1. Problem 2.2.3. (10 points)
2. (a) Problem 2.2.10. (10 points)
(b) Problem 2.2.11. (10 points)
3. The probability to find a particle at position between $x$ and $x+d x$ is

$$
P(x) d x=A \exp \left(-\alpha x^{2}+\beta x^{3}\right) d x
$$

where $A, \alpha$, and $\beta$ are positive parameters. By the definition of probability,

$$
\int_{-\infty}^{\infty} P(x) d x=1
$$

Treat $\beta$ as a small parameter, i.e., for any given $x$, you can view $P(x)$ as a function of $\beta$ and expand it around $\beta=0$.
(a) Find $A$ to the first order of $\beta$. (15 points)
(b) Find the average position

$$
\bar{x}=\int_{-\infty}^{\infty} x P(x) d x
$$

to the first order of $\beta$. (25 points)
4. A container of volume $V$ encloses a neutrino gas of temperature $T$. The number of neutrinos with energy between $E$ and $E+d E$ is

$$
d N=\left(\frac{4 \pi V}{h^{3} c^{3}}\right) \frac{E^{2}}{\exp [E /(k T)]+1} d E
$$

where $h$ is the Planck constant, $c$ is the speed of light, and $k$ is the Boltzmann constant.
(a) Express the total energy density of the neutrino gas in terms of a dimensional factor multiplying a dimensionless integral. Show that the factor has the correct dimension. (10 points).
(b) Follow the discussion of a photon gas and evaluate the dimensionless integral. (20 points).

