

# MATH 5525- Test 1

February 19, 2020

You may use class notes at your convenience. Please, show all your work.

**Problem 1.** (50 points). Consider the system of differential equations

$$\dot{y} = v, \quad \dot{v} = f(y),$$

where  $f$  is a continuous function  $f : \mathbf{R} \rightarrow \mathbf{R}$ .

- Find a first integral of the system.
- Find the equilibrium points of the system in the case that  $f(y) = \sin y$ . From now on, consider  $f(y) = \sin(y)$ .
- Find the Jacobian matrix of the system at the equilibrium points. (That is, write the linearized system about the equilibrium points).
- Determine the nature of the equilibrium points.
- Sketch the phase plane of the system in the interval  $-\pi \leq y \leq \pi$ .

**Problem 2.** (40 points). Consider the predator-pray system governing the number of individuals  $x$   $y$  of the two species at time  $t > 0$ :

$$\dot{x} = x(1 - x - y), \quad \dot{y} = y(-2 + x).$$

- Find the equilibrium points of the system;
- Find two invariant sets.
- Sketch the phase plane.