

Study notes

Math 2243

Linear Algebra and Differential Equations

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# 1 How to solve some problems

1. Problem gives set  $S$  of vectors  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$  and asks to find basis that span  $S$  consisting of elements from  $S$ . To answer this, write the vectors as columns of matrix  $A$ . Then convert the matrix to Echelon form (Row reduction). The pivot columns in  $A$  are the basis.
2. Problem gives set  $S$  of vectors  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$  and just asks to find basis that span  $S$ . It does not say consisting of elements from  $S$ . This was not clear and I asked about it. The answer I got is to use same method as above, and that will work also.
3. Problem gives set  $S$  of vectors  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$  say in  $\mathbb{R}^3$  and asks to find basis for  $\mathbb{R}^3$  that contains  $S$ . Here, we also set up the matrix  $A$  where the first 3 columns are these vectors, but also add 3 more columns, which are  $(1, 0, 0), (0, 1, 0), (0, 0, 1)$  and now convert  $A$  to Echelon form and the pivot columns of  $A$  are the basis. The difference between this and above, is that we append the elementary basis for  $\mathbb{R}^3$  to the matrix  $A$  before starting.
4. Problem gives  $A$  matrix and asks to find is NULL space. This is asking for basis of solution space for  $A\vec{x} = \vec{0}$ . To solve, convert  $A$  to Echelon form. (no need to do reduced Echelon form). Then the number of the free variables is the dimension of the NULL space. So if we have 2 free variables, the NULL space is 2 dimensions. Call the free variables  $s, t$  and so on. Then solve for the leading variables in terms of the free variables. Then at end let  $t = 1, r = 1$  and this gives the basis for the NULL space.
5. Problem gives set of vectors  $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$  and asks if they are linearly independent or not. If the dimension of each vector is the same as the number of the vectors, then make a square matrix  $A$  of the vectors as its columns and find the determinant. If  $|A| = 0$  then the vectors are linearly independent, else they are not.

Another way to do this is to write  $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$  and solve for  $c_i$  and see if the only solution is  $c_i = 0$ . If so, then linearly independent, else not.

6. Problems gives  $A$  matrix and asks for its column space and its row space. To solve, reduce the matrix to Echelon form. The row space are those rows which are not all zeros. The column space are the pivot columns in the original  $A$  (not the pivot columns in the final Echelon form matrix).
7. Problem gives set of vectors  $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$  and one vector  $\vec{w}$  and asks if  $\vec{w}$  is linear combinations of the vectors in  $S$ . To solve, write  $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{w}$  and set up  $A\vec{x} = \vec{b}$ . Then set up the augmented matrix  $[A|\vec{b}]$ . Reduce to Echelon form. Now see if it is consistent or not. If not consistent, then there is no solution and they it means  $\vec{w}$  can not be written as linear combination. If consistent, then this means we can write  $\vec{w}$  as linear combination (there can be infinite ways to do this).
8. Problem gives set of vectors  $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$  and asks if  $S$  spans all of  $\mathbb{R}^n$ ? Let  $n = 3$  for example. If we can find 3 of vectors from  $S$  that are linearly independent, then the answer is yes. Otherwise no.
9. Problem gives set of vectors  $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$  and asks if these vectors are basis for  $\mathbb{R}^n$ ? This is similar to the above. The difference is that the set  $S$  must contain only 3 vectors and no more, which are linearly independent. These they are basis. This means any vector  $\vec{w}$  can be expressed as linear combination of the basis in one unique way.
10. Problem gives square matrix  $A$  and asks to find its inverse  $A^{-1}$ . To solve, set up the augmented matrix by appending to the right side the identity matrix  $I$ . Then convert the whole augmented matrix to Echelon form, and now convert this to reduced Echelon form. When done, the right side (which was  $I$  initially) is  $A^{-1}$ .
11. Problem gives square matrix  $A$  and asks to find its determinant. To solve, look first

if possible to do any row operations to increase the number of zeros in the matrix. Then expand along one row or one column that has most zeros in it. Remember the sign is found using  $(-1)^{m+n}$  where  $m$  is row number and  $n$  is column number.

12. Problem gives  $A\vec{x} = \vec{b}$  and ask what kind of solutions are possible? There are only three possible solutions: No solution, one unique solution, or an infinite number of solutions. So was can not have for example 2 or 3 solutions. This is not possible.
13. Problem gives matrix  $A, B$  and asks to find matrix  $X$  such that  $AX = B$ . To solve, premultiply both sides by  $A^{-1}$  to get  $X = A^{-1}B$ . So we need to find  $A^{-1}$  then do matrix multiplication to find  $X$ .

## 2 Some definitions

span of set of vectors given set  $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$ , then the span of  $S$  is the set  $W$  of all possible linear combinations of elements of  $S$ .