EE 3015
Midterm I
Oct 10th 2001

Name :

Problem 1. Obtain the impulse and step responses for the LTI system described by:

A. **(15 pts)** $h(n) = (0.5)^n u(n)$ $\quad y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$ $\quad x(n) \to \boxed{h(n)} \to y(n)$

B. **(15 pts)** $h(t) = \exp(-0.5\,t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau \qquad x(t) \to \boxed{h(t)} \to y(t)$$

$x(n) = \delta(n)$ impulse
$x(n) = u(n)$ step
$x(t) = \delta(t)$ impulse
$x(t) = u(t)$ step

Problem 2. Given the frequency response of a LTI system $H(e^{j\Omega})$, for the following input signal find the steady state expression of the out put signal

A. **(10 pts)** $x(n) = 2\cos((\pi/6)^* n + \pi/5)$
B. **(10 pts)** $x(n) = 5\sin((\pi/3)^* n + \pi/8)$

$$e^{j\Omega n} \to \boxed{h(n)} \to y(n) \qquad y(n) = \underbrace{e^{j\Omega n}}_{x(n)} \cdot H(e^{j\Omega})\Big|_{\Omega = \pi/6}$$

3. Compute Fourier series coeff. For the following signals:

A. **(15 pts)** $x(n) = 2\sin((\pi/3)^*n + \pi/2) + 3\cos((\pi/6)^*n + \pi/5)$ $\Rightarrow$ Convert From Sine to Cosine

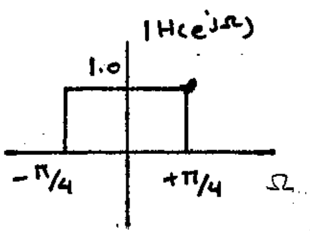
$$\sin(\theta) = \cos(\pi/2 - x) = \cos(x - \pi/2)$$

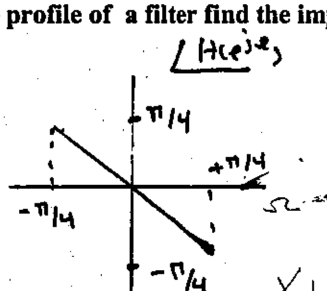
B. **(15 pts)** $x(t) = \exp(j2\pi t) + \exp(j3\pi t)$

$$\therefore x(n) = 2\cos(\pi/3 \cdot n + \pi/2 - \pi/2) + 3\cos(\pi/6 \cdot n + \pi/5)$$

Fundamental $\Omega_0 = \pi/6$.

4. (20 pts)Given the magnitude & phase profile of a filter find the impulse response of this filter.





Ω : rad/sample Discrete frequency

$$\measuredangle H(e^{j\Omega}) = -\Omega$$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H(e^{j\Omega})\cdot e^{j\Omega n}\,d\Omega \qquad \therefore H(e^{j\Omega}) = e^{-j\Omega} \quad -\pi/4 < \Omega < +\pi/4$$
$$= 0 \quad \text{else.}$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j\Omega}\cdot e^{j\Omega n}\,d\Omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{+\pi/4} e^{+j(n-1)\Omega}\,d\Omega = \frac{1}{2\pi}\cdot\frac{1}{j(n-1)}\,e^{j(n-1)\Omega}\Big|_{-\pi/4}^{+\pi/4}$$

$$= \frac{1}{2\pi j(n-1)}\cdot\left[e^{j(n-1)\pi/4} - e^{-j(n-1)\pi/4}\right]$$

Solution to Sample Exam

$$x(t) \to \boxed{h(t)} \to y(t)$$
$$x(n) \to \boxed{h(n)} \to y(n)$$

problem 1.

A. if $h(n) = 0.5\,u(n)$

if $x(n) = \delta(n)$ impulse. Then $y(n) = \sum_{k=-\infty}^{\infty} \delta(k).\,h(n-k) = h(n) = 0.5\,u(n)$ impulse response

if $x(n) = u(n)$ unit step Then $y(n) = \sum_{k=-\infty}^{\infty} u(k).\,h(n-k)$ since $u(k) = \begin{cases} 1 & k \geq 0 \\ 0 & \text{else.} \end{cases}$

$$y(n) = \sum_{k=0}^{\infty} 1.\,(0.5)^{n-k}\,u(n-k) = 0.5^{n}.\sum_{k=0}^{\infty} (0.5)^{-k}.$$

For $n \geq k$.

B. if $x(t) = \delta(t)$ impulse Then $y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} \delta(k).\,h(t-\tau)\,d\tau$
$= h(t)$
$= e^{-0.5t}.$

if $x(t) = u(t)$ Then $y(t) = \int_{-\infty}^{\infty} u(\tau)\,e^{-0.5(t-\tau)}\,d\tau$

$$= \int_{0}^{\infty} e^{-0.5t}.\,e^{0.5\tau}\,d\tau = e^{-0.5t}.\int_{0}^{\infty} e^{0.5\tau}\,d\tau. = e^{-0.5t}.\,\frac{e^{0.5\tau}}{0.5}\Big|_0^{\infty}$$
$$= e^{-0.5t}\left[e^{0.5(\infty)} - 1\right]$$

problem 2.

A. $x(n) = 2\cos(\pi/6\,n + \pi/5)$

$$y(n) = 2.\,|H(e^{j\Omega})|_{\Omega=\pi/6}\,.\cos\left(\pi/6.n + \pi/5 + \measuredangle H(e^{j\Omega})\Big|_{\Omega=\pi/6}\right).$$

B. $x(n) = 5\sin(\pi/3\,n + \pi/8) = 5\cos(\pi/3\,n + \pi/8 - \pi/2)$

$$\therefore\ y(n) = 5.\,|H(e^{j\Omega})|_{\Omega=\pi/3}\,.\cos\left(\pi/3.n + \pi/8 - \pi/2 + \measuredangle H(e^{j\Omega})\Big|_{\Omega=\pi/3}\right)$$

problem 3.

A. $X(n) = 2\sin\left(\pi/3 \cdot n + \pi/2\right) + 3\cos\left(\pi/6 \cdot n + \pi/5\right)$

Find fundamental freq. $\Omega_1 = \pi/3$ $\quad \Omega_2 = \pi/6$ $\quad \Omega_0$: fundamental freq. $= \pi/6$.

$X(n) = 2\cos\left(\pi/3 \cdot n + \cancel{\pi/2} - \cancel{\pi/2}\right) + 3\cos\left(\pi/6 \cdot n + \pi/5\right)$

Fourier Coeff

use Euler formula $X(n) = e^{j(\pi/3 \cdot n)} + e^{-j(\pi/3 \cdot n)} + \frac{3}{2} e^{j(\pi/6 \cdot n + \pi/5)} + \frac{3}{2} e^{-j(\pi/6 n + \pi/5)}$

if $X(n) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{j\Omega_0 k n} = e^{j(2\Omega_0 n)} + e^{-j(2\Omega_0 n)} + \frac{3}{2} \cdot e^{j\pi/5} \cdot e^{j\Omega_0 n} + \frac{3}{2} e^{-j\pi/5} \cdot e^{j\Omega_0 n}$

Then by inspection

For $k=1$ $\quad a_1 = \frac{3}{2} e^{j\pi/5}$ $\quad a_{-1} = \frac{3}{2} e^{-j\pi/5}$

$k=2$ $\quad a_2 = 1$ $\quad a_{-2} = 1$

For all other k $\quad a_k = 0$

B $\quad X(t) = e^{j2\pi t} + e^{j3\pi t}$ $\quad$ use. $\quad a_k = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} dt$

$\omega_1 = 2\pi$ $\quad \omega_2 = 3\pi$

$\omega_0 = \pi$ Fundamental freq. $\omega_0 = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\pi} = 2$.

$\therefore a_k = \frac{1}{2} \int_0^2 e^{j2\pi t} \cdot e^{-jk\pi t} dt + \frac{1}{2} \int_0^2 e^{j3\pi t} \cdot e^{-jk\pi t} dt$

evaluate the integral

problem 4.

$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} d\Omega = \frac{1}{2\pi j (n-1)} \left[e^{j(n-1)\pi/4} - e^{-j(n-1)\pi/4} \right]$

$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} H(e^{j\Omega}) \cdot e^{j\Omega n} d\Omega$