1. (30 points)

(1) Define the Fourier series over the interval $-c < x < c$ corresponding to piecewise continuous function $f(x)$.

(2) State the convergence theorem for such Fourier series.

(3) For what value a does the Fourier series over the interval $-1 < x < 1$ corresponding to the function

$$f(x) = e^x + a\,x$$

converge to $f(x)$ at $x = 1$.

2. (30 points)

Find eigenvalues and corresponding eigenfunctions.

$$X''(x) + \lambda X(x) = 0, \quad 0 < x < 1$$

subject to the boundary conditions $X'(0) = 0$ and $X(1) = 0$.

3. (40 points)

Solve the boundary value problem

$$y_{tt}(x,t) = y_{xx}(x,t) - y(x,t),\ 0 < x < \pi,\ t > 0;$$

$$y(0,t) = y(\pi,t) = 0;\ y(x,0) = 0,\ y_t(x,0) = 1.$$