Study notes

EE 3015
Signals and Systems

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Contents

1 When input is complex exponential

When input is $x[n] = e^{j\Omega_0 n}$ and system is given by $H(\Omega)$ then the output is $y[n] = e^{j\Omega_0 n} H(\Omega_0)$ which is the same as $y[n] = e^{j\Omega_0 n} \left|H(\Omega_0)\right| e^{j \arg H(\Omega_0)}$.

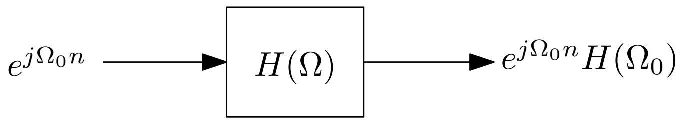

Figure 1: Output when input is complex exponential

Hence when the input is linear combination of complex exponentials

$$\begin{aligned}
A\cos(\Omega_0 n + \theta) &= \frac{A}{2}\left(e^{j(\Omega_0 n+\theta)} + e^{-j(\Omega_0 n+\theta)}\right) \\
&= \left(\frac{A}{2}e^{j\theta}\right)e^{j\Omega_0 n} + \left(\frac{A}{2}e^{-j\theta}\right)e^{-j\Omega_0 n}
\end{aligned}$$

Then, and since the system is linear, then the output will be scaled and linear sum of each output corresponding to each term above. In other words, when the input is $\left(\frac{A}{2}e^{j\theta}\right)e^{j\Omega_0 n}$ then the output is

$$\begin{aligned}
y_1[n] &= \left(\frac{A}{2}e^{j\theta}\right)e^{j\Omega_0 n}\left|H(\Omega_0)\right| e^{j\arg H(\Omega_0)} \\
&= \left|H(\Omega_0)\right| \frac{A}{2} e^{j(\Omega_0 n + \theta + \arg H(\Omega_0))} \qquad (1)
\end{aligned}$$

And when the input is $\left(\frac{A}{2}e^{-j\theta}\right)e^{-j\Omega_0 n}$ then the output is

$$\begin{aligned}
y_2[n] &= \left(\frac{A}{2}e^{-j\theta}\right)e^{-j\Omega_0 n}\left|H(-\Omega_0)\right| e^{j\arg H(-\Omega_0)} \\
&= \left|H(-\Omega_0)\right| \frac{A}{2} e^{-j(\Omega_0 n + \theta - \arg H(-\Omega_0))}
\end{aligned}$$

But for real input, which is the case here, $\left|H(\Omega_0)\right|$ is symmetrical. Hence $\left|H(\Omega_0)\right| = \left|H(-\Omega_0)\right|$ and $\arg H(-\Omega_0) = -\arg H(\Omega_0)$ (see table 4.6 for these properties). Hence

$$y_2[n] = \left|H(\Omega_0)\right| \frac{A}{2} e^{-j(\Omega_0 n + \theta + \arg H(\Omega_0))} \qquad (2)$$

Therefore, by linearity, $y[n] = y_1[n] + y_2[n]$ or by adding (1) and (2)

$$\begin{aligned}
y[n] &= \left|H(\Omega_0)\right| \frac{A}{2} e^{j(\Omega_0 n + \theta + \arg H(\Omega_0))} + \left|H(\Omega_0)\right| \frac{A}{2} e^{-j(\Omega_0 n + \theta + \arg H(\Omega_0))} \\
&= \left|H(\Omega_0)\right| A\left(\frac{e^{j(\Omega_0 n + \theta + \arg H(\Omega_0))} + e^{-j(\Omega_0 n + \theta + \arg H(\Omega_0))}}{2}\right) \\
&= \left|H(\Omega_0)\right| A\cos(\Omega_0 n + \theta + \arg H(\Omega_0))
\end{aligned}$$