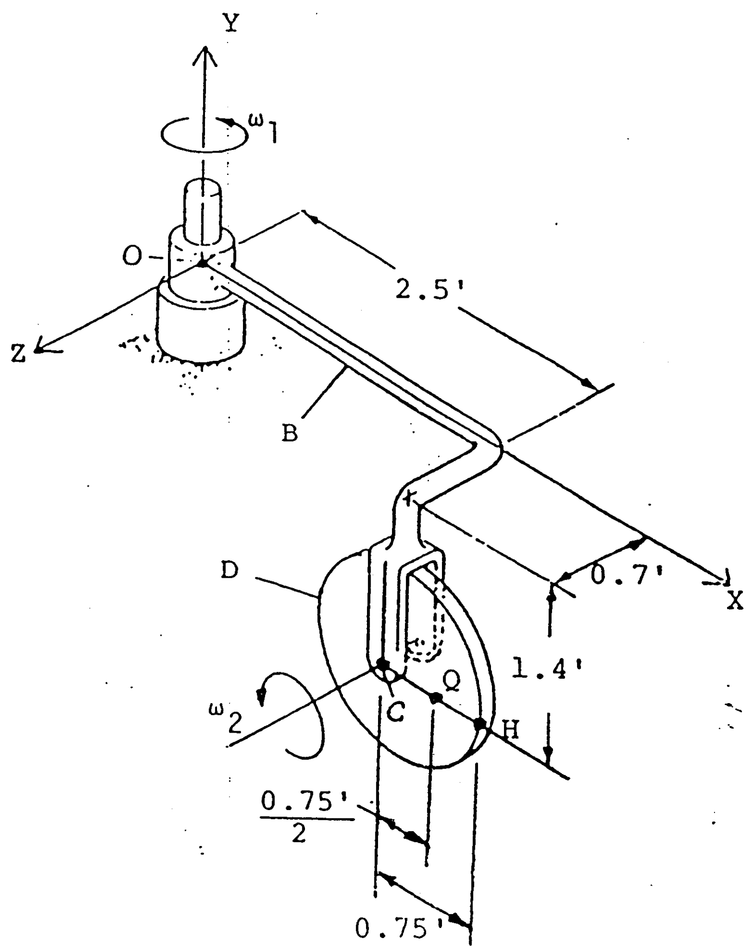
EMA 542

Hwk.

(16)

A disk D of radius 0.75 ft spins with an angular speed $\omega_2 = 0.5$ r/s with respect to the rigid but bent bar B. The angular speed ω_2 is increasing at a rate $\dot{\omega}_2 = 0.25$ r/s^2. Body B turns about a vertical axis through O at a rate $\omega_1 = 1.2$ r/s which is increasing at a rate $\dot{\omega}_1 = 0.6$ r/s^2. A fly is moving on the surface of the disk D from point C to H, at a rate of 1.5 ft/sec which is increasing at a rate of 0.8 ft/sec^2. Determine the absolute velocity and acceleration of the fly when the fly is at point Q.



SOLUTION TO PROBLEM 16

Velocity

$$\vec{V}_Q = \dot{\vec{R}} + \vec{\omega} \times \vec{\rho} + \dot{\vec{\rho}}_r \qquad \vec{\omega} = \vec{\omega}_1$$

where $\dot{\vec{R}} = \vec{V}_c = \vec{\omega}_1 \times \vec{r}_c$

$$\therefore \dot{\vec{R}} = 1.2\bar{j} \times [2.5\bar{i} - 1.4\bar{j} + .7\bar{k}]$$

$$\Rightarrow \underline{\underline{\dot{\vec{R}}}} = .84\bar{i} - 3\vec{k}$$

$$\vec{\rho} = \tfrac{1}{2}(.75)\bar{i}$$

$$\dot{\vec{\rho}}_r = \dot{\rho}_r \vec{e}_\rho + \vec{\omega}_2 \times \vec{\rho}$$

$$\dot{\vec{\rho}}_r = 1.5\bar{i} + .5\bar{k} \times \tfrac{1}{2}(.75)\bar{i}$$

$$\therefore \underline{\underline{\dot{\vec{\rho}}_r}} = 1.5\bar{i} + .1875\bar{j}$$

$$\vec{\omega} \times \vec{\rho} = \vec{\omega}_1 \times \vec{\rho} = 1.2\bar{j} \times \tfrac{1}{2}(.75)\bar{i}$$

$$\therefore \underline{\underline{\vec{\omega} \times \vec{\rho}}} = -.45\bar{k}$$

$$\therefore \boxed{\vec{V}_Q = 2.34\bar{i} + .1875\bar{j} - 3.45\bar{k}}$$

ACCELERATION

$$\vec{a}_G = \ddot{\vec{R}} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \dot{\vec{\omega}} \times \vec{\rho} + \ddot{\vec{\rho}}_r + 2\vec{\omega} \times \dot{\vec{\rho}}_r$$

$$\ddot{\vec{R}} = \vec{a}_c = \dot{\vec{\omega}}_1 \times \vec{r}_c + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_c)$$

$$= .6\bar{j} \times [2.5\bar{i} - 1.4\bar{j} + .7\bar{k}] + (1.2\bar{j}) \times [.84\bar{i} - 3\bar{k}]$$

$$= -1.5\bar{k} + .42\bar{i} - 1.008\bar{k} - 3.6\bar{i}$$

$$\therefore \underline{\underline{\ddot{\vec{R}}}} = -3.18\bar{i} - 2.508\bar{k}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{\rho}) = \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{\rho}) = 1.2\bar{j} \times [-.45\bar{k}]$$

$$\therefore \underline{\underline{\vec{\omega} \times (\vec{\omega} \times \vec{\rho})}} = -.54\bar{i}$$

$$\dot{\vec{\omega}} \times \vec{\rho} = \dot{\vec{\omega}}_1 \times \vec{\rho} = .6\bar{j} \times \tfrac{1}{2}(.75)\bar{i}$$

$$\therefore \underline{\underline{\dot{\vec{\omega}} \times \vec{\rho}}} = -.225\bar{k}$$

$$\ddot{\vec{\rho}}_r = \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{\rho}) + \dot{\vec{\omega}}_2 \times \vec{\rho} + \ddot{\vec{\rho}}_{rr} + 2\vec{\omega}_2 \times \dot{\vec{\rho}}_{rr}$$

$$=$$

$$\ddot{\vec{\rho}}_r = .5\vec{k} \times (.1875\bar{j}) + .25\vec{k} \times \bar{j}(.75)\bar{i}$$

$$+ .8\bar{i} + 2(.5)\vec{k} \times 1.5\bar{i}$$

$$= -.0938\bar{i} + .0938\bar{j} + .8\bar{i} + 1.5\bar{j}$$

$$\therefore \underline{\underline{\ddot{\vec{\rho}}_r}} = .7062\bar{i} + 1.5938\bar{j}$$

$$2\vec{\omega} \times \dot{\vec{\rho}}_r = 2(1.2)\bar{j} \times \left[1.5\bar{i} + .1875\bar{j}\right]$$

$$\underline{\underline{2\vec{\omega} \times \dot{\vec{\rho}}_r}} = -3.6\bar{k}$$

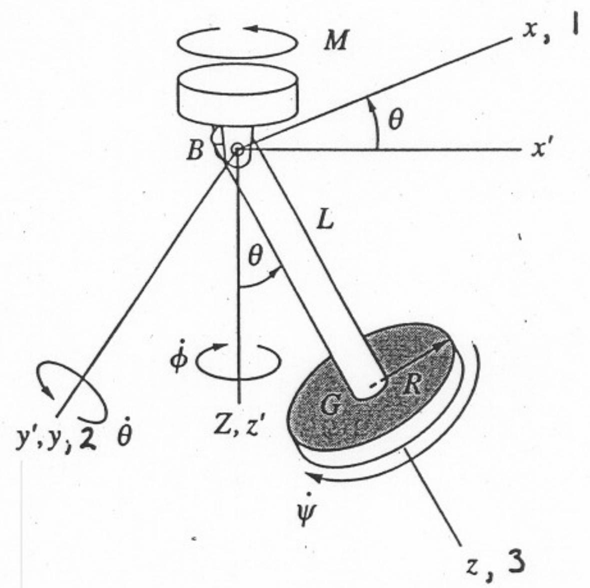
$$\therefore \vec{a}_G = \left[-3.18 - .54 + .7062\right]\bar{i}$$

$$+ \left[1.5938\right]\bar{j} + \left[-2.508 - .225 - 3.6\right]\bar{k}$$

$$\therefore \boxed{\vec{a}_G = -3.0138\bar{i} + 1.5938\bar{j} - 6.333\bar{k}}$$

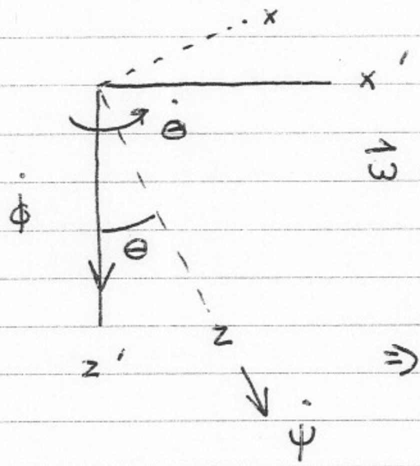
EMA 542 - Homework to Hand In

3B. A gyropendulum, consisting of a disk of radius R, rotates with a constant spin rate $\dot{\psi}$ about the shaft BG of length L. The shaft is pivoted to another vertical shaft at B which rotates with the constant rate $\dot{\phi}$. The pivot, angle θ changes at the constant rate $\dot{\theta}$ as shown. The Z coordinate axis is fixed in space. The xyz coordinate system is attached to the shaft BG. The 123 coordinate system is attached to the disk. At the instant shown, 123 is aligned with xyz. Compute the total angular velocity and angular acceleration of the disk and express them in terms of the 123 body coordinates. Your solution should be in terms of ψ, θ, ϕ and their corresponding time derivatives.



SOLUTION TO 1

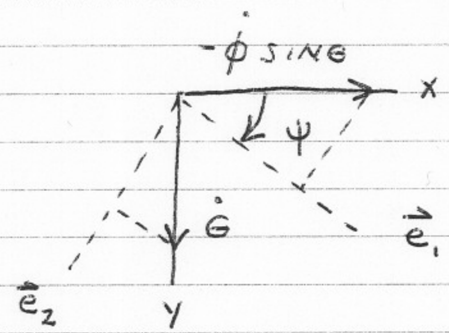
FIRST FORMULATE IN XYZ COORDINATES



$$\vec{\omega} = \dot\phi \cos\theta\, \bar{k} - \dot\phi \sin\theta\, \bar{i} + \dot\theta\, \bar{j} + \dot\psi\, \bar{k}$$

$$\Rightarrow \vec{\omega} = -\dot\phi \sin\theta\, \bar{i} + \dot\theta\, \bar{j} + (\dot\psi + \dot\phi\cos\theta)\,\bar{k}$$

TRANSFORM TO BODY COORDS. 123:



$$\Rightarrow \vec{\omega} = -\dot\phi \sin\theta \cos\psi\, \vec{e}_1 + \dot\phi \sin\theta \sin\psi\, \vec{e}_2 + \dot\theta \sin\psi\, \vec{e}_1 + \dot\theta \cos\psi\, \vec{e}_2 + (\dot\psi + \dot\phi\cos\theta)\,\vec{e}_3$$

$$\Rightarrow \vec{\omega} = (\dot\theta \sin\psi - \dot\phi \sin\theta\cos\psi)\,\vec{e}_1 + (\dot\phi\sin\theta\sin\psi + \dot\theta\cos\psi)\,\vec{e}_2 + (\dot\psi + \dot\phi\cos\theta)\,\vec{e}_3$$

ASSUME EULER RATES ARE CONSTANT

COMPUTE ANGULAR ACCELERATION AND EXPRESS IN BODY COORDINATES

TIME DERIVATIVE CAN BE COMPUTED IN INERTIAL COORDINATES OR BODY COORDINATES

TAKE TIME DERIVATIVE IN BODY COORDINATES

$$\Rightarrow \quad \vec{\alpha} = \dot{\vec{\omega}} = \left(\dot{\theta}\dot{\psi}\cos\psi - \dot{\phi}\dot{\theta}\underbrace{\cos\theta}_{\cos\psi} + \dot{\phi}\dot{\psi}\sin\theta\sin\psi\right)\vec{e}_1$$

$$+ \left(\dot{\phi}\dot{\theta}\cos\theta\sin\psi + \dot{\phi}\dot{\psi}\sin\theta\cos\psi - \dot{\theta}\dot{\psi}\sin\psi\right)\vec{e}_2$$

$$- \dot{\phi}\dot{\theta}\sin\theta\,\vec{e}_3$$