

my solution to practice exam midterm 1 2018

EE 3015  
Signals and Systems

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### 0.1 Problem 1

Given the discrete time system with input  $x[n]$  and impulse response  $h[n]$  obtain the output sequence  $y[n]$  by applying discrete convolution.

$x[n] = [1, 0, -1, 1], h[n] = [1, 1, -1, -1]$ . Both sequences are positive and start with  $n = 0$  position.

Solution

$$y[n] = x[n] \otimes h[n]$$

Folding  $x[-n]$ . When  $n = 0, y[0] = 1$ . When  $n = 1$ , then  $y[1] = (0)(1) + (1)(1) = 1$ . When  $n = 2, y[2] = (-1)(1) + (0)(1) + (1)(-1) = -2$ . When  $n = 3, y[3] = (1)(1) + (-1)(1) + (0)(-1) + (1)(-1) = 1 - 1 - 1 = -1$ . When  $n = 4, y[4] = 1 + 1 = 2$ . When  $n = 5, y[5] = -1 + 1 = 0$ , when  $n = 6, y[6] = -1$ . When  $n > 6, y[n] = 0$ .

Hence

$$y[n] = [1, 1, -2, -1, 2, 0, -1]$$

### 0.2 Problem 2

The impulse response of LTI system is given by  $h(t) = u(t) - 2u(t-1) + u(t-2)$  where  $u(t)$  is unit step signal. Determine the output of this  $y(t)$  where its input  $x(t)$  is given as  $x(t) = e^{-t}u(t+1)$ .

Solution

By folding  $x(t)$ . See key solution. Used same method.

### 0.3 Problem 3

A discrete periodic sequence is given as  $x[n] = 2 \sin\left(\frac{3\pi}{8}n + \frac{\pi}{2}\right) + \cos\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)$ . (a) Find fundamental frequency of this signal. (b) Fourier series coefficients for  $x[n]$ . (c) if  $x[n] = \cos\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)$  is an input to system with frequency response  $H(\Omega) = \frac{1-e^{-j\Omega}}{2+e^{-2j\Omega}}$ , obtain expression for  $y[n]$

Solution

**Part a** For  $\sin\left(\frac{3\pi}{8}n + \frac{\pi}{2}\right)$ , we need  $\frac{3}{8}\pi N = m2\pi$  or  $\frac{m}{N} = \frac{3}{16}$ . Since relatively prime, hence  $N = 16$ . For  $\cos\left(\frac{\pi}{4}n + \frac{\pi}{3}\right)$  we need  $\frac{\pi}{4}N = m2\pi$  or  $\frac{m}{N} = \frac{1}{8}$ . Hence  $N = 8$ . The least common multiplier is 16. Hence fundamental period is  $N = 16$ . Therefore  $\Omega_0 = \frac{2\pi}{N} = \frac{\pi}{8}$ .

**Part b** Since input is periodic, then

$$x[n] = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 n} \quad (1)$$

By writing the input, using Euler relation, we can compare the input to the above and read off  $a_k$ . First we rewrite the input using common  $\Omega_0$  as

$$x[n] = 2 \sin\left(3\Omega_0 n + \frac{\pi}{2}\right) + \cos\left(2\Omega_0 n + \frac{\pi}{3}\right)$$

Hence

$$\begin{aligned} x[n] &= 2 \left( \frac{e^{j(3\Omega_0 n + \frac{\pi}{2})} - e^{-j(3\Omega_0 n + \frac{\pi}{2})}}{2j} \right) + \frac{e^{j(2\Omega_0 n + \frac{\pi}{3})} + e^{-j(2\Omega_0 n + \frac{\pi}{3})}}{2} \\ &= \frac{1}{j} e^{j(3\Omega_0 n + \frac{\pi}{2})} - \frac{1}{j} e^{-j(3\Omega_0 n + \frac{\pi}{2})} + \frac{1}{2} e^{j(2\Omega_0 n + \frac{\pi}{3})} + \frac{1}{2} e^{-j(2\Omega_0 n + \frac{\pi}{3})} \\ &= \frac{1}{j} e^{j\frac{\pi}{2}} e^{j3\Omega_0 n} - \frac{1}{j} e^{-j\frac{\pi}{2}} e^{-j3\Omega_0 n} + \frac{1}{2} e^{j\frac{\pi}{3}} e^{j2\Omega_0 n} + \frac{1}{2} e^{-j\frac{\pi}{3}} e^{-j2\Omega_0 n} \end{aligned}$$

But  $e^{j\frac{\pi}{2}} = j \sin \frac{\pi}{2} = j$  and  $e^{-j\frac{\pi}{2}} = -j \sin \frac{\pi}{2} = -j$  and  $e^{j\frac{\pi}{3}} = \cos\left(\frac{\pi}{3}\right) + j \sin\left(\frac{\pi}{3}\right) = \frac{1}{2}\sqrt{3}j + \frac{1}{2}$  and  $e^{-j\frac{\pi}{3}} = \cos\left(\frac{\pi}{3}\right) - j \sin\left(\frac{\pi}{3}\right) = \frac{1}{2} - \frac{1}{2}\sqrt{3}j$ . Hence the above simplifies to

$$x[n] = e^{j3\Omega_0 n} + e^{-j3\Omega_0 n} + \frac{1}{4}(1 + \sqrt{3}j)e^{j2\Omega_0 n} - \frac{1}{4}(1 - \sqrt{3}j)e^{-j2\Omega_0 n} \quad (2)$$

Comparing (2) to (1) shows that

$$\begin{aligned} a_3 &= 1 \\ a_{-3} &= 1 \\ a_2 &= \frac{1}{2}e^{j\frac{\pi}{3}} = \frac{1}{4}(1 + \sqrt{3}j) \\ a_{-2} &= \frac{1}{2}e^{-j\frac{\pi}{3}} = -\frac{1}{4}(1 - \sqrt{3}j) \end{aligned}$$

**Part c** The output is

$$y[n] = |H(\Omega)|_{\Omega=\frac{\pi}{4}} \cos\left(\frac{\pi}{4}n + \frac{\pi}{3} + \arg H(\Omega)_{\Omega=\frac{\pi}{4}}\right) \quad (1)$$

But

$$\begin{aligned} |H(\Omega)| &= \left| \frac{1 - e^{-j\Omega}}{2 + e^{-2j\Omega}} \right| \\ &= \frac{|1 - e^{-j\Omega}|}{|2 + e^{-2j\Omega}|} \\ &= \frac{\sqrt{(1 - \cos \Omega)^2 + \sin^2 \Omega}}{\sqrt{(2 + \cos 2\Omega)^2 + \sin^2(2\Omega)}} \end{aligned}$$

When  $\Omega = \frac{\pi}{4}$  the above becomes

$$\begin{aligned}
 \left| H\left(\frac{\pi}{4}\right) \right| &= \frac{\sqrt{\left(1 - \cos\left(\frac{\pi}{4}\right)\right)^2 + \sin^2\left(\frac{\pi}{4}\right)}}{\sqrt{\left(2 + \cos\left(\frac{\pi}{2}\right)\right)^2 + \sin^2\left(\frac{\pi}{2}\right)}} \\
 &= \frac{\sqrt{\left(1 - \cos\left(\frac{\pi}{4}\right)\right)^2 + \sin^2\left(\frac{\pi}{4}\right)}}{\sqrt{4 + 1}} \\
 &= \frac{\sqrt{\frac{3}{2} - \sqrt{2} + \frac{1}{2}}}{\sqrt{5}} \\
 &= \frac{\sqrt{2 - \sqrt{2}}}{\sqrt{5}} \\
 &= 0.34228
 \end{aligned}$$

And

$$\begin{aligned}
 \arg H(\Omega) &= \arg \frac{1 - e^{-j\Omega}}{2 + e^{-2j\Omega}} \\
 &= \arg \frac{(1 - \cos \Omega) + j \sin \Omega}{(2 + \cos(2\Omega)) - j \sin(2\Omega)} \\
 &= \arctan\left(\frac{\sin \Omega}{1 - \cos \Omega}\right) - \arctan\left(\frac{-\sin(2\Omega)}{2 + \cos(2\Omega)}\right)
 \end{aligned}$$

When  $\Omega = \frac{\pi}{4}$  the above becomes

$$\begin{aligned}
 \arg H\left(\frac{\pi}{4}\right) &= \arctan\left(\frac{\sin\left(\frac{\pi}{4}\right)}{1 - \cos\left(\frac{\pi}{4}\right)}\right) - \arctan\left(\frac{-\sin\left(\frac{\pi}{2}\right)}{2 + \cos\left(\frac{\pi}{2}\right)}\right) \\
 &= \arctan\left(\frac{\sin\left(\frac{\pi}{4}\right)}{1 - \cos\left(\frac{\pi}{4}\right)}\right) - \arctan\left(\frac{-1}{2}\right) \\
 &= \arctan\left(\frac{\sin\left(\frac{\pi}{4}\right)}{1 - \cos\left(\frac{\pi}{4}\right)}\right) + \arctan\left(\frac{1}{2}\right) \\
 &= \arctan(2.4142) + 0.46365 \\
 &= 1.1781 + 0.46365 \\
 &= 1.6418 \text{ rad}
 \end{aligned}$$

Hence (1) becomes

$$y[n] = 0.3423 \cos\left(\frac{\pi}{4}n + \frac{\pi}{3} + 1.6418\right)$$