ECE 3015 - Final Exam
Sample 1

1. **(25 points)** The Laplace transforms of the natural and forced response for a system are given by

$$Y^{(n)}(s) = \frac{3s+5}{s^2+3s+2}; \quad Y^{(f)}(s) = \frac{6\,\cancel{36}}{s^3+3s^2+2s}$$

(a) Find the natural response $y^{(n)}(t)$.

$$Y^{(n)}(s) = \frac{3s+5}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1} \qquad A = \left.\frac{3s+5}{s+1}\right|_{s=-2} = \frac{-1}{-1} = 1$$

$$B = \left.\frac{3s+5}{s+2}\right|_{s=-1} = \frac{2}{1} = 2$$

$$y^{(n)}(t) = e^{-2t}u(t) + 2e^{-t}u(t)$$

(b) Find the forced response $y^{(f)}(t)$. $\quad Y^{(f)}(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$ $\quad$ (-12)

$$A = \left.\frac{6\,\cancel{36}}{(s+2)(s+1)}\right|_{s=0} = 3 \qquad B = \left.\frac{6\,\cancel{36}}{s(s+2)}\right|_{s=-1} = -6$$

$$C = \left.\frac{6\,\cancel{36}}{s(s+1)}\right|_{s=-2} = 3$$

$$y^{(f)}(t) = 3u(t) - 6e^{-t}u(t) + 3e^{-2t}u(t)$$

(-12) (9)

$c_p = 3$

Homogen. $\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 0$

(c) Find the differential equation description for this system.

Let RHS be $A x(t) + B\frac{d}{dt}x(t)$ $\quad 2c_p = 6\alpha \Rightarrow \alpha = 1$

$$\frac{6\,\cancel{36}}{s} = A X(s) + B\,sX(s) - B\,x(0^+) \qquad X(s) = \frac{\alpha}{s} \quad x(0^+) = 3$$

$$= A\frac{1}{s} + 3B - 3B$$

$$A = 6 \qquad \frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 6x(t)$$

(d) Find the initial conditions and the input. $\quad x(t) = u(t)$

$$y(0^+) = 3$$

$$\left.\frac{d}{dt}y(t)\right|_{t=0^+} = \left.-2e^{-2t} - 2e^{-t}\right|_{t=0^+} = -4$$

2. (15 points) Find the Fourier Transform of the signal

$$x(t) = \left(\frac{d}{dt}\left[te^{-|t-1|}\right]\right) * \left(\cos(2t)e^{-t}u(t)\right)$$

$$e^{-|t|} \xleftrightarrow{FT} \frac{2}{1+\omega^2}$$

$$e^{-|t-1|} \xleftrightarrow{FT} \frac{2e^{-j\omega}}{1+\omega^2}$$

$$te^{-|t-1|} \xleftrightarrow{FT} \frac{d}{d\omega}\left\{\frac{2e^{-j\omega}}{1+\omega^2}\right\} = \frac{-2je^{-j\omega}}{1+\omega^2} - \frac{2e^{-j\omega}\,2\omega}{(1+\omega^2)^2}$$

$$\frac{d}{dt}\left[te^{-|t-1|}\right] \xleftrightarrow{FT} j\omega\frac{d}{d\omega}\left\{\frac{2e^{-j\omega}}{1+\omega^2}\right\}$$

$$e^{-t}u(t) \xleftrightarrow{FT} \frac{1}{j\omega+1}$$

$$\cos 2t\, e^{-t}u(t) \longleftrightarrow \frac{1}{2}\left(\frac{1}{j(\omega-2)+1}\right) + \frac{1}{2}\left(\frac{1}{j(\omega+2)+1}\right)$$

$$X(j\omega) = j\frac{\omega}{2}\frac{d}{d\omega}\left\{\frac{2e^{-j\omega}}{1+\omega^2}\right\}\left[\frac{1}{j(\omega-2)+1} + \frac{1}{j(\omega+2)+1}\right]$$

3. (20 points) A system has impulse response

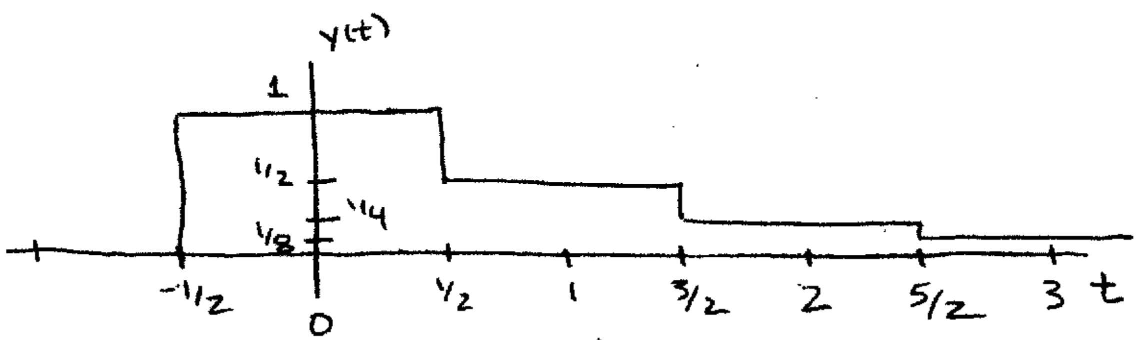
$$h(t) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \delta(t-n)$$

Use convolution to find the output $y(t)$ for an input $x(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$. Sketch $y(t)$ in the space provided.

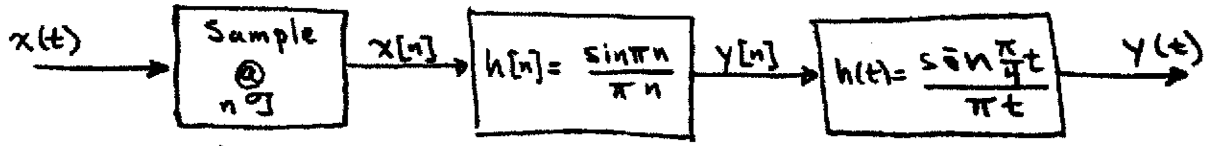
since $\delta(t-n) * x(t) = x(t-n)$

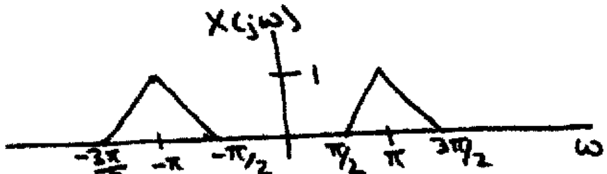
$$\sum \left(\tfrac{1}{2}\right)^n \delta(t-n) * x(t) = \sum \left(\tfrac{1}{2}\right)^n x(t-n)$$

$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n \left[u(t-n+\tfrac{1}{2}) - u(t-n-\tfrac{1}{2})\right]$$



4. (25 points) Consider the system depicted below. We have $x[n] = x(nT)$, $y[n] = h[n] * x[n]$, and $y(t) = h(t) * y_s(t)$.

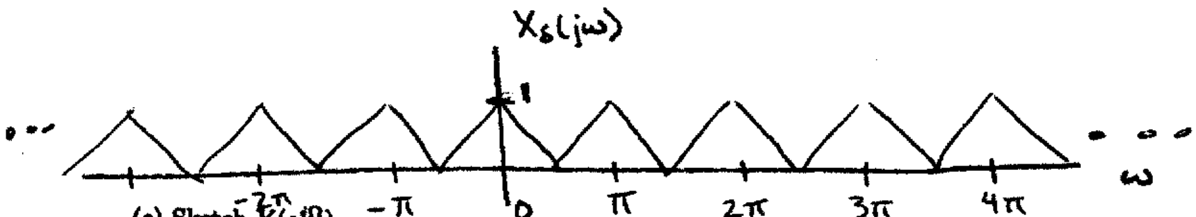




(a) Find the maximum T so that $x(t)$ can be reconstructed from $x[n]$ using a lowpass reconstruction filter.

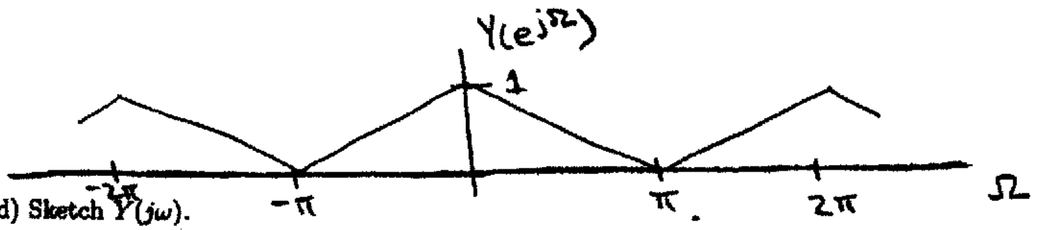
Sampling Thm $\quad \frac{2\pi}{T} > 2\left(\frac{3\pi}{2}\right) \quad T < 2/3$

(b) Let $T = 2$. Sketch $X_s(j\omega)$ in the space provided. $\quad X_s(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s)) \qquad \omega_s = \pi$



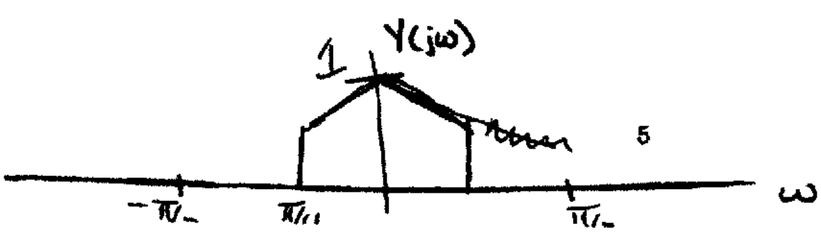
(c) Sketch $Y(e^{j\Omega})$.

$h[n] = \delta[n]$ so $y[n] = x[n] \qquad Y(e^{j\Omega}) = X_s(j\omega)\Big|_{\omega = \frac{\Omega}{T}}$

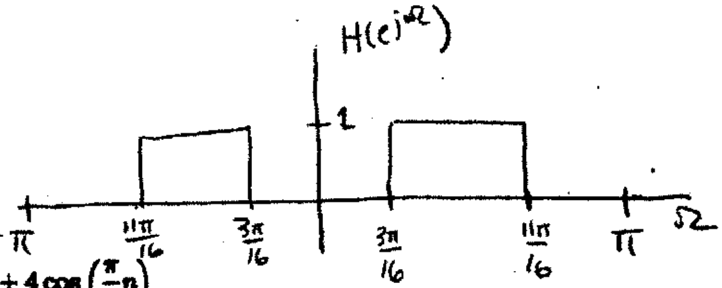


(d) Sketch $Y(j\omega)$.

$$H(j\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{4} \\ 0 & \text{other} \end{cases}$$

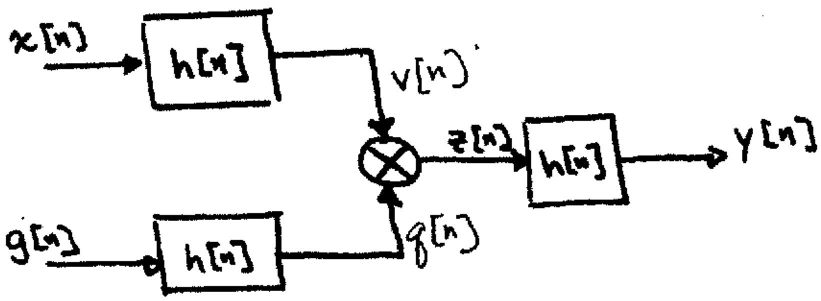


5. (15 points) A discrete-time system is depicted below. We have

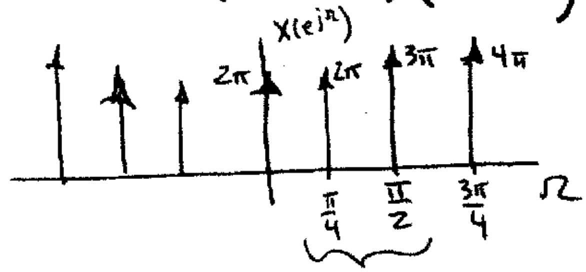


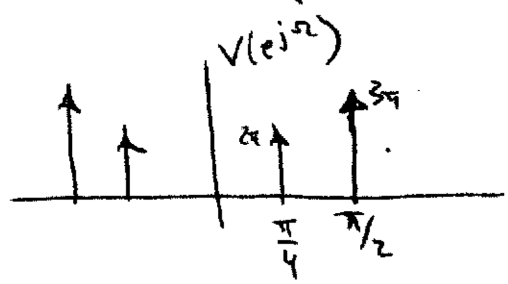
$$x[n] = \sum_{k=0}^{3}(1+k)\cos\left(k\frac{\pi}{4}n\right), \quad g[n] = 2 + 4\cos\left(\frac{\pi}{4}n\right)$$

$$h[n] = \frac{2\sin\left(\frac{\pi}{4}n\right)}{\pi n}\cos\left(\frac{7\pi}{16}n\right)$$



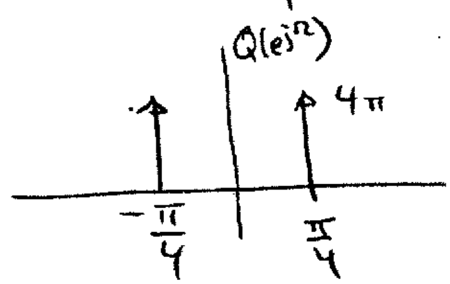
$z[n] = (h[n]*x[n])(g[n]*h[n])$

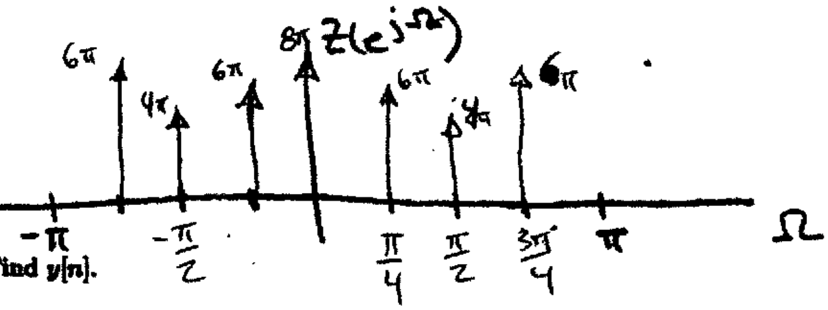




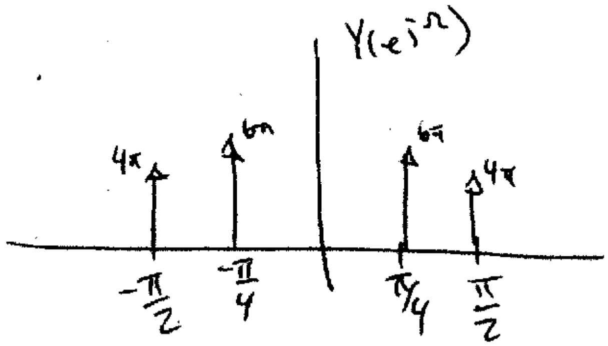
(a) Sketch $Z(e^{j\Omega})$.

$$Z(e^{j\Omega}) = \frac{1}{2\pi} V(e^{j\Omega}) \circledast Q(e^{j\Omega})$$





(b) Find $y[n]$.



$$y[n] = 6\cos\frac{\pi}{4}n + 4\cos\frac{\pi}{2}n$$