Single tone FM modulation

$$x_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t .$$

↓

$$S_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[ \delta(f - (f_c + nf_m)) + \delta(f + (f_c + nf_m)) \right]$$

for one sided

$$= A_c \sum_{n=0}^{\infty} J_n(\beta) \left[ \delta(f - (f_c + nf_m)) - \cdots \right]$$

$$(BT)_{Carson} = 2(f_m + \Delta f)$$

$$= 2f_m(\beta + 1)$$

$\beta \to f_m$

$$(BT)_{\%} = 2\, n_{max} f_m$$

$$\text{Power in Carrier} = \frac{A_c^2}{2} J_0^2(\beta)$$

$$P_{Total} = \frac{A_c^2}{2} \Big[ \underbrace{J_0^2(\beta) + 2J_1^2(\beta) + \cdots}_{=1} \Big]$$

$J_{-n}(\beta) = J_n(\beta)$ n even

$J_{-n}(\beta) = -J_n(\beta)$ n odd.

## single tone

$$FM(t) = A_c \cos\Big(\overbrace{\omega_c t + \underbrace{\frac{k_f A_m}{\omega_m} \sin(\omega_m t)}_{\phi(t)}}^{\theta_i(t)}\Big)$$

$\phi(t)$ (phase deviation)

freq deviation $\frac{d\phi(t)}{dt} = k_f\, m(t)$

so $\Delta\omega$ = max freq. deviation = $\boxed{k_f A_m}$

$$\boxed{\beta = \frac{k_f A_m}{\omega_m}} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

$k_f = 2\pi f_d$ — $f_d$ called deviation constant

$$\boxed{\beta = \frac{f_d A_m}{f_m}}$$

## HW7

(1) $m_p = 16V$.
$x = -8.7V$.

(a) $N = 8$ bits. Find ~~offset~~ offset binary code.

$\Delta = \frac{16}{2^7} = \boxed{0.125}$

quantization level $= \left(\frac{Abs(x)}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$

Since $x$ is negative, then Code $= 2^7 - 70 = 128 - 70 = \boxed{58}$

in binary this is $\boxed{0011\ 1010}$

(b) sign/magnitude..

since $x < 0$ then Code $= 2^7 + 70 = 128 + 70 = 198$

which in binary is $\boxed{1100\ 0110}$

(c) 2's complement.

since $x < -\frac{\Delta}{2}$ then Code. $= 2^8 - 70 = 256 - 70$
$= 186$

which in binary is $\boxed{1011\ 1010}$

(d) 1's complement.

since $x < 0$ Then

code $(2^8 - 1) - 70 = 255 - 70 = 185$

which in binary is $\boxed{1011\ 1001}$

extra: to illustrate this more, this is the Calculations assuming $x = +8.7V$.

offset binary

$$\Delta = \frac{16}{2^7} = 0.125$$

$$\text{Level} = \text{round}\left(\frac{8.7}{\Delta}\right) = 69.6 \rightarrow \boxed{70}$$

since $x > 0$ then code $(70)_2 = 0100\ 0110$

sign magnitude

since $x > 0$ then code $(70)_2 = 0100\ 0110$

2's Complement

since $x > 0$ then code $= (70)_2 = 0100\ 0110$

1's Complement

since $x > 0$, then code $= (70)_2 = 0100\ 0110$.

---

$$SNR = \frac{\overline{m^2(t)}}{\text{noise Pow}} = \frac{E(m^2)}{\frac{m_p^2}{(3)(2^{2N})}} \qquad \int_{-m_p}^{m_p} m^2 f_X(x)\,dx$$

use diagp f (Nois

Handout 7/6/2010. ECE 405

| n | β = 11.0000 | 12.0000 | 13.0000 | 14.0000 | 15.0000 |
|---|---|---|---|---|---|
| 0 | -0.1712 | 0.0477 | 0.2069 | 0.1711 | -0.0142 |
| 1.0000 | -0.1768 | -0.2234 | -0.0703 | 0.1334 | 0.2051 |
| 2.0000 | 0.1390 | -0.0849 | -0.2177 | -0.1520 | 0.0416 |
| 3.0000 | 0.2273 | 0.1951 | 0.0033 | -0.1768 | -0.1940 |
| 4.0000 | -0.0150 | 0.1825 | 0.2193 | 0.0762 | -0.1192 |
| 5.0000 | -0.2383 | -0.0735 | 0.1316 | 0.2204 | 0.1305 |
| 6.0000 | -0.2016 | -0.2437 | -0.1180 | 0.0812 | 0.2061 |
| 7.0000 | 0.0184 | -0.1703 | -0.2406 | -0.1508 | 0.0345 |
| 8.0000 | 0.2250 | 0.0451 | -0.1410 | -0.2320 | -0.1740 |
| 9.0000 | 0.3089 | 0.2304 | 0.0670 | -0.1143 | -0.2200 |
| 10.0000 | 0.2804 | 0.3005 | 0.2338 | 0.0850 | -0.0901 |
| 11.0000 | 0.2010 | 0.2704 | 0.2927 | 0.2357 | 0.1000 |
| 12.0000 | 0.1216 | 0.1953 | 0.2615 | 0.2855 | 0.2367 |
| 13.0000 | 0.0643 | 0.1201 | 0.1901 | 0.2536 | 0.2787 |
| 14.0000 | 0.0304 | 0.0650 | 0.1188 | 0.1855 | 0.2464 |
| 15.0000 | 0.0130 | 0.0316 | 0.0656 | 0.1174 | 0.1813 |
| 16.0000 | 0.0051 | 0.0140 | 0.0327 | 0.0661 | 0.1162 |
| 17.0000 | 0.0019 | 0.0057 | 0.0149 | 0.0337 | 0.0665 |
| 18.0000 | 0.0006 | 0.0022 | 0.0063 | 0.0158 | 0.0346 |
| 19.0000 | 0.0002 | 0.0008 | 0.0025 | 0.0068 | 0.0166 |
| 20.0000 | 0.0001 | 0.0003 | 0.0009 | 0.0028 | 0.0074 |
| 21.0000 | 0.0000 | 0.0001 | 0.0003 | 0.0010 | 0.0031 |
| 22.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0004 | 0.0012 |
| 23.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0004 |
| 24.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 |
| 25.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |

| n | β = 16.0000 | 17.0000 | 18.0000 | 19.0000 | 20.0000 |
|---|---|---|---|---|---|
| 0 | -0.1749 | -0.1699 | -0.0134 | 0.1466 | 0.1670 |
| 1.0000 | 0.0904 | -0.0977 | -0.1880 | -0.1057 | 0.0668 |
| 2.0000 | 0.1862 | 0.1584 | -0.0075 | -0.1578 | -0.1603 |
| 3.0000 | -0.0438 | 0.1349 | 0.1863 | 0.0725 | -0.0989 |
| 4.0000 | -0.2026 | -0.1107 | 0.0696 | 0.1806 | 0.1307 |
| 5.0000 | -0.0575 | -0.1870 | -0.1554 | 0.0036 | 0.1512 |
| 6.0000 | 0.1667 | 0.0007 | -0.1560 | -0.1788 | -0.0551 |
| 7.0000 | 0.1825 | 0.1875 | 0.0514 | -0.1165 | -0.1842 |
| 8.0000 | -0.0070 | 0.1537 | 0.1959 | 0.0929 | -0.0739 |
| 9.0000 | -0.1895 | -0.0429 | 0.1228 | 0.1947 | 0.1251 |
| 10.0000 | -0.2062 | -0.1991 | -0.0732 | 0.0916 | 0.1865 |
| 11.0000 | -0.0682 | -0.1914 | -0.2041 | -0.0984 | 0.0614 |
| 12.0000 | 0.1124 | -0.0486 | -0.1762 | -0.2055 | -0.1190 |
| 13.0000 | 0.2368 | 0.1228 | -0.0309 | -0.1612 | -0.2041 |
| 14.0000 | 0.2724 | 0.2364 | 0.1316 | -0.0151 | -0.1464 |
| 15.0000 | 0.2399 | 0.2666 | 0.2356 | 0.1389 | -0.0008 |
| 16.0000 | 0.1775 | 0.2340 | 0.2611 | 0.2345 | 0.1452 |
| 17.0000 | 0.1150 | 0.1739 | 0.2286 | 0.2559 | 0.2331 |
| 18.0000 | 0.0668 | 0.1138 | 0.1706 | 0.2235 | 0.2511 |
| 19.0000 | 0.0354 | 0.0671 | 0.1127 | 0.1676 | 0.2189 |
| 20.0000 | 0.0173 | 0.0362 | 0.0673 | 0.1116 | 0.1647 |
| 21.0000 | 0.0079 | 0.0180 | 0.0369 | 0.0675 | 0.1106 |
| 22.0000 | 0.0034 | 0.0084 | 0.0187 | 0.0375 | 0.0676 |
| 23.0000 | 0.0013 | 0.0037 | 0.0089 | 0.0193 | 0.0380 |
| 24.0000 | 0.0005 | 0.0015 | 0.0039 | 0.0093 | 0.0199 |
| 25.0000 | 0.0002 | 0.0006 | 0.0017 | 0.0042 | 0.0098 |
| 26.0000 | 0.0001 | 0.0002 | 0.0007 | 0.0018 | 0.0045 |
| 27.0000 | 0.0000 | 0.0001 | 0.0003 | 0.0007 | 0.0020 |
| 28.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0003 | 0.0008 |
| 29.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0003 |
| 30.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |

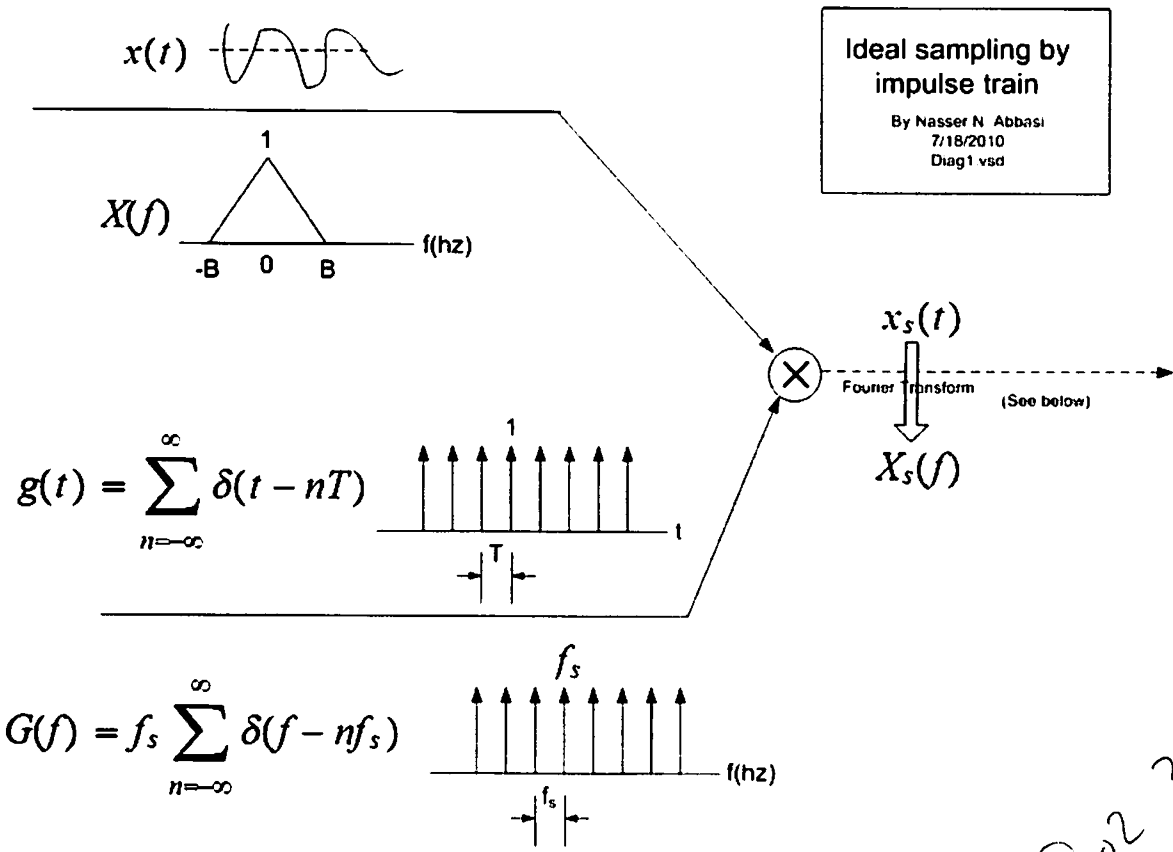
**Bessel Function Table** Dr. James S. Kang, Professor, Cal Poly Pomona

| | | | β | | | |
|---|---|---|---|---|---|---|
| n | 0.1000 | 0.2000 | 0.3000 | 0.4000 | 0.5000 | 0.6000 |
| 0 | 0.9975 | 0.9900 | 0.9776 | 0.9604 | 0.9385 | 0.9120 |
| 1.0000 | 0.0499 | 0.0995 | 0.1483 | 0.1960 | 0.2423 | 0.2867 |
| 2.0000 | 0.0012 | 0.0050 | 0.0112 | 0.0197 | 0.0306 | 0.0437 |
| 3.0000 | 0.0000 | 0.0002 | 0.0006 | 0.0013 | 0.0026 | 0.0044 |
| 4.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0003 |
| 5.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

| | | | β | | | |
|---|---|---|---|---|---|---|
| n | 0.7000 | 0.8000 | 0.9000 | 1.0000 | 1.1000 | 1.2000 |
| 0 | 0.8812 | 0.8463 | 0.8075 | 0.7652 | 0.7196 | 0.6711 |
| 1.0000 | 0.3290 | 0.3688 | 0.4059 | 0.4401 | 0.4709 | 0.4983 |
| 2.0000 | 0.0588 | 0.0758 | 0.0946 | 0.1149 | 0.1366 | 0.1593 |
| 3.0000 | 0.0069 | 0.0102 | 0.0144 | 0.0196 | 0.0257 | 0.0329 |
| 4.0000 | 0.0006 | 0.0010 | 0.0016 | 0.0025 | 0.0036 | 0.0050 |
| 5.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0002 | 0.0004 | 0.0006 |
| 6.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |

| | | | β | | |
|---|---|---|---|---|---|
| n | 1.0000 | 2.0000 | 3.0000 | 4.0000 | 5.0000 |
| 0 | 0.7652 | 0.2239 | -0.2601 | -0.3971 | -0.1776 |
| 1.0000 | 0.4401 | 0.5767 | 0.3391 | -0.0660 | -0.3276 |
| 2.0000 | 0.1149 | 0.3528 | 0.4861 | 0.3641 | 0.0466 |
| 3.0000 | 0.0196 | 0.1289 | 0.3091 | 0.4302 | 0.3648 |
| 4.0000 | 0.0025 | 0.0340 | 0.1320 | 0.2811 | 0.3912 |
| 5.0000 | 0.0002 | 0.0070 | 0.0430 | 0.1321 | 0.2611 |
| 6.0000 | 0.0000 | 0.0012 | 0.0114 | 0.0491 | 0.1310 |
| 7.0000 | 0.0000 | 0.0002 | 0.0025 | 0.0152 | 0.0534 |
| 8.0000 | 0.0000 | 0.0000 | 0.0005 | 0.0040 | 0.0184 |
| 9.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0009 | 0.0055 |
| 10.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0015 |
| 11.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0004 |
| 12.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |

| | | | β | | |
|---|---|---|---|---|---|
| n | 6.0000 | 7.0000 | 8.0000 | 9.0000 | 10.0000 |
| 0 | 0.1506 | 0.3001 | 0.1717 | -0.0903 | -0.2459 |
| 1.0000 | -0.2767 | -0.0047 | 0.2346 | 0.2453 | 0.0435 |
| 2.0000 | -0.2429 | -0.3014 | -0.1130 | 0.1448 | 0.2546 |
| 3.0000 | 0.1148 | -0.1676 | -0.2911 | -0.1809 | 0.0584 |
| 4.0000 | 0.3576 | 0.1578 | -0.1054 | -0.2655 | -0.2196 |
| 5.0000 | 0.3621 | 0.3479 | 0.1858 | -0.0550 | -0.2341 |
| 6.0000 | 0.2458 | 0.3392 | 0.3376 | 0.2043 | -0.0145 |
| 7.0000 | 0.1296 | 0.2336 | 0.3206 | 0.3275 | 0.2167 |
| 8.0000 | 0.0565 | 0.1280 | 0.2235 | 0.3051 | 0.3179 |
| 9.0000 | 0.0212 | 0.0589 | 0.1263 | 0.2149 | 0.2919 |
| 10.0000 | 0.0070 | 0.0235 | 0.0608 | 0.1247 | 0.2075 |
| 11.0000 | 0.0020 | 0.0083 | 0.0256 | 0.0622 | 0.1231 |
| 12.0000 | 0.0005 | 0.0027 | 0.0096 | 0.0274 | 0.0634 |
| 13.0000 | 0.0001 | 0.0008 | 0.0033 | 0.0108 | 0.0290 |
| 14.0000 | 0.0000 | 0.0002 | 0.0010 | 0.0039 | 0.0120 |
| 15.0000 | 0.0000 | 0.0001 | 0.0003 | 0.0013 | 0.0045 |
| 16.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0004 | 0.0016 |
| 17.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0005 |
| 18.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 |




Alternative way to write the sampled signal x(t)

$$x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

Fourier series approx

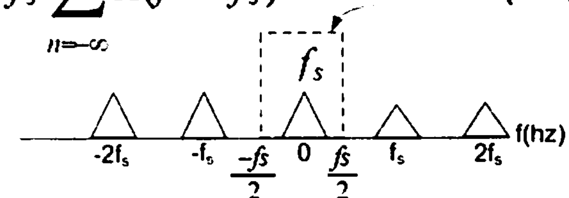
$$x_s(t) \approx x(t)\left( f_s \sum_{n=-\infty}^{\infty} e^{j\frac{2\pi}{T}nt} \right)$$

Fourier series approximation of the pulse train

Fourier Transform

$$X_s(f) = X(f) \circledast \underbrace{\left( f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right)}_{G(f)} = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

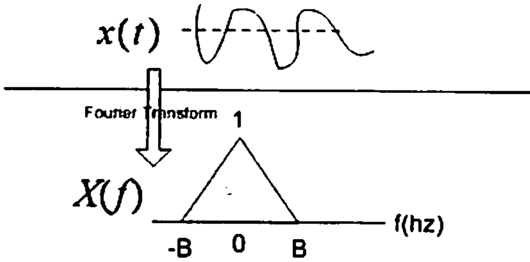
ideal filter (height=$\frac{1}{f_s}$)

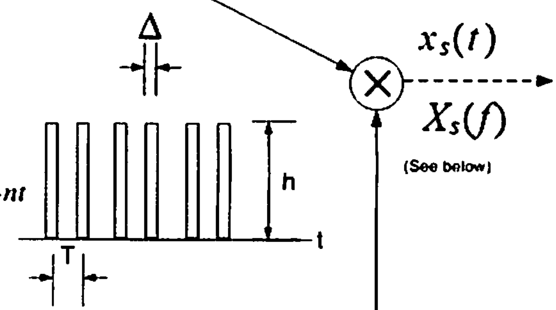


## 2 Practical sampling

practical sampling by rect train

By Nasser M. Abbasi 7/18/2010 Diag2.vsd



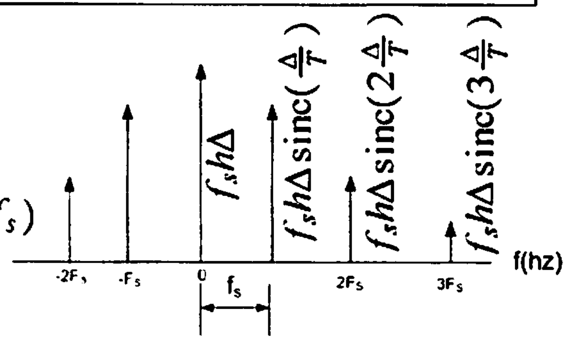




Fourier series coeff

$$g(t) = \sum_{n=-\infty}^{\infty} h\, rect\left(\frac{t-nT}{\Delta}\right) \approx \sum_{n=-\infty}^{\infty} \overbrace{h\frac{\Delta}{T}\,\mathrm{sinc}\left(n\frac{\Delta}{T}\right)}^{\text{Fourier series coeff}} e^{j\frac{2\pi}{T}nt}$$

Fourier Transform

$$G(f) = f_s h\Delta \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(n\frac{\Delta}{T}\right)\delta(f-nf_s)$$




Rewrite in fourier series approximation

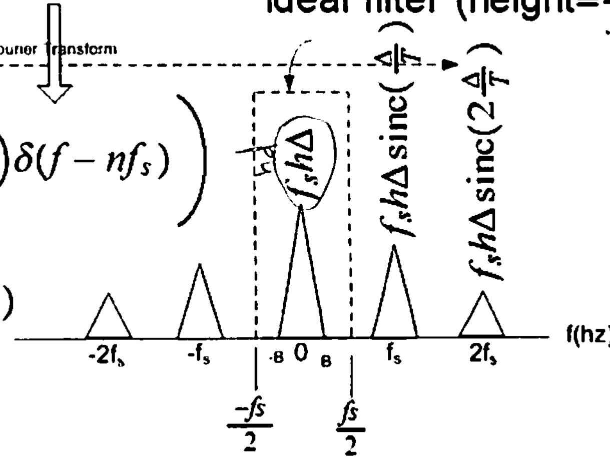
$$x_s(t) = x(t)\left(\sum_{n=-\infty}^{\infty} h\, rect\left(\frac{t-nT}{\Delta}\right)\right) \approx x(t)\left(f_s h\Delta \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(n\frac{\Delta}{T}\right) e^{j\frac{2\pi}{T}nt}\right)$$

Fourier Transform

$$X_s(f) = X(f) \circledast \left(f_s h\Delta \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(n\frac{\Delta}{T}\right)\delta(f-nf_s)\right)$$

$$= f_s h\Delta \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(n\frac{\Delta}{T}\right) X(f-nf_s)$$

ideal filter (height=$\frac{1}{f_s h\Delta}$)



## Welcome to Vibration Data Laplace Transform Table

**Laplace transforms are used to solve differential equations.**
**As an example, Laplace transforms are used to determine the response of a harmonic oscillator to an input signal.**

**By Tom Irvine Email: tomirvine@aol.com**

### Operation Transforms

| N | F(s) | f ( t ) , t > 0 |
|---|---|---|
| 1.1 | $Y(s) = \int_0^{\infty} \exp(-st)\,y(t)\,dt$ | definition of a Laplace transform<br>y(t) |
| 1.2 | Y(s) | inversion formula<br>$y(t) = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} \exp(st)\,Y(s)\,ds$ |
| 1.3 | sY(s) - y(0) | first derivative<br>$y'(t)$ |
| 1.4 | $s^2\,Y(s) - s\,y(0) - y'(0)$ | second derivative<br>$y''(t)$ |
| 1.5 | $s^n\,Y(s) - s^{n-1}[y(0)]$<br>$-s^{n-2}[y'(0)] - \ldots - s\left[y^{(n-2)}(0)\right]$<br>$-[y^{(n-1)}(0)]$ | nth derivative<br>$y^{(n)}(t)$ |
| 1.6 | (1/s) F(s) | integration<br>$\int_0^t Y(\tau)\,d\tau$ |
| 1.7 | F(s)G(s) | convolution integral<br>$\int_0^t f(t-\tau)\,g(\tau)\,d\tau$ |
| 1.8 | $\frac{1}{\alpha} F\left(\frac{s}{\alpha}\right)$ | f (at) |
| 1.9 | F(s - a) | shifting in the s-plane |

| | | |
|---|---|---|
| | | exp(-at) f(t) |
| 1.10 | $\frac{1}{1-\exp(-sT)}\int_0^T \exp(-st)f(t)\,dt$ | f(t) has period T, such that<br>f( t + T ) = f (t) |
| 1.11 | $\frac{1}{1+\exp(-sT)}\int_0^T \exp(-st)g(t)\,dt$ | g(t) has period T, such that<br>g(t + T ) = - g(t) |

**Function Transforms**

| N | F(s) | f ( t ) , t > 0 |
|---|---|---|
| 2.1 | 1 | d(t)<br>unit impulse at t = 0 |
| 2.2 | s | $\frac{d}{dt}\delta(t)$<br>double impulse at t = 0 |
| 2.3 | $\exp(-\alpha s),\quad \alpha \geq 0$ | d(t-a) |
| 2.4a | 1/s | unit step<br>u(t) |
| 2.4b | $\frac{1}{s}\left[\exp(-as)-\exp(-bs)\right]$ | 0 t < a<br>1 a < t < b<br>0 t > b |
| 2.5 | $\frac{1}{s}\exp(-\alpha s)$ | u(t-a) |
| 2.6 | $\frac{1}{s^2}$ | t |
| 2.7a | $\frac{1}{s^n},\quad n = 1, 2, 3, \ldots$ | $\frac{t^{n-1}}{(n-1)!}$ |
| 2.7b | $\frac{n!}{s^{n+1}}$ , n=1, 2, 3,?. | $t^n$ |
| 2.8 | $\frac{1}{s^k}$ , k is any real number > 0 | |

| | | |
|---|---|---|
| | | $\frac{t^{k-1}}{\Gamma(k)}$ <br> the Gamma function is given in Appendix A |
| 2.9 | $\frac{1}{s+\alpha}$ | exp(-at) |
| 2.10 | $\frac{1}{(s+\alpha)^2}$ | t exp(-at) |

| | | |
|---|---|---|
| 2.11 | $\frac{1}{(s+\alpha)^n}$ ; $n = 1, 2, 3, \ldots$ | $\left[\frac{t^{n-1}}{(n-1)!}\right]\exp(-\alpha t)$ |
| 2.12 | $\frac{\alpha}{s(s+\alpha)}$ | 1 - exp(-at) |
| 2.13 | $\frac{1}{(s+\alpha)(s+\beta)}$, $\beta \neq \alpha$ | $\frac{1}{(\beta-\alpha)}[\exp(-\alpha t)-\exp(-\beta t)]$ |
| 2.14 | $\frac{1}{s(s+\alpha)(s+\beta)}$, $\beta \neq \alpha$ | $\frac{1}{\alpha\beta}+\frac{\exp(-\alpha t)}{\alpha(\alpha-\beta)}+\frac{\exp(-\beta t)}{\beta(\beta-\alpha)}$ |
| 2.15 | $\frac{s}{(s+\alpha)(s+\beta)}$, $\beta \neq \alpha$ | $\frac{1}{(\alpha-\beta)}[\alpha\exp(-\alpha t)-\beta\exp(-\beta t)]$ |
| 2.16a | $\frac{\alpha}{s^2+\alpha^2}$ | sin(at) |
| 2.16b | $\frac{[\sin(\phi)]s+\alpha[\cos(\phi)]}{s^2+\alpha^2}$ | sin(at + f) |
| 2.17 | $\frac{s}{s^2+\alpha^2}$ | cos(at) |
| 2.18 | $\frac{s^2-\alpha^2}{[s^2+\alpha^2]^2}$ | t cos(at) |

| | | |
|---|---|---|
| 2.19 | $\frac{1}{s\left(s^2+\alpha^2\right)}$ | $\frac{1}{\alpha^2}[1-\cos(\alpha t)]$ |
| 2.20 | $\frac{1}{\left(s^2+\alpha^2\right)^2}$ | $\frac{1}{2\alpha^3}[\sin(\alpha t)-\alpha t\ \cos(\alpha t)]$ |
| 2.21 | $\frac{s}{\left(s^2+\alpha^2\right)^2}$ | $\frac{1}{2\alpha}[t\sin(\alpha t)]$ |
| 2.22 | $\frac{s^2}{\left(s^2+\alpha^2\right)^2}$ | $\frac{1}{2\alpha}[\sin(\alpha t)+\alpha t\cos(\alpha t)]$ |

| | | |
|---|---|---|
| 2.23 | $\frac{1}{\left(s^2+\omega^2\right)\left(s^2+\alpha^2\right)},\quad \alpha\neq\omega$ | $\left\{\frac{1}{\omega^2-\alpha^2}\right\}\left\{\frac{1}{\alpha}\sin(\alpha t)-\frac{1}{\omega}\sin(\omega t)\right\}$ |
| 2.24 | $\frac{\alpha}{s^2(s+\alpha)}$ | $t-\frac{1}{\alpha}[1-\exp(-\alpha t)]$ |
| 2.25 | $\frac{\beta}{(s+\alpha)^2+\beta^2}$ | exp(-at)sin(bt) |
| 2.26 | $\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$ | exp(-at)cos(bt) |
| 2.27 | $\frac{s+\lambda}{(s+\alpha)^2+\beta^2}$ | $\exp(-\alpha t)\left\{\cos(\beta t)+\left[\frac{\lambda-\alpha}{\beta}\right]\sin(\beta t)\right\}$ |
| 2.28 | $\frac{s+\alpha}{s^2+\beta^2}$ | $\frac{\sqrt{\alpha^2+\beta^2}}{\beta}\sin(\beta t+\phi),\quad \phi=\arctan\left(\frac{\beta}{\alpha}\right)$ |
| 2.29 | $\frac{1}{s^2-\alpha^2}$ | $\frac{1}{\alpha}\sinh(\alpha t)$ |
| 2.30 | $\frac{s}{s^2-\alpha^2}$ | cosh(at) |
| 2.31 | $\arctan\left(\frac{\alpha}{s}\right)$ | $\frac{1}{t}\sin(\alpha t)$ |
| 2.32 | $\frac{1}{\sqrt{s}}$ | $\frac{1}{\sqrt{\pi t}}$ |

| | | |
|---|---|---|
| 2.33 | $\frac{1}{\sqrt{s+\alpha}}$ | $\frac{1}{\sqrt{\pi t}}\exp[-\alpha t]$ |

| | | |
|---|---|---|
| 2.34 | $\frac{1}{\sqrt{s^3}}$ | $2\sqrt{\frac{t}{\pi}}$ |
| 2.35 | $\frac{1}{\sqrt{s^2+\alpha^2}}$ | $J_o(\alpha t)$<br>Bessel function given in Appendix A |
| 2.36 | $\frac{1}{\left(s^2+\alpha^2\right)^{3/2}}$ | $\left(\frac{t}{\alpha}\right)J_1(\alpha t)$ |
| 2.37 | $\frac{1}{\sqrt{s^2-\alpha^2}}$ | $I_o(\alpha t)$<br>Modified Bessel function given in Appendix A |
| 2.38 | $\frac{1}{\left(s^2-\alpha^2\right)^{3/2}}$ | $\left(\frac{t}{\alpha}\right)I_1(\alpha t)$ |
| 2.39 | $\sqrt{s-\alpha}-\sqrt{s-\beta}$ | $\frac{1}{2t\sqrt{\pi t}}[\exp(\beta t)-\exp(\alpha t)]$ |

**Examples of the Laplace Transform as a Solution for Mechanical Shock and Vibration Problems:**

**Free Vibration of a Single-Degree-of-Freedom System:** free.pdf

**Response of a Single-degree-of-freedom System Subjected to a Unit Step Displacement:** unit_step.pdf

**Response of a Single-degree-of-freedom System Subjected to a Classical Pulse Base Excitation:** sbase.pdf

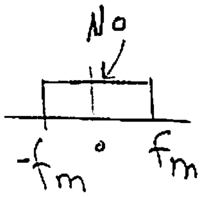
**Partial Fractions in Shock and Vibration Analysis:** partial.pdf

**References**

1. Jan Tuma, Engineering Mathematics Handbook, McGraw-Hill, New York, 1979.

**2. F. Oberhettinger and L. Badii, Table of Laplace Transforms, Springer-Verlag, N.Y., 1972.**

## FM SNR




$S_i = \frac{A_c^2}{2}$ watt  →  **FM demod**  →  $S_o = K_f^2 \frac{A_m^2}{2}$

$P_i = N_o f_m$

$$P_o = \frac{8\pi^2 N_o f_m^3}{3 A_c^2}$$

$$(SNR)_i = \frac{A_c^2}{2 N_o f_m}$$

$$SNR_o = \frac{3}{4} \frac{A_c^2 \beta^2}{N_o f_m}$$

$$\boxed{\beta = \frac{K_f A_m}{2\pi f_m}}$$

## AM, $\mu = 1$

$S_i = \frac{A_c^2}{2}$  →  **AM demodulator**  →  $S_o = \frac{A_c^2}{2}$

$P_i = N_o f_m$

$P_o = 2 N_o f_m$

$$(SNR)_{o,AM} = \frac{A_c^2}{4 N_o f_m}$$