EMA 542

Home Work to be Handed In

5A) A device for simulating conditions in space allows rotations about three orthogonal axes as illustrated in the figure.
At this instant, the astronaut is moving as shown with a velocity $v_r = 5.0$ ft/sec and an acceleration $a_r = 32.0$ ft/sec^2, both relative to the capsule. Use the method of *multiple-rotating-coordinate* systems, with at least two rotating coordinate systems, to determine for the instant pictured:

(a) the inertial velocity of the astronaut's head;
(b) the inertial acceleration of the astronaut's head;

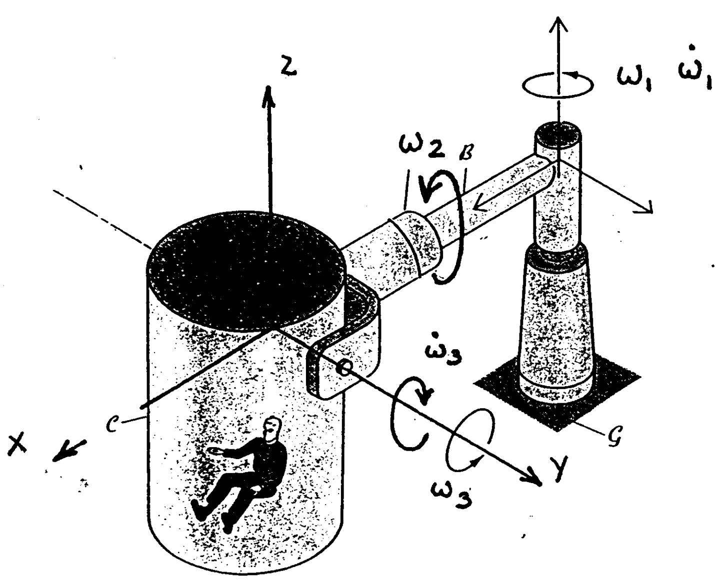
given the data in the figures.

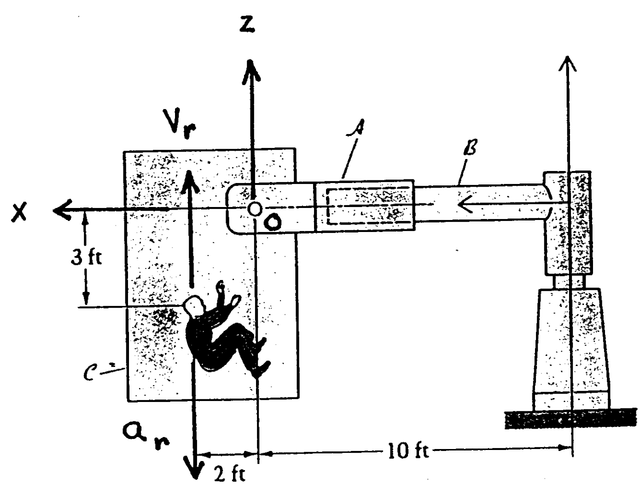
$v_r = 5.0\ ft/sec$ $\quad a_r = 32.0\ ft/sec^2$

$\omega_1 = 4.0\ rad/sec$ $\quad \dot{\omega}_1 = 3.0\ rad/sec^2$

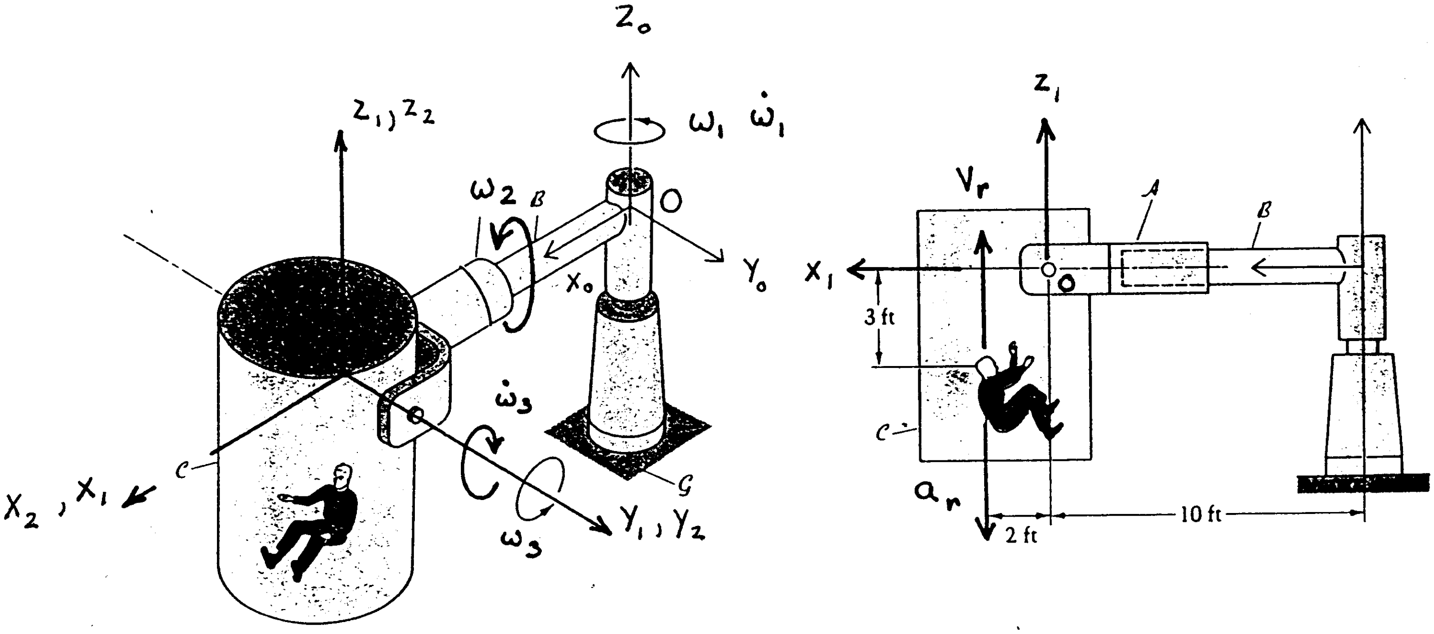
$\omega_2 = 5.0\ rad/sec$ $\quad \dot{\omega}_2 = 0.0\ rad/sec^2$

$\omega_3 = 6.0\ rad/sec$ $\quad \dot{\omega}_3 = 2.0\ rad/sec^2$





SOLUTION TO 5A - EMA 542



USE 2 ROTATING COORDINATE SYSTEMS, 1 & 2 AS SHOWN ABOVE.

$$\vec{\omega}_{1/0} = \omega_1 \bar{k} + \omega_2 \bar{i} = 4\bar{k} + 5\bar{i}$$

$$\vec{\omega}_{2/1} = \omega_3 \bar{j} = 6\bar{j}$$

$$\mathring{\vec{\omega}}_{1/0} = \dot{\omega}_1 \bar{k} + \omega_1 \bar{k} \times \omega_2 \bar{i} = 3\bar{k} + 20\bar{j}$$

$$\dot{\vec{\omega}}_{2/1} = \dot{\omega}_3 \bar{j} = -2\bar{j}$$

MOTION IN ① COORDINATE SYSTEM:

$$\vec{V}_1 = \mathring{\vec{R}}_2 + \vec{\omega}_{2/1} \times \vec{\rho}_2 + \dot{\vec{\rho}}_{2r} \qquad (i)$$

$$\dot{\vec{R}}_2 = 0 \qquad \vec{\rho}_2 = 2\bar{i} - 3\bar{k}$$

$$\vec{\omega}_{2/1} \times \vec{\rho}_2 = 6\bar{j} \times (2\bar{i} - 3\bar{k}) = -12\bar{k} - 18\bar{i}$$

$$\dot{\vec{\rho}}_{2r} = 5\bar{k}$$

$$\Rightarrow \vec{V}_1 = -18\bar{i} - 7\bar{k} \qquad (2)$$

$$\vec{a}_1 = \ddot{\vec{R}}_2 + \vec{\omega}_{2/1} \times (\vec{\omega}_{2/1} \times \vec{\rho}_2) + \dot{\vec{\omega}}_{2/1} \times \vec{\rho}_2$$

$$+ 2\,\vec{\omega}_{2/1} \times \dot{\vec{\rho}}_{2r} + \ddot{\vec{\rho}}_{2r}$$

$$\ddot{\vec{R}}_2 = 0 \qquad \vec{\omega}_{2/1} \times (\vec{\omega}_{2/1} \times \vec{\rho}_2) = 6\bar{j} \times (-12\bar{k} - 18\bar{i})$$

$$\therefore \vec{\omega}_{2/1} \times (\vec{\omega}_{2/1} \times \vec{\rho}_2) = -72\bar{i} + 108\bar{k} \qquad (3)$$

$$\dot{\vec{\omega}}_{2/1} \times \vec{\rho}_2 = -2\bar{j} \times (2\bar{i} - 3\bar{k}) = 4\bar{k} + 6\bar{i} \qquad (4)$$

$$2\,\vec{\omega}_{2/1} \times \dot{\vec{\rho}}_{2r} = 2(6\bar{j}) \times 5\bar{k} = 60\bar{i} \qquad (5)$$

$$\ddot{\vec{\rho}}_{2r} = -32\bar{k}$$

$$\therefore \vec{a}_1 = (-72 + 6 + 60)\bar{i} + (108 + 4 - 32)\bar{k}$$

$$\vec{a}_1 = -6\bar{i} + 80\bar{k} \qquad (6)$$

MOTION IN O OR FIXED COORDINATE SYSTEM:

$$\vec{V}_0 = \dot{\vec{R}}_1 + \vec{\omega}_{1/0} \times \vec{\rho}_1 + \dot{\vec{\rho}}_{1r} \qquad \vec{\rho}_1 = 2\bar{i} - 3\bar{k}$$

$$\dot{\vec{R}}_1 = \omega_1 (10)\bar{j} = 40\bar{j} \qquad (7)$$

$$\vec{\omega}_{1/0} \times \vec{\rho}_1 = (4\bar{k} + 5\bar{i}) \times (2\bar{i} - 3\bar{k})$$

$$= 8\bar{j} + 15\bar{j} = 23\bar{j} \qquad (8)$$

$$\dot{\vec{\rho}}_{1r} = \vec{v}_1 = -18\bar{i} - 7\bar{k}$$

$$\therefore \quad \underline{\vec{V}_0} = -18\bar{i} + 63\bar{j} - 7\bar{k} \qquad (9)$$

$$\vec{a}_0 = \ddot{\vec{R}}_1 + \vec{\omega}_{1/0} \times (\vec{\omega}_{1/0} \times \vec{\rho}_1) + \dot{\vec{\omega}}_{1/0} \times \vec{\rho}_1$$

$$+ 2\vec{\omega}_{1/0} \times \dot{\vec{\rho}}_{1r} + \ddot{\vec{\rho}}_{1r}$$

$$\ddot{\vec{R}}_1 = 10\dot{\omega}_1 \bar{j} - 10(\omega_1^2)\bar{i} = 30\bar{j} - 160\bar{i} \qquad (10)$$

$$\vec{\omega}_{1/0} \times (\vec{\omega}_{1/0} \times \vec{\rho}_1) = (4\bar{k} + 5\bar{i}) \times 23\bar{j} = -92\bar{i} + 115\bar{k} \qquad (11)$$

$$\dot{\vec{\omega}}_{1/0} \times \vec{\rho}_1 = (3\bar{k} + 20\bar{j}) \times (2\bar{i} - 3\bar{k})$$

$$\Rightarrow \quad \dot{\vec{\omega}}_{1/0} \times \vec{\rho}_1 = 6\bar{j} - 40\bar{k} - 60\bar{i} \qquad (12)$$

$$2\,\vec{\omega}_{1/0} \times \dot{\vec{\rho}}_{1r} = 2(4\bar{k} + 5\bar{i}) \times (-18\bar{i} - 7\bar{k})$$

$$= -144\bar{j} + 70\bar{j} = -74\bar{j} \qquad (13)$$

$$\ddot{\vec{\rho}}_{1r} = \vec{a}_1 = -6\bar{i} + 80\bar{k}$$

$$\therefore \quad \vec{a}_0 = (-160 - 92 - 60 - 6)\bar{i}$$

$$+ (30 + 6 - 74)\bar{j} + (115 - 40 + 80)\bar{k}$$

$$\Rightarrow \quad \underline{\vec{a}_0} = -318\bar{i} - 38\bar{j} + 155\bar{k} \qquad (14)$$

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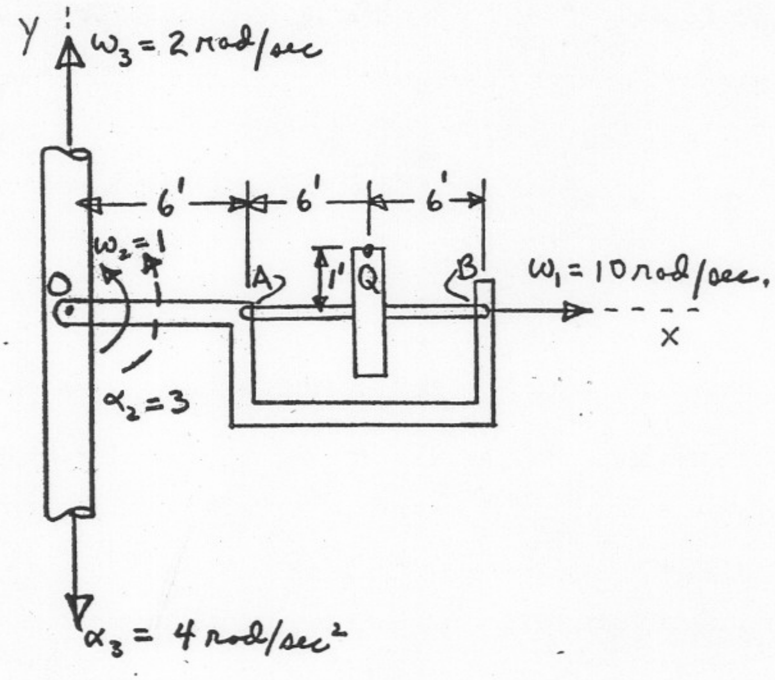
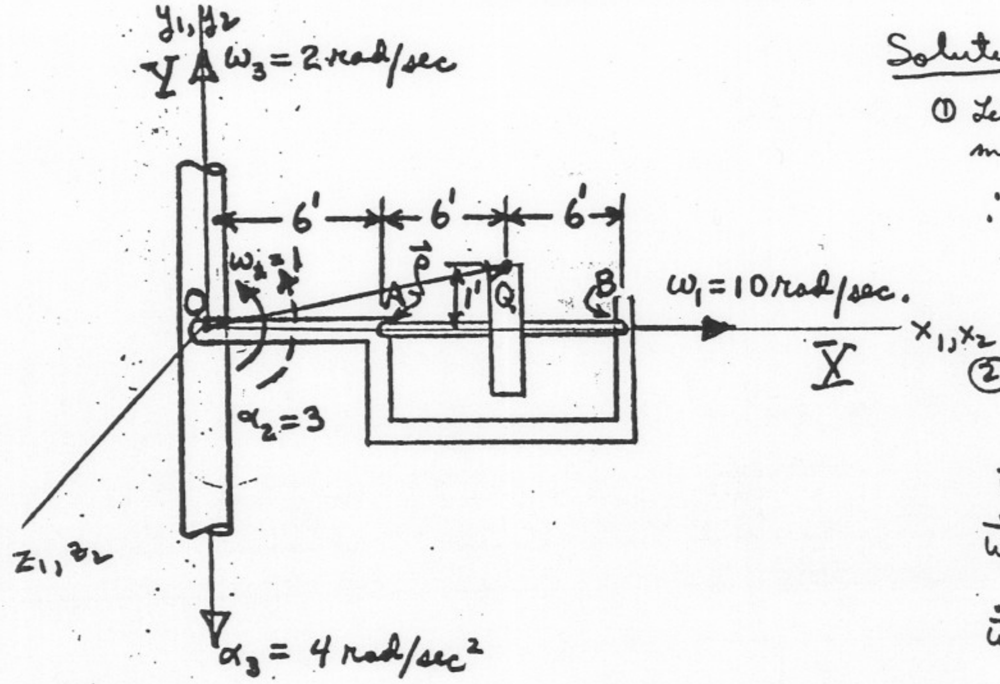
5B) The thin disc of radius 1 ft. rotates with a constant angular velocity $\omega_1 = 10$ rad/sec in bearings A and B. The weightless arm containing the bearings rotates about the fixed point O as shown with the angular velocity $\omega_2 = 1$ rad/sec and angular acceleration $\alpha_2 = 3$ rad/sec**2. The vertical shaft CD rotates as shown with an angular velocity $\omega_3 = 2$rad/sec and and angular acceleration $\alpha_3 = 4$ rad/sec**2. Calculate the absolute velocity and acceleration of point Q at the top of the disk for the position shown.





Solution

① Let axes x_1, y_1, z_1 be attached and moving with the vertical shaft

$\therefore \ \vec{\omega}_{1/XYZ} = 2\vec{j}$

$\dot{\vec{\omega}}_{1/XYZ} = -4\vec{j}$

② Let axes $x_2 y_2 z_2$ be attached to the horizontal arm so that relative to the frame $x_1 y_1$:

$\vec{\omega}_{2/1} = \vec{k}$

$\dot{\vec{\omega}}_{2/1} = 3\vec{k}$

Ⓐ $\therefore \ \vec{v}_Q = \dot{\vec{R}}_{1/0} + \vec{\omega}_{1/XYZ} \times \vec{\rho}_{P/1} + (\dot{\vec{\rho}}_{P/1})_r$

where $\dot{\vec{R}}_{1/0} = 0$

$\vec{\rho}_{P/1} = 12\vec{i} + \vec{j}$

$\vec{\omega}_1 \times \vec{\rho}_{P/1} = -24\vec{k}$

$(\dot{\vec{\rho}}_{P/1})_r = \dot{\vec{R}}_{2/1} + \vec{\omega}_{2/1} \times \vec{\rho}_{P/2} + (\dot{\vec{\rho}}_{P/2})_r = -\vec{i} + 12\vec{j} + 10\vec{k}$

where $\dot{\vec{R}}_{2/1} = 0$

$\vec{\rho}_{P/1} = \vec{\rho}_{P/1} = 12\vec{i} + \vec{j}$

$\vec{\omega}_{2/1} \times \vec{\rho}_{P/2} = +12\vec{j} - \vec{i}$

$(\dot{\vec{\rho}}_{P/2})_r = 10\vec{k}$

$\therefore \ \boxed{\vec{v}_Q = -\vec{i} + 12\vec{j} - 14\vec{k}}$

Let $\vec{\omega}_{1/xyz} = \vec{\omega}_{1/0}$; $\dot{\vec{\omega}}_{1/xyz} = \dot{\vec{\omega}}_{1/0}$ to simplify notation

(B) $$\vec{a}_Q = \ddot{\vec{R}}_{1/0} + \vec{\omega}_{1/0} \times (\vec{\omega}_{1/0} \times \vec{\rho}_{P/1}) + \dot{\vec{\omega}}_{1/0} \times \vec{\rho}_{P/1} + (\ddot{\vec{\rho}}_{P/1})_r + 2\vec{\omega}_{1/0} \times (\dot{\vec{\rho}}_{P/1})_r$$

<u>where</u> $\underline{\underline{\ddot{\vec{R}}_{1/0} = 0}}$

$$\underline{\underline{\vec{\omega}_{1/0} \times (\vec{\omega}_{1/0} \times \vec{\rho}_1) = -48\vec{i}}}$$

$$\underline{\underline{\dot{\vec{\omega}}_{1/0} \times \vec{\rho}_1 = +48\vec{k}}}$$

$$2\vec{\omega}_{1/0} \times \dot{\vec{\rho}}_{P/1} = 2(2\vec{j}) \times (-\vec{i} + 12\vec{j} + 10\vec{k}) = \underline{\underline{4\vec{k} + 40\vec{i}}}$$

$$(\ddot{\vec{\rho}}_{P/1})_r = \ddot{\vec{R}}_{2/1} + (\ddot{\vec{\rho}}_{P/2})_r + \vec{\omega}_{2/1} \times (\vec{\omega}_{2/1} \times \vec{\rho}_{P/2}) + \dot{\vec{\omega}}_{2/1} \times \vec{\rho}_{P/2} + 2\vec{\omega}_{2/1} \times (\dot{\vec{\rho}}_{P/2})_r$$

where: $\ddot{\vec{R}}_{2/1} = 0$

$$\vec{\omega}_{2/1} \times (\vec{\omega}_{2/1} \times \vec{\rho}_{P/2}) = -12\vec{i} - \vec{j}$$

$$\dot{\vec{\omega}}_{2/1} \times \vec{\rho}_{P/2} = 36\vec{j} - 3\vec{i}$$

$$(\ddot{\vec{\rho}}_{P/2})_r = -100\vec{j}$$

$$2\vec{\omega}_{2/1} \times (\dot{\vec{\rho}}_{P/2})_r = 2(3\vec{k}) \times (10\vec{k}) = 0$$

$$\underline{\underline{= -15\vec{i} - 65\vec{j}}}$$

$$\therefore \boxed{\vec{a}_Q = -23\vec{i} - 65\vec{j} + 52\vec{k}}$$