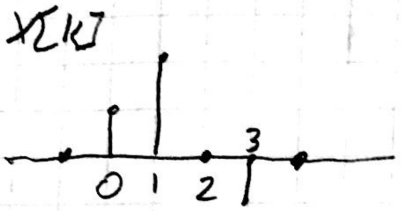
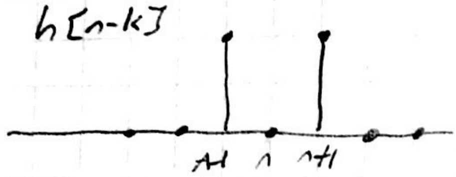
Homework 2 solutions:

Email me at leex8370@umn.edu if you see any mistakes.

2.1 a) $y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$





$n < -1 \quad y_1[n] = 0$

$n = -1 \quad y_1[n] = 2$

$n = 0 \quad y_1[n] = 4$

$n = 1 \quad y_1[n] = 2$

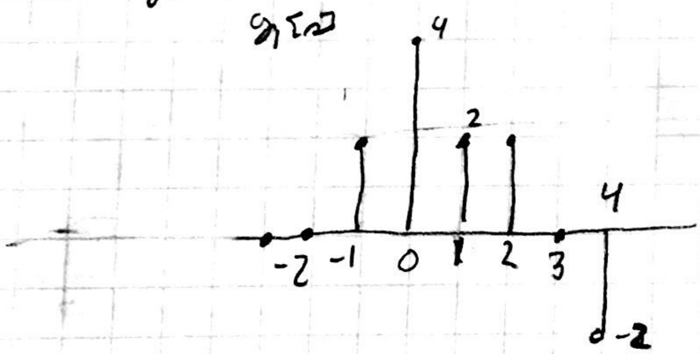
$n = 2 \quad y_1[n] = 2$

$n = 3 \quad y_2[n] = 0$

$n = 4 \quad y_4[n] = -2$

$n > 4 \quad y_1[n] = 0$

$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$

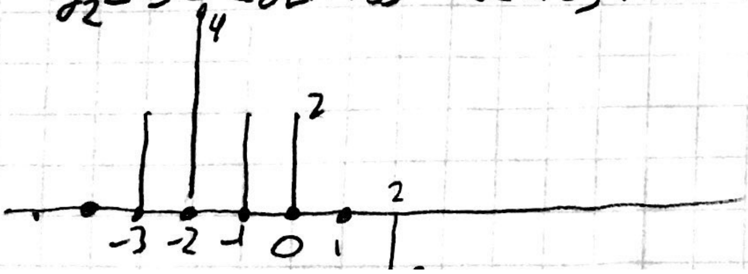


$y_2[n] = x[n+2] * h[n] = \delta[n+2] * x[n] * h[n]$

$= \delta[n+2] * y_1[n] = y_1[n+2]$

Shift by 2:

$y_2[n] = 2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n+2]$

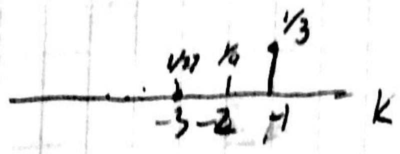


2.6 compute & plot $y[n] = x[n] * h[n]$

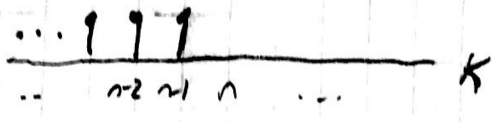
$x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1] \qquad h[n] = u[n-1]$

0 for $n \geq +1$

$x[k] =$



$h[n-k]$



for $n-1 \geq -1$, we have total overlap.

Then $y[n] = \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k}$

let $k = -p-1$

$$y[n] = \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{(p+1)}$$

$$= \frac{1}{3} \cdot \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p} = \frac{1}{3} \cdot \frac{1}{1-\frac{1}{3}}$$

$$= \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$$

for $n < 0$

we have $y[n] = \sum_{k=-\infty}^{n-1} \left(\frac{1}{3}\right)^{-k}$

let $k = -p+n-1$

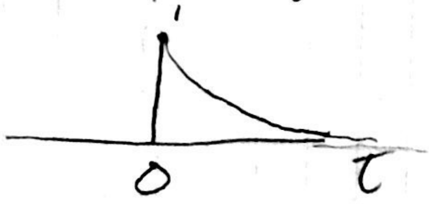
$$\sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p-n+1}$$

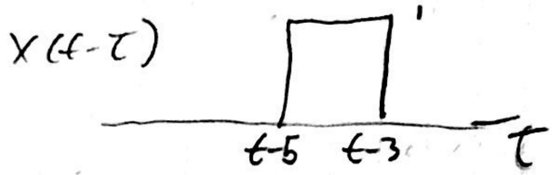
$$= 3^n \sum_{p=0}^{\infty} \left(\frac{1}{3}\right)^{p+1} = \frac{1}{2} \cdot 3^n$$

2.11 $x(t) = u(t-3) - u(t-5)$

$h(t) = e^{-3t}u(t)$

a) compute $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$





$t < 3$ no overlap $y(t) = 0$

$3 < t < 5$ $\quad y(t) = \int_0^{t-3} e^{-3\tau}\,d\tau = \dfrac{-e^{-3\tau}}{3}\Big|_0^{t-3} = \dfrac{1 - e^{-3(t-3)}}{3}$

$t > 5$ $\quad y(t) = \int_{t-5}^{t-3} e^{-3t}\,dt = \dfrac{-e^{-3t}}{3}\Big|_{t-5}^{t-3} = \dfrac{e^{-3(t-5)} - e^{-3(t-3)}}{3}$

b) $g(t) = \dfrac{dx(t)}{dt} * h(t)$

$\dfrac{dx(t)}{dt} = \dfrac{du(t-3)}{dt} - \dfrac{du(t-5)}{dt} = \delta(t-3) - \delta(t-5)$

$g(t) = (\delta(t-3) - \delta(t-5)) * h(t)$

$= h(t-3) - h(t-5)$

↳ convolving with delta results in time shift.

$= e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)$

c $\quad \dfrac{dy(t)}{dt} = \begin{cases} 0 & t<0 \\ e^{-3(t-3)} & 3<t<5 \\ e^{-3(t-5)} - e^{-3(t-3)} & t>5 \end{cases} = g(t)$

so $g(t) = \dfrac{dy(t)}{dt}$

2.4.

$h_2[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$

$x[n] \rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n]$

overall system response $= h_1[n] * h_2[n] * h_2[n]$

$h_2[n] * h_2[n] = (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1])$

$= \delta[n] + 2\delta[n-1] + \delta[n-2]$

$h[n] = h_1[n] * (\delta[n] + 2\delta[n-1] + \delta[n-2])$

$= h_1[n] + 2h[n-1] + h[n-2]$

$h_1[n]$ is causal, so $h_1[n] = 0$ for $n < 0$.

Then $h[0] = h_1[0] = 1$

$h[1] = h_1[1] + 2h_1[0] = 5$

$\Rightarrow h_1[1] = 3$

$h[2] = h_1[2] + 2h_1[1] + h_1[0] = 10$

$\Rightarrow h_1[2] = 3$

$h[3] = h_1[3] + 2h_1[2] + h_1[1] = 11$

$h_1[3] = 2$

$h[4] = h_1[4] + 2h_1[3] + h_1[2] = 8$

$h_1[4] = 1$

$h[5] = h_1[5] + 2h_1[4] + h_1[3] = 4$

$h_1[5] = 0$

$h[6] = h_1[6] + 2h_1[5] + h_1[4] = 1$

$h_1[6] = 0$

$h[7] = h_1[7] + 2h_1[6] + h_1[5] = 0$

$\vdots$

b) $y[n] = h[n] * (\delta[n] - \delta[n-1]) = h[n] - h[n-1]$

4. $y[n] - \frac{1}{2}y[n-1] = x[n]$

$x[n] = (1/3)^n u[n]$

Assume there's a particular & homogeneous soln satisfying

$y_h[n] - 1/2\, y_h[n-1] = 0$

a) verify that $y_h[n] = A(1/2)^n$

$A(1/2)^n - 1/2 A(1/2)^{n-1} = A(1/2)^n - 2 \cdot 1/2 A(1/2)^n = 0$

b) consider $y_p[n]$ st

$y_p[n] - 1/2\, y_p[n-1] = (1/3)^n u[n]$

Assume $y_p[n]$ has form $B(1/3)^n$, find B

$n \geq 0$

$B(1/3)^n - 1/2 B(1/3)^{n-1} = (1/3)^n$

$B(1/3)^n - 3/2\, B(1/3)^n = (1/3)^n$

$B - 3/2 B = 1$

$-1/2 B = 1$

$B = -2$

c) given that $y[0] = x[0] + 1/2\, y[-1]$

$= x[0] = 1$

$y[0] = A(1/2)^0 + B(1/3)^0 = 1$

$= A + B = 1$

$A = 1 - B = 1 - -2 = 3$

2.42 $x(t) = u(t+0.5) - u(t-0.5)$

a) $h(t) = e^{j\omega_0 t}$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)$$

$$= \int_{-\infty}^{\infty} [u(t-0.5-\tau) - u(t+0.5-\tau)] e^{j\omega_0 \tau} d\tau$$

$$= \int_{t-0.5}^{t+0.5} e^{j\omega_0 \tau} d\tau$$

only interested in $y(0)$

$$y(0) = \int_{-0.5}^{0.5} e^{j\omega_0 \tau} d\tau$$

$$= \frac{e^{j\omega_0 \tau}}{j\omega_0}\Bigg|_{\tau=-0.5}^{0.5} = \frac{e^{j\omega_0 .5} - e^{-j\omega_0 .5}}{j\omega_0}$$

$$= \frac{2}{\omega_0}\left[\frac{e^{j\omega_0 .5} - e^{-j\omega_0 .5}}{2j}\right]$$

$$= \frac{2}{\omega_0} \sin(\omega_0/2)$$

for this to equal to zero we need $\sin(\omega_0/2) = 0$, & ω_0 not $= 0$

$\sin(\omega_0) = 0$ when $\omega_0 = 2\pi n$ for $n \in \mathbb{Z}$

To see what happens for $\omega_0 = 0$, take the limit

l'Hopital's rule

$$\lim_{\omega_0 \to 0} 2\frac{\sin(\omega_0/2)}{\omega_0} = \lim_{\omega_0 \to 0} 2\frac{\cos(\omega_0/2)}{1} = 2$$

which is why we cannot have $n = 0$.

b) the answer is not unique. $\omega_0 = 2\pi n$, $n \in \mathbb{Z} - \{0\}$

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