Homework 13

As we are rapidly approaching the final exam, this homework will not be collected or graded. These problems are simply to aid you in studying for the exam. As such, I will not be providing a specific list of textbook problems beyond that on the review sheet. If you can comfortably do two or three of each of the problem types from the textbook, you are probably safe to skip the rest. I will only type up solutions for the additional problems, but I encourage you to check your answers for the textbook problems with the back of the book.

Textbook Problems:

- §2.2: 1-12, 20-22
- §2.3: 1-3, 9-12
- §2.4: 1-10

Additional Problems:

1. Write a differential equation of the form $\frac{dx}{dt} = f(x)$ that has a stable equilibrium at $x = 5$ and an unstable equilibrium at $x = 1$.
2. Write a differential equation of the form $\frac{dx}{dt} = f(x)$ that has stable equilibria at $x = 5$ and $x = 7$ and an unstable equilibrium at $x = 1$.
3. Formula One cars have a feature called the drag reduction system (DRS) which opens a flap on the rear wing to decrease drag at particular points in the race and facilitate overtaking. For the purposes of this problem, we will assume that the cars have constant acceleration and drag proportional to velocity, so that

$$\frac{dv}{dt} = a - \rho v$$

where ρ is a positive constant called the *drag coefficient*. The *top speed* of a car is defined to be $\lim_{t\to\infty} v(t)$, otherwise known as the *terminal velocity.*

   (a) The differential equation $\frac{dv}{dt} = a - \rho v$ is separable. Solve it (leaving a and ρ as constants), and find the particular solution when $v(0) = v_0$. Find the top speed $\lim_{t\to\infty} v(t)$.

   (b) Under normal conditions, the top speed of a car is 85 m/s. With the DRS active, the top speed increases to 90 m/s. If the car's engine provides a constant acceleration of 14 m/s^2, what is the drag coefficient with and without DRS?

   (c) Two cars exit a corner at 25 m/s, with car A 10 meters behind car B. At the same time, they both begin accelerating at 14 m/s^2. However, car A has DRS enabled while car B does not. Using the drag coefficients calculated in the previous problem, determine how much time it will take for car A to be right next to car B. *(The position equations will involve both exponential and polynomial terms. I recommend using a computer algebra system to solve for the time where $x_A(t) = x_B(t)$)*

There are 500 meters from the corner exit to where the drivers must begin braking for the next corner. Which car will be ahead when they brake for the next corner?

(d) Formula One cars are exceptionally "draggy," and at high speed can get more deceleration from just their aerodynamics than a road car gets from slamming on the brakes. With no throttle or brakes, we only have drag acting on the car, so that

$$\frac{dv}{dt} = -\rho v$$

Say that a driver's brakes have failed during a race. If they can slow the car to 10 m/s by the time they reach the pit lane, their mechanics will be able to safely bring the car to a stop. If the driver is currently traveling at 80 m/s, how far before the pit lane must they begin coasting? Use the non-DRS drag coefficient you calculated in part (b).