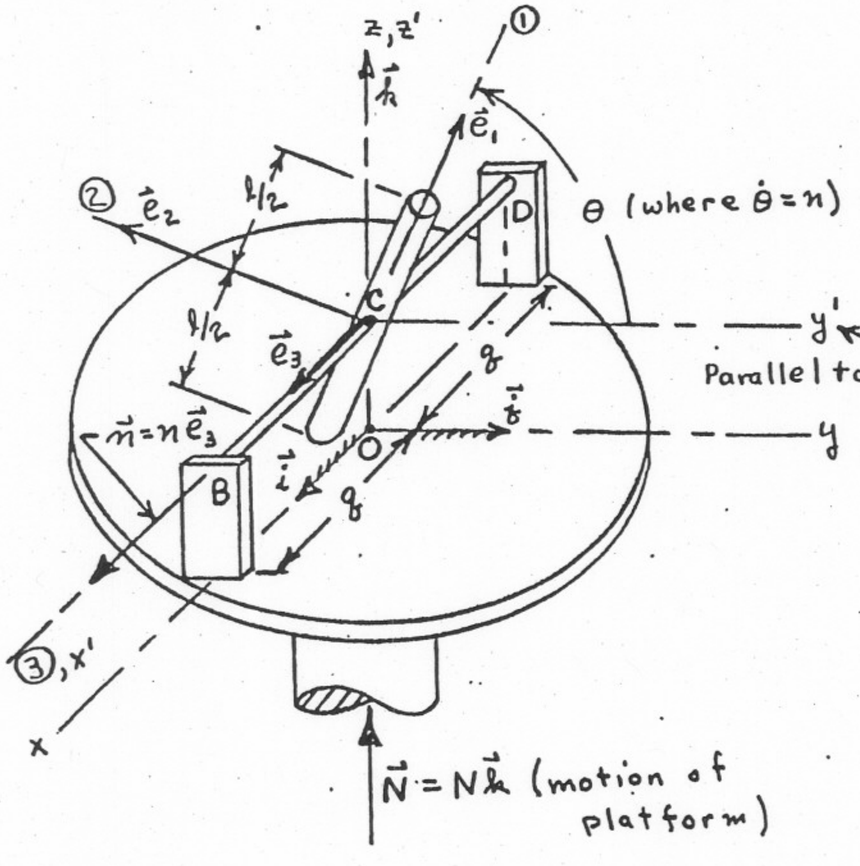
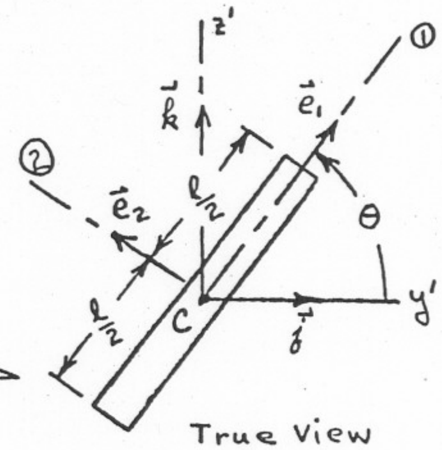
EM 542

Turntable A rotates at constant angular velocity N about the vertical z axis and the x, y, z axes are attached to the turntable. The slender rod of mass m and length ℓ is forced to rotate at constant angular velocity n about axis 3 relative to the platform. [a] Determine the resultant moment $\vec{M}_C$ that must be applied to the system at point C in order to sustain this motion. Give your answer in terms of components along axes x', y', z' (i.e., $\vec{M}_C = M_{x'}\vec{i} + M_{y'}\vec{j} + M_{z'}\vec{k}$). [b] Determine the vertical components of the bearing reactions acting on the shaft at B and D and clearly show the direction of your answers on the sketch below.





True View of rod

Solution

(a)

(6) $\begin{cases} \omega_1 = N\sin\theta \\ \omega_2 = N\cos\theta \end{cases}$ (4) $\omega_3 = n$ | (6) $\begin{cases} \dot\omega_1 = Nn\cos\theta \\ \dot\omega_2 = -Nn\sin\theta \end{cases}$ $\dot\omega_3 = 0$ (3) | $I_1 = 0$ (3) $I_2 = \frac{1}{12}ml^2$ (3) $I_3 = \frac{1}{12}ml^2$ (3)

a) M_1 (5) $= I_1\dot\omega_1 + \omega_2\omega_3(I_3 - I_2) = 0$

b) M_2 (5) $= I_2\dot\omega_2 + \omega_1\omega_3(I_1 - I_3) = -\frac{1}{12}ml^2 Nn\sin\theta + Nn\sin\theta\left(-\frac{1}{12}ml^2\right)$

$\therefore\ M_2 = -\frac{1}{6}ml^2 Nn\sin\theta$

c) M_3 (5) $= I_3\dot\omega_3 + \omega_1\omega_2(I_2 - I_1) = N^2\sin\theta\cos\theta\left(\frac{1}{12}ml^2\right)$

$\therefore\ M_3 = \frac{1}{12}ml^2 N^2\sin\theta\cos\theta$

(5) $M_x = M_3 = \frac{1}{12}ml^2N^2\sin\theta\cos\theta$

(6) $M_y = M_1\cos\theta - M_2\sin\theta = +\frac{1}{6}ml^2 Nn\sin^2\theta$

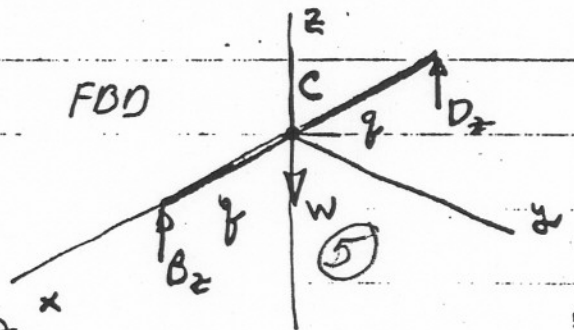
(6) $M_z = M_1\sin\theta + M_2\cos\theta = -\frac{1}{6}ml^2 Nn\sin\theta\cos\theta$

(b)

a) $B_z + D_z \overset{-Mg}{=} 0$ (5)

$B_z = -D_z + Mg$

FBD



$D_z(q) - B_z(q) = M_y = \frac{1}{6}ml^2 Nn\sin^2\theta$ (6)

$\therefore D_z(2q) = \frac{1}{6}ml^2Nn\sin^2\theta + Mgq$

$D_z = -\underset{-Mg}{B_z} = \frac{1}{12}\frac{ml^2}{q}Nn\sin^2\theta + \frac{1}{2}Mg$ (2)

$B_z = \frac{1}{2}Mg - \frac{1}{12}M\frac{l^2}{q}Nn\sin^2\theta$ (2)

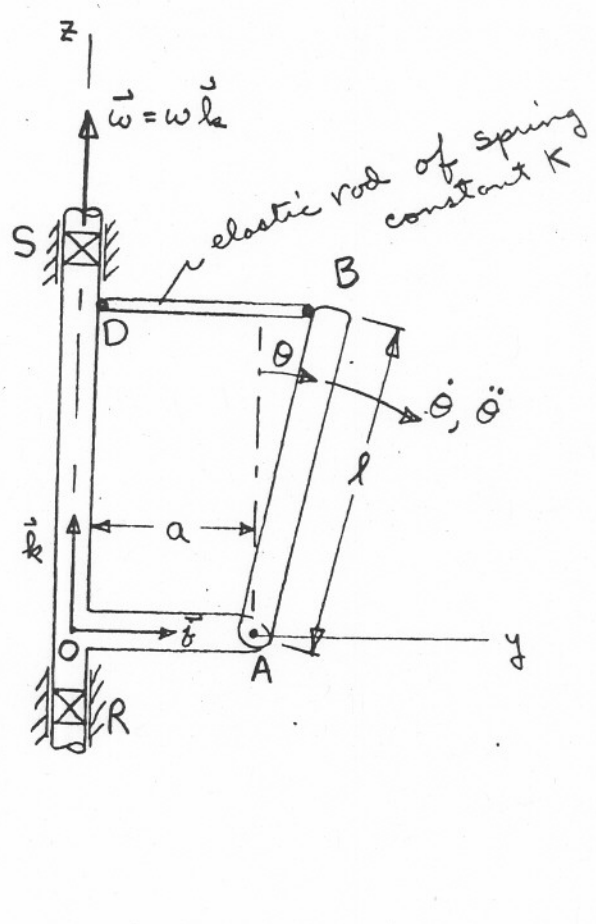
EMA 542

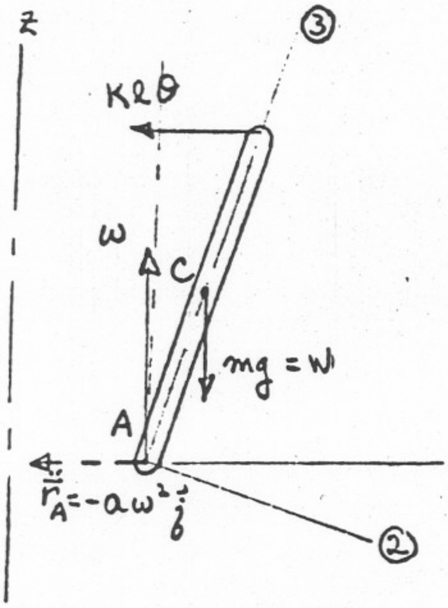
Home Work to be Handed In

15) Frame SRA rotates at a constant angular velocity $\vec{\omega}$ about the vertical z axis. Bar AB of total mass m and length l is hinged to the frame at A by a bearing which allows it to rotate in the *SRA* plane at an anglar velocity $\dot{\theta}$,and an angular acceleration $\ddot{\theta}$ relative to the *SRA* frame. The motion of the bar *AB* is restrained by a massless, elastic rod *DB* which has an unstretched length a and a spring constant $K = AE/a$.

a. Determine the complete rotational equation of motion of bar *AB* as it vibrates through small angles θ about point *A* by using the relative angular momentum method and rigid body moments of inertia.

b. Determine the resultant moments exerted by bearing *A* on bar *AB*.





$$\omega_1 = -\dot{\theta} \qquad \dot{\omega}_1 = -\ddot{\theta} \qquad I_1 = \tfrac{1}{3} m l^2$$
$$\omega_2 = -\omega \sin\theta \qquad \dot{\omega}_2 = -\omega\dot{\theta}\cos\theta \qquad I_2 = \tfrac{1}{3} m l^2$$
$$\omega_3 = \omega\cos\theta \qquad \dot{\omega}_3 = -\omega\dot{\theta}\sin\theta \qquad I_3 = 0$$

[Must Use Eqs. (k) since point A is a moving point]

$$\therefore M_1 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2) + m\left\{0 - \left(\tfrac{l}{2}\right)(-a\omega^2\cos\theta)\right\}$$
$$= -\tfrac{1}{3} m l^2 \ddot{\theta} + (-\omega\sin\theta)(\omega\cos\theta)\left[0 - \tfrac{1}{3} m l^2\right] + m\tfrac{l}{2} a\omega^2\cos\theta$$

$$\therefore M_1 = -\tfrac{1}{3} m l^2\ddot{\theta} + \tfrac{1}{3} m l^2\omega^2\sin\theta\cos\theta + \tfrac{1}{2} m l a\omega^2\cos\theta$$

Since the external moments about axis ① for small angles are given by

$$M_1 \approx + K l^2\theta - mg\tfrac{l}{2}\theta$$

we have for small angles:

$$K l^2\theta - mg\tfrac{l}{2}\theta = -\tfrac{1}{3} m l^2\ddot{\theta} + \tfrac{1}{3} m l^2\omega^2\theta + \tfrac{1}{2} m l a\omega^2$$

$$\therefore \tfrac{1}{3} m l\ddot{\theta} + \left[Kl - \tfrac{mg}{2} - \tfrac{1}{3} m l\omega^2\right]\theta = \tfrac{1}{2} m a\omega^2 \quad \checkmark$$

$$M_2 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3) + m(0-0)$$
$$= -\tfrac{1}{3}ml^2\omega\dot{\theta}\cos\theta + (-\dot{\theta})(\omega\cos\theta)(\tfrac{1}{3}ml^2 - 0)$$

$$M_2 = -\tfrac{2}{3}ml^2\omega\dot{\theta}\cos\theta$$

$$M_3 = 0 = I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) + m(0-0)$$

$y_c = 0$ for axis ②

$x_c = 0$ for axis ①