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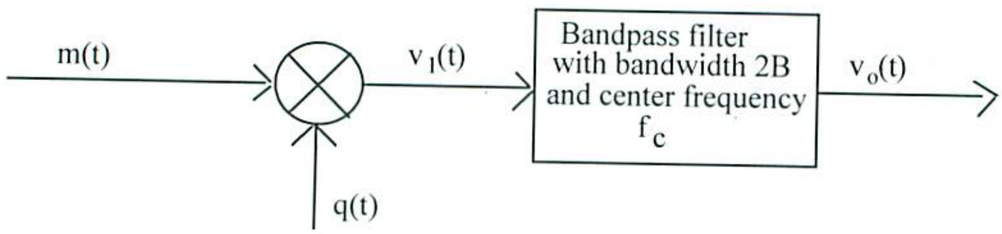
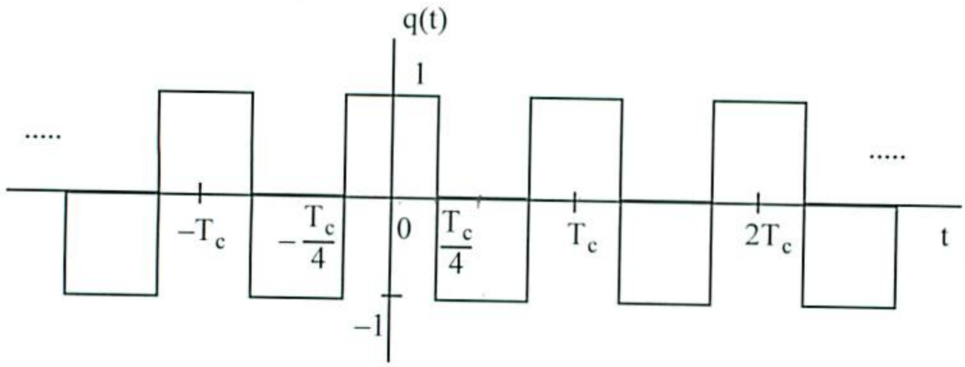
ECE 405 **QUIZ #1** **20 POINTS** **SUMMER 2010**

14/20

A model of a balanced modulator is shown below. Let the message signal be $m(t) = 2\cos(2\pi\times10000t)$. The frequency of carrier is $f_c = 100000$ Hz so that $T_c = 1/100000$ s. B = 25 kHz.

(a) Plot m(t) in the time domain for $0 \le t \le 0.2$ms.
(b) Plot the spectrum $M(f) = F[m(t)]$ in the frequency domain.
(c) Find the exponential Fourier coefficients Q_n of q(t) and represent q(t) by its exponential Fourier series.
(d) Plot the spectrum $Q(f) = F[q(t)]$ in the frequency domain.
(e) Plot $v_1(t)$ in the time domain for $0 \le t \le 0.2$ms.
(f) Plot the spectrum $V_1(f) = F[v_1(t)]$ in the frequency domain for -600 kHz $\le f \le$ 600 kHz.
(g) Plot $v_o(t)$ in the time domain for $0 \le t \le 0.2$ms.
(h) Plot the spectrum $V_o(f) = F[v_o(t)]$ in the frequency domain for -600 kHz $\le f \le$ 600 kHz.
(i) The center frequency of the bandpass filter is changed to $5f_c$ with bandwidth 50 kHz, find the expression for $v_o(t)$.



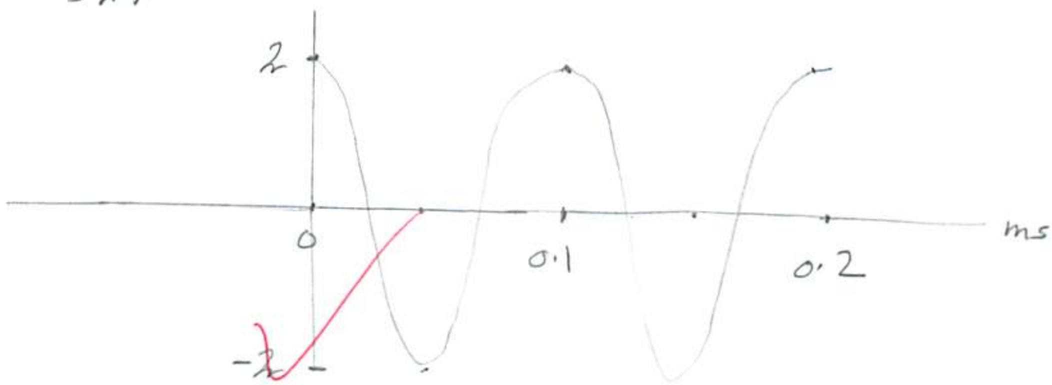


$m(t) = 2\cos(2\pi f_m t)$ where $f_m = 10{,}000$ Hz.

$f_c = 100{,}000$ Hz

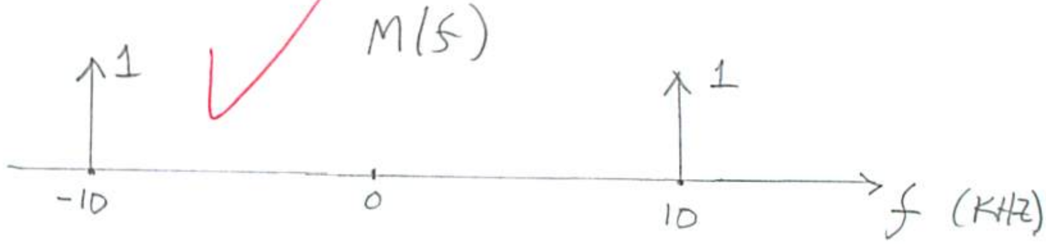
period of $m(t) = \frac{1}{f_m} = 0.1$ ms.

(a)



(b) $F[m(t)] = \frac{A_m}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]$ where $A_m = 2$ here

$= \delta(f - 10k) + \delta(f + 10k)$



(c) $q(t)$: period T_c, $h = 1$, $\tau = \frac{T_c}{2}$.

$$q(t) = \sum_{n=-\infty}^{\infty} Q_n e^{j\frac{2\pi}{T_c}nt}$$

where $\boxed{Q_n = \frac{1}{T_c}\int_{T_c} q(t)\, e^{-j\frac{2\pi}{T_c}nt}\, dt}$

So

$$Q_n = h d \, \mathrm{sinc}(nd)$$

where $h = 2$, $d = \frac{\tau}{T} = \frac{T_c}{2 T_c} = \frac{1}{2}$

so $$Q_n = \frac{2}{2} \, \mathrm{sinc}\left(\frac{n}{2}\right) = \mathrm{sinc}\left(\frac{n}{2}\right).$$

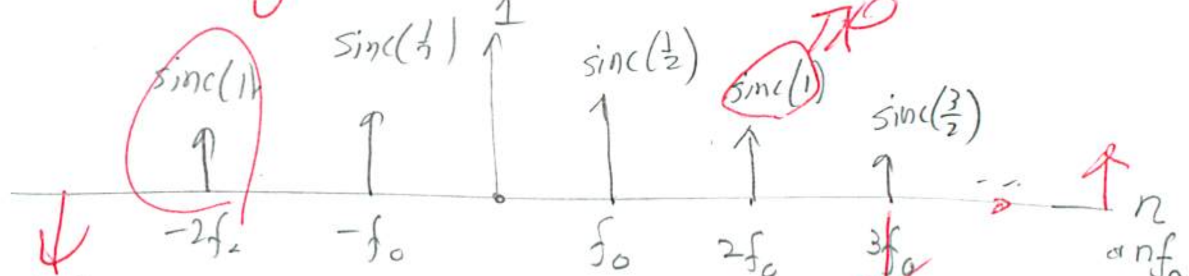
so $$q(t) = \sum Q_n \, e^{j \frac{2\pi}{T_c} n t}$$

$$\boxed{q(t) = \sum \mathrm{sinc}\left(\frac{n}{2}\right) e^{j \frac{2\pi}{T_c} n t}}$$

where $T_c = 0.001$ ms

(d) $$F\{q(t)\} = \sum \mathrm{sinc}\left(\frac{n}{2}\right) F\left[1 \cdot e^{j \frac{2\pi}{T_c} n t}\right]$$ $\quad f_0 = \frac{2\pi}{T_c}$

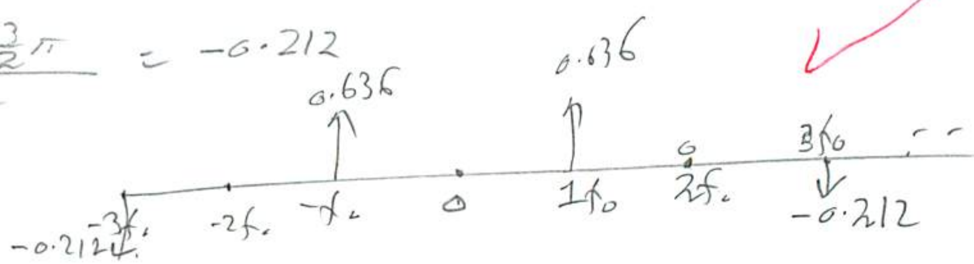
$$F[q(t)] = \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(\frac{n}{2}\right) \delta\left(f - \frac{2\pi}{T_c} n\right) = \sum \mathrm{sinc}\left(\frac{n}{2}\right) \delta(f - n f_0)$$



$$\mathrm{sinc}\left(\tfrac{1}{2}\right) = \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = \frac{2}{\pi} = 0.636$$

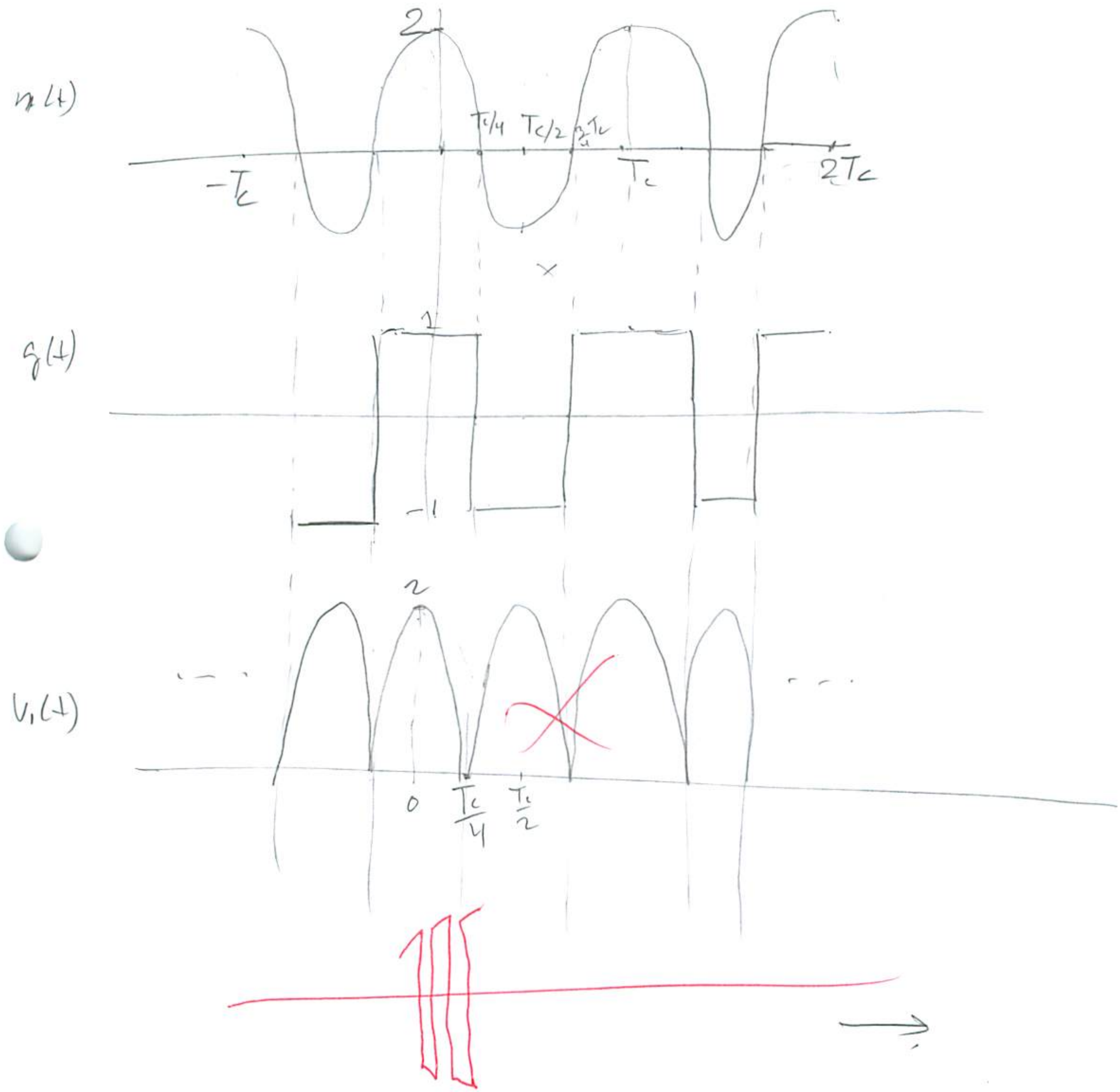
$$\mathrm{sinc}(1) = \frac{\sin \pi}{\pi} = 0$$

$$\mathrm{sinc}\left(\tfrac{3}{2}\right) = \frac{\sin \frac{3}{2}\pi}{\frac{3}{2}\pi} = -0.212$$



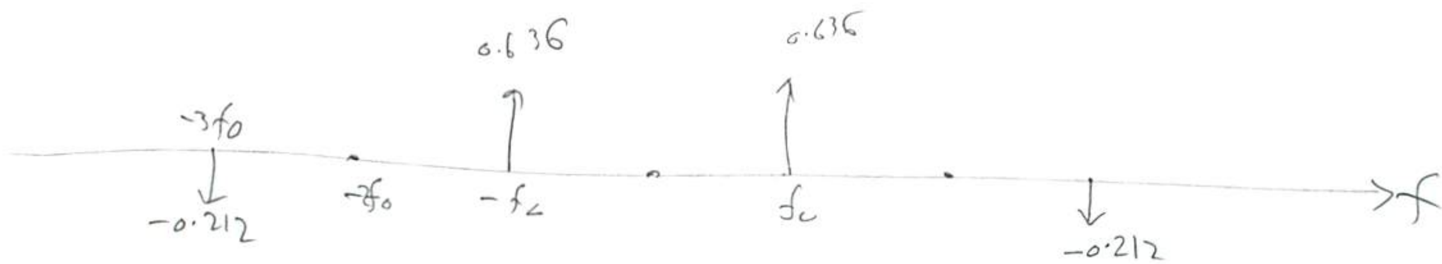
(e) $v_1 = m(t) \cdot g(t)$.

$$g(t) = \sum \text{sinc}\left(\frac{n}{2}\right) e^{j\frac{2\pi}{T_c}nt} = \sum \text{sinc}\left(\frac{n}{2}\right) e^{jf_0 nt}.$$

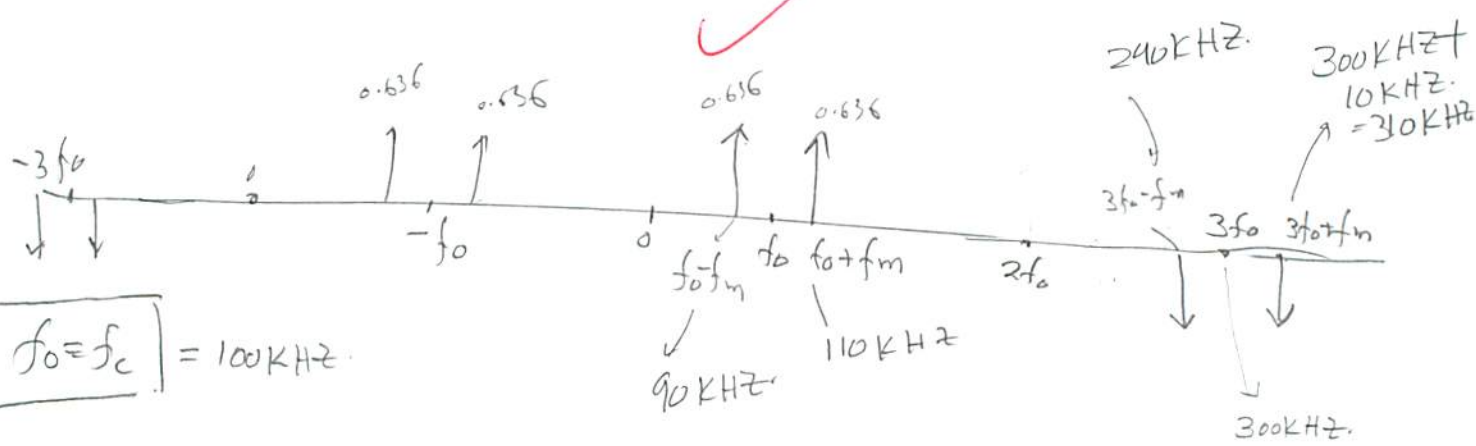


(f) the spectrum of $v_1(t)$ is $F(m(t)) \circledast F(q(t))$ (4)

$\circledast$



$\Downarrow$



$\boxed{f_0 = f_c} = 100KHZ$

(a) ✗
(b) ✗
(c) ✗