M + \frac{m}{2}\right) + Mgz + m\frac{z^2}{2l}g}$

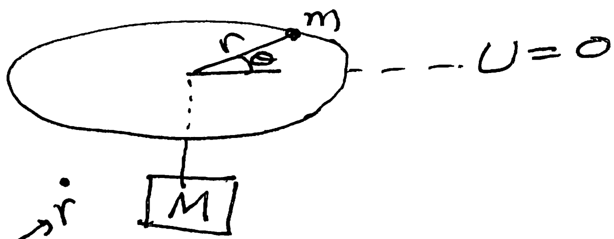
(c) $\left.\begin{aligned} \frac{\partial L}{\partial z} &= Mg + \frac{mz}{l}g \\ \frac{\partial L}{\partial \dot{z}} &= 2\dot{z}\left(M + \frac{m}{2}\right) \end{aligned}\right\} \Rightarrow \boxed{2\ddot{z}\left(M + \frac{m}{2}\right) - \left(Mg + \frac{mz}{l}g\right) = 0}$

$\Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{z}} - \frac{\partial L}{\partial z} = 0$

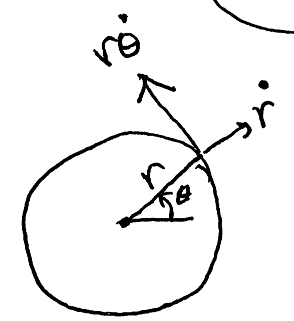
(d) From (c). $\ddot{z} = \dfrac{g\left(M + \frac{mz}{l}\right)}{2\left(M + \frac{m}{2}\right)}$. When $m=0$ we obtain

$$\ddot{z} = \frac{gM}{2M} = \boxed{\frac{g}{2}}$$

problem 2



Top view:



Velocity diagram

$$\Rightarrow v^2 = (\dot r\cos\theta - r\dot\theta\sin\theta)^2 + (\dot r\sin\theta + r\dot\theta\cos\theta)^2$$

$$\boxed{v^2 = \dot r^2 + r^2\dot\theta^2}$$ ← velocity of small m

large M has same radial speed, since string do not stretch.

(a) need 2 generalized coordinates, $\boxed{r, \theta}$

(b) $U = -Mg(l-r)$ assuming string has total length $= l$.

$$T = \tfrac{1}{2}mv^2 + \tfrac{1}{2}M\dot r^2$$
$$= \tfrac{1}{2}m(\dot r^2 + r^2\dot\theta^2) + \tfrac{1}{2}M\dot r^2$$

so $$\boxed{L = T - U = \tfrac{1}{2}m(\dot r^2 + r^2\dot\theta^2) + \tfrac{1}{2}M\dot r^2 + Mg(l-r)}$$

(c) For r $\frac{\partial L}{\partial r} = mr\dot\theta^2 - Mg$

$$\frac{\partial L}{\partial \dot r} = m\dot r + M\dot r = \dot r(M+m).$$

so $\frac{d}{dt}\frac{\partial L}{\partial \dot r} - \frac{\partial L}{\partial r} = 0 \Rightarrow \boxed{\ddot r(M+m) = mr\dot\theta^2 - Mg}$ (1)

for θ $\frac{\partial L}{\partial \theta} = 0$. $\frac{\partial L}{\partial \dot\theta} = mr^2\dot\theta$

so $\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} - \frac{\partial L}{\partial \theta} = 0 \Rightarrow \ddot\theta mr^2 = 0$ or $\boxed{\ddot\theta = 0}$ (2)

(d) when r constant, then (1) becomes $0 = mr\dot\theta^2 - Mg = 0$

$\dot\theta^2 = \frac{Mg}{mr}$ or $\dot\theta = \pm\sqrt{\frac{Mg}{mr}} \rightarrow \boxed{\theta = \pm\sqrt{\frac{Mg}{mr}}\,t + \text{Constant}}$

when r constant, then $\dot\theta$ is also constant.

(e) From (2), integral of motion is $\frac{\partial L}{\partial \dot\theta} = \text{constant} \Rightarrow mr^2\dot\theta = \text{constant}$ angular momentum.