Physics 3041 (Spring 2021) Homework Set 2 (Due 2/3)

- 1. Problem 2.2.3. (10 points)
- 2. (a) Problem 2.2.10. (10 points)
- (b) Problem 2.2.11. (10 points)
- 3. The probability to find a particle at position between x and x + dx is

$$P(x)dx = A\exp(-\alpha x^2 + \beta x^3)dx,$$

where A, α , and β are positive parameters. By the definition of probability,

$$\int_{-\infty}^{\infty} P(x)dx = 1.$$

Treat β as a small parameter, i.e., for any given x, you can view P(x) as a function of β and expand it around $\beta = 0$.

- (a) Find A to the first order of β . (15 points)
- (b) Find the average position

$$\bar{x} = \int_{-\infty}^{\infty} x P(x) dx$$

to the first order of β . (25 points)

4. A container of volume V encloses a neutrino gas of temperature T. The number of neutrinos with energy between E and E + dE is

$$dN = \left(\frac{4\pi V}{h^3 c^3}\right) \frac{E^2}{\exp[E/(kT)] + 1} dE,$$

where h is the Planck constant, c is the speed of light, and k is the Boltzmann constant.

(a) Express the total energy density of the neutrino gas in terms of a dimensional factor multiplying a dimensionless integral. Show that the factor has the correct dimension. (10 points).

(b) Follow the discussion of a photon gas and evaluate the dimensionless integral. (20 points).