Homework 12 - Due December 15

Homework instructions: Complete the assigned problems on your own paper. Once you are finished, scan or photograph your work and upload it to Gradescope. When prompted, tell Gradescope where to find each problem.
You are allowed (and in fact encouraged) to work with other students on homework assignments. If you do that, please indicate on each problem who you worked with. If you use sources other than your notes, the textbook, and any resources on Canvas for your homework, you must indicate the source on each problem. You are not permitted to view, request, or look for solutions to any of the homework problems from solutions manuals, homework help websites, online forums, other students, or any other sources.

Textbook Problems:

- §1.4: 4, 17, 19, 33, 43
- §1.5: 3, 17, 37
- §2.1: 15, 16, 17

Additional Problems:

1. This problem will discuss two different ways to solve the differential equation $y' + y = e^x$.

   (a) Using the methods of chapter 5, solve the homogeneous linear differential equation with constant coefficients
   $$y' + y = 0$$

   (b) Use the method of undetermined coefficients to find a particular solution to
   $$y' + y = e^x$$

   (c) Using (a) and (b), write the general solution of
   $$y' + y = e^x$$

   (d) We can also view this differential equation as a first-order linear differential equation of the type discussed in section 1.5. For the differential equation
   $$y' + y = e^x$$
   what are the functions $P(x)$ and $Q(x)$?

   (e) Use the method of integrating factors to solve
   $$y' + y = e^x$$

(f) Compare your answers in (c) and (e). Do they describe the same solutions to the differential equation?

2. This problem will explore the spread of Green's disease (a highly contagious illness that causes green skin and no other symptoms) in a city of 100,000 residents. There is currently no cure, and it appears that those who catch the disease remain infectious forever.

   (a) We will assume that the number $P(t)$ of positive cases satisfies a logistic equation of the form
   $$\frac{dP}{dt} = kP(M - P)$$
   This essentially says that the number of new infections each day depends on the number of currently infected individuals *and* on the number of remaining susceptible individuals.

   On day $t = 0$, there are 5,000 positive cases identified in the city. Leaving k as an unknown constant, what is the initial value problem (differential equation and initial condition) satisfied by $P(t)$?

   (b) Solve the initial value problem you wrote in (a). Show all steps – do not use the formula for solutions of logistic equations given in the textbook.

   (c) On day $t = 0$, there are 500 new cases being identified each day. Determine the value of k.

   (d) After how many days will half the population of this city have contracted Green's disease?

   (e) There's a saying in this area of mathematics that "all models are wrong, but some are useful." In the last year or so, we've seen that even the most sophisticated models of disease spread will never be perfectly accurate. Still, they remain useful tools for policy makers and public health officials.

   In a few sentences, reflect on the model for disease spread you explored in this problem. What useful information does it tell us, and in what ways is the model likely to be wrong?