Fourier Transform Pairs:

$$\text{Integral: } \mathcal{F}\{f(x)\} = F(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx \quad , \quad \mathcal{F}^{-1}\{F(\omega)\} = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega x}\, d\omega$$

$$\text{Cosine: } \mathcal{F}_c\{f(x)\} = F_c(\omega) = \int_{0}^{\infty} f(x)\, \cos(\omega x)\, dx \quad , \quad \mathcal{F}^{-1}\{F_c(\omega)\} = f(x) = \frac{2}{\pi}\int_{0}^{\infty} F_c(\omega)\, \cos(\omega x)\, d\omega$$

$$\text{Sine: } \mathcal{F}_s\{f(x)\} = F_s(\omega) = \int_{0}^{\infty} f(x)\, \sin(\omega x)\, dx \quad , \quad \mathcal{F}^{-1}\{F_c(\omega)\} = f(x) = \frac{2}{\pi}\int_{0}^{\infty} F_s(\omega)\, \sin(\omega x)\, d\omega$$

If $f(x)$ is continuously differentiable and $f(x) \longrightarrow 0$ as $|x| \longrightarrow \infty$, then

$$\mathcal{F}\{f'(x)\} = \int_{-\infty}^{\infty} f'(x)\, e^{-i\omega x}\, dx = i\,\omega\, F(\omega).$$

If $f(x)$ is continuously 2-times differentiable and $f''(x) \longrightarrow 0$ as $|x| \longrightarrow \infty$, then

$$\mathcal{F}\{f''(x)\} = \int_{-\infty}^{\infty} f''(x)\, e^{-i\omega x}\, dx = (i\,\omega)^2\, F(\omega).$$

If $f(x)$ is continuously n-times differentiable and $f^n(x) \longrightarrow 0$ as $|x| \longrightarrow \infty$, then

$$\mathcal{F}\{f^{(n)}(x)\} = \int_{-\infty}^{\infty} f^{(n)}(x)\, e^{-i\omega x}\, dx = (i\,\omega)^n\, F(\omega).$$

Convolution:

$$\mathcal{F}^{-1}\{\hat{f}(\omega)\,\hat{g}(\omega)\} = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\, d\tau = (f * g)\,(t)$$

Fourier Series Expansion:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi t}{p}) + \sum_{n=1}^{\infty} b_n \sin(\frac{n\pi t}{p}) \qquad \text{where } p \text{ is half the period and}$$

$$\text{where } a_0 = \frac{1}{p}\int_{-p}^{p} f(t)\, dt; \quad a_n = \frac{1}{p}\int_{-p}^{p} f(t)\cos(\frac{n\pi t}{p})\, dt; \text{ and } b_n = \frac{1}{p}\int_{-p}^{p} f(t)\sin(\frac{n\pi t}{p})\, dt.$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n\, e^{\frac{in\pi t}{p}} \qquad \text{where} \qquad c_n = \frac{1}{2p}\int_{-p}^{p} f(t) e^{-\frac{in\pi t}{p}}\, dt \qquad \text{and where } p \text{ is half the period.}$$

Expansions and Identities

$$\begin{aligned}
1/(1-x) &= 1 + x + x^2 + x^3 + \cdots \qquad \text{for } |x| < 1 \\
e^x &= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \\
\cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots \\
\sin(x) &= \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots \\
\cosh(x) &= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots \\
\sinh(x) &= \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots \\
\sin(x \pm y) &= \sin(x)\cos(y) \pm \sin(y)\cos(x) \\
\cos(x \pm y) &= \cos(x)\cos(y) \mp \sin(x)\sin(y)
\end{aligned}$$

$$\text{Quadratic Eq.: } az^2 + bz + c = 0 \quad \longrightarrow \quad \text{Roots: } z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**TABLE 9.2**

| | | |
|---|---|---|
| a. $f(t) = \begin{cases} 0 & t < 0 \\ e^{-at} & 0 < t,\ a > 0 \end{cases}$ | | $F(\omega) = \dfrac{1}{a + i\omega}$ |
| b. $f(t) = \begin{cases} e^{at} & t \le 0 \\ e^{-at} & 0 \le t \end{cases}$ | $a > 0$ | $F(\omega) = \dfrac{2a}{a^2 + \omega^2}$ |
| c. $f(t) = \begin{cases} -e^{at} & t < 0 \\ e^{-at} & 0 < t \end{cases}$ | $a > 0$ | $F(\omega) = \dfrac{-2i\omega}{a^2 + \omega^2}$ |
| d. $f(t) = e^{-at} \quad 0 < t$ | $a > 0$ | $F_c(\omega) = \sqrt{\dfrac{2}{\pi}}\dfrac{a}{a^2 + \omega^2}$ |
| e. $f(t) = e^{-at} \quad 0 < t$ | | $F_s(\omega) = \sqrt{\dfrac{2}{\pi}}\dfrac{\omega}{a^2 + \omega^2}$ |
| f. $f(t) = \begin{cases} 0 & -\infty < t < -k \\ a & -k < t < 0 \\ b & 0 < t < l \\ 0 & l < t < \infty \end{cases}$ | | $F(\omega) = \dfrac{1}{i\omega}[(b - a) + ae^{i\omega k} - be^{-i\omega l}]$ |

Our first objective will be to show that the Fourier transformation distributes over sums of functions and commutes with scalars. In particular, we have

[redacted] **(Linearity)** If the Fourier transforms of $f_1$ and $f_2$ exist, then

$$\mathcal{F}(a_1 f_1 + a_2 f_2) = a_1\mathcal{F}(f_1) + a_2\mathcal{F}(f_2) \qquad a_1, a_2 \text{ constants}$$

**PROOF** From the definition of the Fourier transformation $\mathcal{F}$ and familiar properties of integrals, we have

$$\begin{aligned}\mathcal{F}(a_1 f_1 + a_2 f_2) &= \int_{-\infty}^{\infty} [a_1 f_1(t) + a_2 f_2(t)]e^{-i\omega t}\,dt \\ &= a_1\int_{-\infty}^{\infty} f_1(t)e^{-i\omega t}\,dt + a_2\int_{-\infty}^{\infty} f_2(t)e^{-i\omega t}\,dt \\ &= a_1\mathcal{F}(f_1) + a_2\mathcal{F}(f_2)\end{aligned}$$

The property ascribed to the Fourier transformation by Theorem 1 is also valid for the Fourier cosine and sine transformations and, in each case, its extension to a linear combination of more than two functions is immediate. In other words, *all three Fourier transformations are linear operators,* as are their inverses.

[redacted] **(Symmetry)** If $F(\omega)$ is the Fourier transform of $f(t)$, then $2\pi f(-\omega)$ is the transform of $F(t)$.

**PROOF** By hypothesis, Form inverse

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt \qquad \text{and inversely} \qquad f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}\,d\omega$$

If in the latter integral we replace $t$ by $-t$, it becomes

$$2\pi f(-t) = \int_{-\infty}^{\infty} F(\omega)e^{-i\omega t}\,d\omega$$

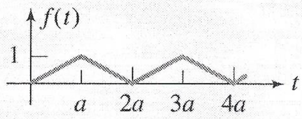
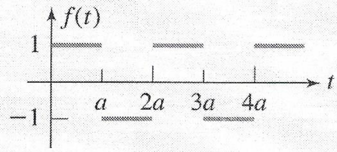
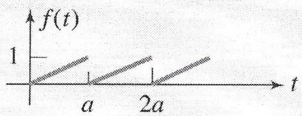
**TABLE 3.1** ***Table of Laplace Transforms of Functions***

| | $f(t)$ | $F(s) = \mathfrak{L}[f(t)](s)$ |
|---|---|---|
| 1. | $1$ | $\dfrac{1}{s}$ |
| 2. | $t$ | $\dfrac{1}{s^2}$ |
| 3. | $t^n (n = 1, 2, 3, \cdots)$ | $\dfrac{n!}{s^{n+1}}$ |
| 4. | $\dfrac{1}{\sqrt{t}}$ | $\sqrt{\dfrac{\pi}{s}}$ |
| 5. | $e^{at}$ | $\dfrac{1}{s-a}$ |
| 6. | $te^{at}$ | $\dfrac{1}{(s-a)^2}$ |
| 7. | $t^n e^{at}$ | $\dfrac{n!}{(s-a)^{n+1}}$ |
| 8. | $\dfrac{1}{a-b}(e^{at} - e^{bt})$ | $\dfrac{1}{(s-a)(s-b)}$ |
| 9. | $\dfrac{1}{a-b}(ae^{at} - be^{bt})$ | $\dfrac{s}{(s-a)(s-b)}$ |
| 10. | $\dfrac{(c-b)e^{at} + (a-c)e^{bt} + (b-a)e^{ct}}{(a-b)(b-c)(c-a)}$ | $\dfrac{1}{(s-a)(s-b)(s-c)}$ |
| 11. | $\sin(at)$ | $\dfrac{a}{s^2+a^2}$ |
| 12. | $\cos(at)$ | $\dfrac{s}{s^2+a^2}$ |
| 13. | $1 - \cos(at)$ | $\dfrac{a^2}{s(s^2+a^2)}$ |
| 14. | $at - \sin(at)$ | $\dfrac{a^3}{s^2(s^2+a^2)}$ |
| 15. | $\sin(at) - at\cos(at)$ | $\dfrac{2a^3}{(s^2+a^2)^2}$ |
| 16 | $\sin(at) + at\cos(at)$ | $\dfrac{2as^2}{(s^2+a^2)^2}$ |
| 17. | $t\sin(at)$ | $\dfrac{2as}{(s^2+a^2)^2}$ |
| 18. | $t\cos(at)$ | $\dfrac{(s^2-a^2)}{(s^2+a^2)^2}$ |
| 19. | $\dfrac{\cos(at) - \cos(bt)}{(b-a)(b+a)}$ | $\dfrac{s}{(s^2+a^2)(s^2+b^2)}$ |
| 20. | $e^{at}\sin(bt)$ | $\dfrac{b}{(s-a)^2+b^2}$ |
| 21. | $e^{at}\cos(bt)$ | $\dfrac{s-a}{(s-a)^2+b^2}$ |
| 22. | $\sinh(at)$ | $\dfrac{a}{s^2-a^2}$ |
| 23. | $\cosh(at)$ | $\dfrac{s}{s^2-a^2}$ |
| 24. | $\sin(at)\cosh(at) - \cos(at)\sinh(at)$ | $\dfrac{4a^3}{s^4+4a^4}$ |
| 25. | $\sin(at)\sinh(at)$ | $\dfrac{2a^2 s}{s^4+4a^4}$ |

**TABLE 3.1 (continued)**

| | $f(t)$ | $F(s) = \mathfrak{L}[f(t)](s)$ |
|---|---|---|
| 26. | $\sinh(at) - \sin(at)$ | $\dfrac{2a^3}{s^4 - a^4}$ |
| 27. | $\cosh(at) - \cos(at)$ | $\dfrac{2a^2 s}{s^4 - a^4}$ |
| 28. | $\dfrac{1}{\sqrt{\pi t}} e^{at}(1 + 2at)$ | $\dfrac{s}{(s-a)^{3/2}}$ |
| 29. | $J_0(at)$ | $\dfrac{1}{\sqrt{s^2 + a^2}}$ |
| 30. | $J_n(at)$ | $\dfrac{1}{a^n} \dfrac{\left(\sqrt{s^2 + a^2} - s\right)^n}{\sqrt{s^2 + a^2}}$ |
| 31. | $J_0(2\sqrt{at})$ | $\dfrac{1}{s} e^{-a/s}$ |
| 32. | $\dfrac{1}{t} \sin(at)$ | $\tan^{-1}\left(\dfrac{a}{s}\right)$ |
| 33. | $\dfrac{2}{t}[1 - \cos(at)]$ | $\ln\left(\dfrac{s^2 + a^2}{s^2}\right)$ |
| 34. | $\dfrac{2}{t}[1 - \cosh(at)]$ | $\ln\left(\dfrac{s^2 - a^2}{s^2}\right)$ |
| 35. | $\dfrac{1}{\sqrt{\pi t}} - a e^{a^2 t} \operatorname{erfc}\left(\dfrac{a}{\sqrt{t}}\right)$ | $\dfrac{1}{\sqrt{s} + a}$ |
| 36. | $\dfrac{1}{\sqrt{\pi t}} + a e^{a^2 t} \operatorname{erf}\left(\dfrac{a}{\sqrt{t}}\right)$ | $\dfrac{\sqrt{s}}{s - a^2}$ |
| 37. | $e^{a^2 t} \operatorname{erf}(a\sqrt{t})$ | $\dfrac{a}{\sqrt{s}(s - a^2)}$ |
| 38. | $e^{a^2 t} \operatorname{erfc}(a\sqrt{t})$ | $\dfrac{1}{\sqrt{s}(\sqrt{s} + a)}$ |
| 39. | $\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$ | $\dfrac{1}{s} e^{-a\sqrt{s}}$ |
| 40. | $\dfrac{1}{\sqrt{\pi t}} e^{-a^2/4t}$ | $\dfrac{1}{\sqrt{s}} e^{-a\sqrt{s}}$ |
| 41. | $\dfrac{1}{\sqrt{\pi(t + a)}}$ | $\dfrac{1}{\sqrt{s}} e^{as} \operatorname{erfc}(\sqrt{as})$ |
| 42. | $\dfrac{1}{\pi t} \sin(2a\sqrt{t})$ | $\operatorname{erf}\left(\dfrac{a}{\sqrt{s}}\right)$ |
| 43. | $f\left(\dfrac{t}{a}\right)$ | $aF(as)$ |
| 44. | $e^{bt/a} f\left(\dfrac{t}{a}\right)$ | $aF(as - b)$ |
| 45. | $\delta_\epsilon(t)$ | $\dfrac{e^{-\epsilon s}(1 - e^{-\epsilon s})}{\epsilon s}$ |
| 46. | $\delta(t - a)$ | $e^{-as}$ |
| 47. | $L_n(t)$ (Laguerre polynomial) | $\dfrac{1}{s}\left(\dfrac{s-1}{s}\right)^n$ |

**TABLE 3.1 (continued)**

| | $f(t)$ | $F(s) = \mathfrak{L}[f(t)](s)$ |
|---|---|---|
| 48. | $\frac{n!}{(2n)!\sqrt{\pi t}} H_{2n}(t)$ (Hermite polynomial) | $\frac{(1-s)^n}{s^{n+1/2}}$ |
| 49. | $\frac{-n!}{\sqrt{\pi}(2n+1)!} H_{2n+1}(t)$ (Hermite polynomial) | $\frac{(1-s)^n}{s^{n+3/2}}$ |
| 50. | triangular wave   | $\frac{1}{as^2}\left[\frac{1-e^{-as}}{1+e^{-as}}\right]\left(=\frac{1}{as^2}\tanh\left(\frac{as}{2}\right)\right)$ |
| 51. | square wave   | $\frac{1}{s}\tanh\left(\frac{as}{2}\right)$ |
| 52. | sawtooth wave   | $\frac{1}{as^2}-\frac{e^{-as}}{s(1-e^{-as})}$ |

## Operational Formulas

| $f(t)$ | $F(s)$ |
|---|---|
| $af(t)+bg(t)$ | $aF(s)+bG(s)$ |
| $f'(t)$ | $sF(s)-f(0+)$ |
| $f^{(n)}(t)$ | $s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$ |
| $\int_0^t f(\tau)\,d\tau$ | $\frac{1}{s}F(s)$ |
| $tf(t)$ | $-F'(s)$ |
| $t^nf(t)$ | $(-1)^nF^{(n)}(s)$ |
| $\frac{1}{t}f(t)$ | $\int_s^\infty F(\sigma)\,d\sigma$ |
| $e^{at}f(t)$ | $F(s-a)$ |
| $f(t-a)H(t-a)$ | $e^{-as}F(s)$ |
| $f(t+T)=f(t)$ (periodic) | $\frac{1}{1-e^{-Ts}}\int_0^T e^{-st}f(t)\,dt$ |

# Appendix B

# Tables of Integral Transforms

In this Appendix we provide a set of *short* tables of integral transforms of the functions that are either cited in the text or in most common use in mathematical, physical and engineering applications. In these tables no attempt is made to give complete lists of transforms. For exhaustive lists of integral transforms, the reader is referred to Erdélyi *et al.* (1954), Campbell and Foster (1948), Ditkin and Prudnikov (1965), Doetsch (1950-56, 1970), Marichev (1983), and Oberhettinger (1972, 1974).

## Table B-1. Fourier Transforms

| | $f(x)$ | $F(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(-ikx)f(x)dx$ |
|---|---|---|
| 1 | $\exp(-a\|x\|), \quad a>0$ | $\left(\sqrt{\frac{2}{\pi}}\right)a\left(a^2+k^2\right)^{-1}$ |
| 2 | $x\exp(-a\|x\|)$ | $\left(\sqrt{\frac{2}{\pi}}\right)(-2aik)\left(a^2+k^2\right)^{-2}$ |
| 3 | $\exp(-ax^2), \quad a>0$ | $\frac{1}{\sqrt{2a}}\exp\left(-\frac{k^2}{4a}\right)$ |
| 4 | $\left(x^2+a^2\right)^{-1}, \quad a>0$ | $\sqrt{\frac{\pi}{2}}\frac{\exp(-a\|k\|)}{a}$ |
| 5 | $x\left(x^2+a^2\right)^{-1}$ | $\sqrt{\frac{\pi}{2}}\left(\frac{ik}{2a}\right)\exp(-a\|k\|)$ |
| 6 | $\begin{cases} c, & a\le x\le b \\ 0, & \text{outside}\end{cases}$ | $\frac{ic}{\sqrt{2\pi}}\frac{1}{k}\left(e^{-ibk}-e^{-iak}\right)$ |
| 7 | $\|x\|\exp(-a\|x\|), \quad a>0$ | $\sqrt{\frac{2}{\pi}}\left(a^2-k^2\right)\left(a^2+k^2\right)^{-2}$ |
| 8 | $\frac{\sin ax}{x}$ | $\sqrt{\frac{\pi}{2}}H(a-\|k\|)$ |
| 9 | $\exp\{-x(a-i\omega)\}H(x)$ | $\frac{1}{\sqrt{2\pi}}\frac{i}{(\omega-k+ia)}$ |
| 10 | $\left(a^2-x^2\right)^{-\frac{1}{2}}H(a-\|x\|)$ | $\sqrt{\frac{\pi}{2}}J_0(ak)$ |
| 11 | $\frac{\sin\left[b\left(x^2+a^2\right)^{\frac{1}{2}}\right]}{\left(x^2+a^2\right)^{\frac{1}{2}}}$ | $\sqrt{\frac{\pi}{2}}J_0\left(a\sqrt{b^2-k^2}\right)H(b-\|k\|)$ |
| 12 | $\frac{\cos\left(b\sqrt{a^2-x^2}\right)}{\left(a^2-x^2\right)^{\frac{1}{2}}}H(a-\|x\|)$ | $\sqrt{\frac{\pi}{2}}J_0\left(a\sqrt{b^2+k^2}\right)$ |
| 13 | $e^{-ax}H(x), \quad a>0$ | $\frac{1}{\sqrt{2\pi}}(a-ik)\left(a^2+k^2\right)^{-1}$ |
| 14 | $\frac{1}{\sqrt{\|x\|}}\exp(-a\|x\|)$ | $\left(a^2+k^2\right)^{-\frac{1}{2}}\left[a+\left(a^2+k^2\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$ |
| 15 | $\delta(x)$ | $\frac{1}{\sqrt{2\pi}}$ |
| 16 | $\delta^{(n)}(x)$ | $\frac{1}{\sqrt{2\pi}}(ik)^n$ |
| 17 | $\delta(x-a)$ | $\frac{1}{\sqrt{2\pi}}\exp(-iak)$ |
| 18 | $\delta^{(n)}(x-a)$ | $\frac{1}{\sqrt{2\pi}}(ik)^n\exp(-iak)$ |
| 19 | $\exp(iax)$ | $\sqrt{2\pi}\,\delta(k-a)$ |

| | $f(x)$ | $F(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(-ikx)f(x)dx$ |
|---|---|---|
| 20 | $1$ | $\sqrt{2\pi}\ \delta(k)$ |
| 21 | $x$ | $\sqrt{2\pi}\ i\,\delta'(k)$ |
| 22 | $x^n$ | $\sqrt{2\pi}\ i^n\ \delta^{(n)}(k)$ |
| 23 | $H(x)$ | $\sqrt{\frac{\pi}{2}}\left[\frac{1}{i\pi k}+\delta(k)\right]$ |
| 24 | $H(x-a)$ | $\sqrt{\frac{\pi}{2}}\left[\frac{\exp(-ika)}{\pi ik}+\delta(k)\right]$ |
| 25 | $H(x)-H(-x)$ | $\sqrt{\frac{2}{\pi}}\left(-\frac{i}{k}\right)$ |
| 26 | $x^n\exp(iax)$ | $\sqrt{2\pi}\ i^n\ \delta^{(n)}(k-a)$ |
| 27 | $\lvert x\rvert^{-1}$ | $\frac{1}{\sqrt{2\pi}}(A-2\log\lvert k\rvert)$, $A$ is a constant |
| 28 | $\log(\lvert x\rvert)$ | $-\sqrt{\frac{\pi}{2}}\frac{1}{\lvert k\rvert}$ |
| 29 | $H(a-\lvert x\rvert)$ | $\sqrt{\frac{2}{\pi}}\left(\frac{\sin ak}{k}\right)$ |
| 30 | $\lvert x\rvert^{\alpha}$ ($\alpha$ not integer) | $\sqrt{\frac{2}{\pi}}\ \Gamma(\alpha+1)\ \lvert k\rvert^{-(1+\alpha)}$ $\times\cos\left[\frac{\pi}{2}(\alpha+1)\right]$ |
| 31 | $sgn\ x$ | $\sqrt{\frac{2}{\pi}}\frac{1}{(ik)}$ |
| 32 | $x^{-n-1}\ sgn\ x$ | $\frac{1}{\sqrt{2\pi}}\frac{(-ik)^n}{n!}(A-2\log\lvert k\rvert)$ |

| | $f(x)$ | $F(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(-ikx)f(x)dx$ |
|---|---|---|
| 33 | $\frac{1}{x}$ | $-i\sqrt{\frac{\pi}{2}}\ sgn\ k$ |
| 34 | $\frac{1}{x^n}$ | $-i\sqrt{\frac{\pi}{2}}\left[\frac{(-ik)^{n-1}}{(n-1)!}\ sgn\ k\right]$ |
| 35 | $x^n\exp(iax)$ | $\sqrt{2\pi}\ i^n\delta^{(n)}(k-a)$ |
| 36 | $x^{\alpha}H(x)$, $\alpha$ is not an integer | $\frac{\Gamma(\alpha+1)}{\sqrt{2\pi}}\lvert k\rvert^{-(\alpha+1)}$ $\times\exp\left[-\left(\frac{\pi i}{2}\right)(\alpha+1)\ sgn\ k\right]$ |
| 37 | $x^n\exp(iax)\ H(x)$ | $\sqrt{\frac{\pi}{2}}\left[\frac{n!}{i\pi(k-a)^{n+1}}+i^n\ \delta^{(n)}(k-a)\right]$ |
| 38 | $\exp(iax)\ H(x-b)$ | $\sqrt{\frac{\pi}{2}}\left[\frac{\exp[-ib(k-a)]}{i\pi(k-a)}+\delta(k-a)\right]$ |
| 39 | $\frac{1}{x-a}$ | $-i\sqrt{\frac{\pi}{2}}\exp(-iak)\,sgn\ k$ |
| 40 | $\frac{1}{(x-a)^n}$ | $-i\sqrt{\frac{\pi}{2}}\exp(-iak)\frac{(-ik)^{n-1}}{(n-1)!}\ sgn\ k$ |
| 41 | $\frac{e^{iax}}{(x-b)}$ | $i\sqrt{\frac{\pi}{2}}\exp[ib(a-k)][1-2H(k-a)]$ |
| 42 | $\frac{e^{iax}}{(x-b)^n}$ | $i\sqrt{\frac{\pi}{2}}[1-2H(k-a)]$ $\times\frac{\exp\{ib(a-k)\}}{(n-1)!}[-i(k-a)]^{n-1}$ |
| 43 | $\lvert x\rvert^{\alpha}\ sgn\ x$ ($\alpha$ not integer) | $\sqrt{\frac{2}{\pi}}\frac{(-i)\Gamma(\alpha+1)}{\lvert k\rvert^{\alpha+1}}\cos\left(\frac{\pi\alpha}{2}\right)sgn\,k$ |