EM 542 - Homework

Problem (18a)

A projectile is fired vertically upward with an initial velocity v_o at a latitude θ. Determine where it lands (i.e. where it crosses the xy plane immediately before striking).

Homework

(18-a)

Example A projectile is fired vertically upward with an initial velocity v_0 at a latitude θ. Determine where it lands (i.e. where it crosses the xy plane immediately before striking)

Solution:

Referring to Eqs.(1-85) :
$$\begin{cases} x_0 = 0 \; ; \; \dot{x}_0 = 0 \\ y_0 = 0 \; ; \; \dot{y}_0 = 0 \\ z_0 = 0 \; ; \; \dot{z}_0 = v_0 \end{cases}$$

$$\begin{cases} x = \frac{\omega g t^3}{3}\cos\theta - \omega t^2 v_0 \cos\theta \\ \quad = \omega t^2 \cos\theta\left(\frac{gt}{3} - v_0\right) \\ y = 0 \\ z = -\frac{1}{2}gt^2 + v_0 t \end{cases}$$

Upon crossing the xy plane, $z = 0 = -\frac{1}{2}gt^2 + v_0 t$

$$\therefore \; 0 = t\left(v_0 - \frac{1}{2}gt\right)$$

$$\therefore \; t = 0, \; \frac{2v_0}{g}$$

Therefore, $x = \omega \frac{4v_0^2}{g^2}\cos\theta\left[\frac{2v_0}{3} - v_0\right]$

$$\text{or} \quad x = -\frac{4}{3}\frac{\omega v_0^3 \cos\theta}{g^2} \quad (\therefore \text{Drift is westerly})$$

* Example for $v_0 = 1000\,\text{ft/sec}$, $\theta = 0$, ; $x \approx -94$ ft (westerly)

EMA 542

Home Work to be Handed In

6) A projectile is fired at latitude λ with an initial velocity vector $v_o = \dot{y}_o\vec{j} + \dot{z}_o\vec{k}$ and $x_o = y_o = z_o = \dot{x}_o = 0$. It is desired to fire the projectile at an angle $\alpha = tan^{-1}(\dot{z}_o / \dot{y}_o)$ so that it again crosses the same meridian plane just before it strikes the Earth (i.e., when $z = 0.0$).

a) Determine the required firing angle α in terms of the latitude λ.

b) For $\dot{y}_o$= 2,000 ft/sec, and a latitude of 40°, make a 3-D computer plot of the projectile's complete trajectory as seen by an observer on the Earth.

(b) A projectile is fired at latitude λ with $v_0 = \dot{y}_0 \vec{j} + \dot{z}_0 \vec{k}$ with $x_0 = y_0 = z_0 = \dot{x}_0 = 0$. It is desired to fire the projectile at an angle $\alpha = \tan^{-1}\left(\frac{\dot{z}_0}{\dot{y}_0}\right)$ so it crosses the same meridian plane just before striking the earth.

using eqn's 1-85...

$$x = \frac{\omega_e g t^3}{3}\cos\lambda + \omega_e t^2(\dot{y}_0 \sin\lambda - \dot{z}_0 \cos\lambda)$$

$$y = \dot{y}_0 t$$

$$z = -\frac{gt^2}{2} + \dot{z}_0 t$$

when it crosses the same meridian plane ($x = x_0$)

$$z = z_0 = 0 = -\frac{gt^2}{2} + \dot{z}_0 t \quad \text{(when it crosses xy plane)}$$

$$\therefore \quad t = \frac{2\dot{z}_0}{g}$$

$$y = \dot{y}_0\left(\frac{2\dot{z}_0}{g}\right) = \frac{2\dot{y}_0 \dot{z}_0}{g}$$

$$x = \frac{\omega_e g}{3}\left(\frac{8\dot{z}_0^3}{g^3}\right)\cos\lambda + \omega_e\left(\frac{4\dot{z}_0^2}{g^2}\right)(\dot{y}_0 \sin\lambda - \dot{z}_0\cos\lambda)$$

$$= \frac{8\omega_e}{3}\frac{\dot{z}_0^3}{g^2}\cos\lambda + \frac{4\omega_e}{g^2}\dot{z}_0^2\dot{y}_0 \sin\lambda - \frac{4\omega_e}{g^2}\dot{z}_0^3\cos\lambda$$

Prob 18 cont'd^2... 10/19/91

$$x = -\frac{4}{3}\frac{\omega_e \dot{z}_0^3 \cos\lambda}{g^2} + \frac{4\omega_e}{g^2}\dot{z}_0^2 \dot{y}_0 \sin\lambda = 0$$

$$+\frac{\dot{z}_0^3 \cos\lambda}{3} = \frac{\dot{z}_0^2 \dot{y}_0 \sin\lambda}{1}$$

$$\frac{\dot{z}_0 \cos\lambda}{3} = \dot{y}_0 \sin\lambda$$

$$\frac{\dot{z}_0}{\dot{y}_0} = \frac{(\sin\lambda)\,3}{(\cos\lambda)} = 3\tan\lambda$$

$$\therefore \quad \alpha = \tan^{-1} \dot{z}_0/\dot{y}_0$$

$$\alpha = \tan^{-1}(3\tan\lambda)$$

$$\therefore \boxed{\tan\alpha = 3\tan\lambda}$$

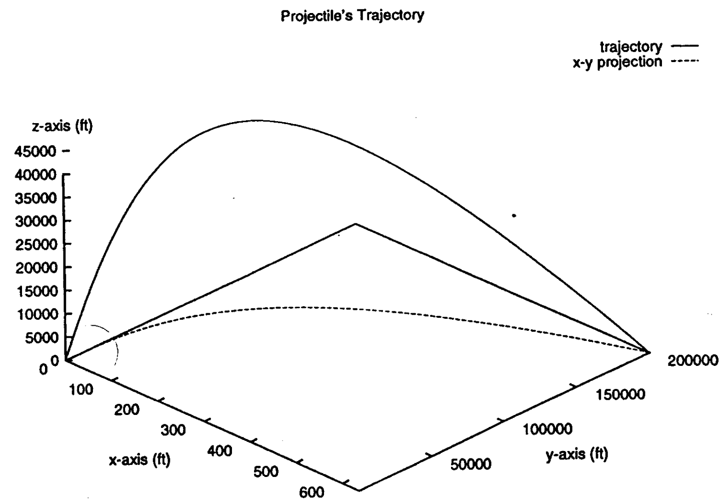

Figure 1: Particle Trajectory $y_{vel} = 2000ft/sec$, $z_{vel} = 1610ft/sec$

If we want the particle to hit on the same meridian plane and it is going to be fired from $\lambda = 40°$, then we have to fire the projectile at an angle of $\alpha = \tan^{-1}(3tan(\lambda))$. If we have a y velocity of $2000ft/sec$ then we need to have a z velocity of $5034ft/sec$. The following figure depicts this scenario.

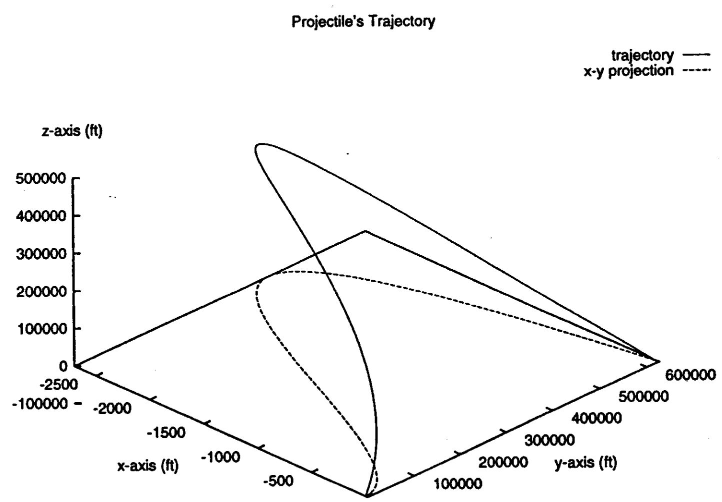

Figure 2: Particle Trajectory $y_{vel} = 2000ft/sec$,$z_{vel} = 5034ft/sec$