The University of California, Davis
Department of Mechanical and Aeronautical Engineering
MAE 222 Advanced Dynamics

Lecture: TR 10:00-11:50, 1007 Giedt Hall

Fall 2010 Prof. Karnopp, Rm. 2012 Bainer, 752-3606, dckarnopp@ucdavis.edu

Text: *Dynamics of Mechanical and Electromechanical Systems*, Crandall, Karnopp, Kurtz, Pridmore-Brown; Krieger Publishing Co.

Grading: Midterm Exam. 30%, Final Exam. 60%, Homework10%

Lecture	Date	Topic	Reading	Problems
1	9/23	Variational principles	pp.1-22	1-1,2,9,10
2	9/28	Variational principles	pp.22-34	1-14,17,20,22
3	9/30	for dynamics		
4	10/5	Kinematics	pp.42-68	2-1,3,5,6,7,13,16
5	10/7			
6	10/12	State functions	pp. 68-110	2-19,20,24,25,26,27
7	10/14			
8	10/19	Hamilton's principle,	pp. 110-146	2-41,47,49,50
9	10/21	Lagrange's equations		
10	10/26	Rigid bodies	pp. 165-186	3-1,2,3,7,14
11	10/28	.		
12	11/2	Inertia tensor	pp. 186-202	3-18,19,23
13	11/4	MIDTERM EXAM		
14	11/9	Rigid body dynamics	pp.208-226	4-1,2,14
15	11/11	HOLIDAY (Veteran's Day)		
16	11/16			
17	11/18	Euler's equations	pp. 226-246	4-15,17,18,20,25
18	11/23	Gyroscopes		
19	11/25	HOLIDAY (Thanksgiving)		
20	11/30	Review		
21	12/2	FINAL EXAM		

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Notes for MAE 222 Advanced Dynamics

This course will emphasize the connections between two quite different ways to arrive at mathematical descriptions of dynamic mechanical systems. (For those interested, the text also shows that two similarly distinct methods apply to electromechanical and other mixed energy systems.)

The "direct" method, based on Newton's law is universally used in basic courses in dynamics such as our own E102, but we will also investigate the "indirect" method based on variational principles that often leads to simpler formulations of the equations of motion for a system. The derivation of the variational method involves the calculus of variations rather than the more familiar differential calculus. Typically, engineering students do not encounter this topic until graduate school. Part of the point of this course is to demonstrate that just because a topic is relegated to graduate education does not mean that it is necessarily more difficult to understand than more conventional undergraduate topics. As you will learn, there are many dynamic systems that are easy to describe with Lagrange's equations, which arise from variational principles, and yet are very hard to describe correctly using the apparently straightforward Newton's laws.

An inspirational book on variational principles is *The Variational Principles of Mechanics*, by Cornelius Lanczos, Mathematical Expositions No. 4, the University of Toronto Press, 1949.

The opening quote in this book is a well known one (at least in Germany) from *Faust, Part I*, by **Johann Wolfgang von Goethe, 1749-1832:**

> *Was du ererbt von deinen Vätern hast,*
> *Erwirb es um es zu besitzen.*
>
> (What you have inherited from your fathers
> must be earned in order to be possessed.)

In the preface, Lanczos writes:

The variational principles of mechanics are firmly rooted in the soil of that great century of Liberalism which starts with Descartes and ends with the French revolution and which has witnessed the lives of Leibniz, Spinoza, Goethe, and Johan Sebastian Bach. It is the only period of cosmic thinking in the entire history of Europe since the time of the Greeks. If the author has succeeded in conveying and inkling of this cosmic spirit, his efforts will be amply rewarded.

Short Sketches of the Lives of Some Important Figures in the Historical Development of Dynamics

Many of the famous scientists of the past did not confine themselves to scientific work but also were prominent members of society at large. For this reason, it is interesting to know a little about the lives of people whose names are associated with the theorems, procedures and principles we will be studying. It goes without saying that the non-scientific activities of the prominent scientists of the past were not always completely successful. For example, this is what Napoleon had to say about **Pierre-Simon Laplace 1749-1827** who was Minister of the Interior of France for only six weeks in 1799:

[that] mathematician of the first order, Laplace, rapidly revealed himself as a mediocre administrator; he carried into administration the spirit of the infinitely small.

Here are some of the people whose names are associated with the development of the topics we will be studying. If you are interested, more extensive biographies are easily discovered on the internet.

Rene Descartes 1596-1650, philosopher and scientist. He was French, but retired to Holland in 1628 and was called to Sweden by Queen Christina in 1649 where he was unable to endure the climate and died. He originated Cartesian coordinates and exponential notation and was the founder of analytical geometry.

Gottfried William, Baron von Leibniz 1646-1716. A philosopher and mathematician, he studied in Leipzig and Jena and received the Doctor of Law title in Altdorf in 1666. He never had an academic position but worked for the Elector of Mainz. He tried to persuade Louis XIV of France to attack Eqypt and to leave Germany alone. In the period1672-1676 he developed the infinitesimal calculus independently from Newton. His work was published in 1648, three years before Newton's work was published. We presently use his notation for derivatives.

Newton was concerned with the rate of change of momentum but Leibniz preferred to deal with another quantity that he called "vis viva" which, except for a factor of ½ , is what we call "kinetic energy. His idea was that the change in kinetic energy was equal to the work done by the force. He also worked with a quantity that we now call "potential energy". His idea was to consider the kinetic and potential energies of a system as a whole and to apply a variational principle to these quantities. Newton's ideas lead to the vectorial treatment of mechanics, while Leibniz's concepts formed the basis of analytical mechanics.

Francois Marie Aronet (pen name **Voltaire**) **1694-1778** satirized Leibniz's "best of all possible worlds" in *Candide.* The comic figure Dr. Pangloss continually insisted that everything was for the best no matter what disaster befell the poor Candide. This was an unfair extension of the variational mathematics used in describing mechanics to a description of the natural world in which it was imagined that whatever actually occurred in nature was in a sense optimum. To Voltaire this was obviously nonsense.

The concept of enlarging reality by including tentative possibilities and selecting one of these by a minimization or optimization process leads to a variational as opposed to a causal approach to the natural world. Leibniz also proposed a "principle of continuity" *i.e.* "nature makes no leaps".

Sir Isaac Newton 1642-1724 was a professor at Cambridge 1669-1701. His most important work was done in the period 1665-1666. His most famous work *Philosophiae naturalis principia mathematica* was published in 1687.

He became a member of parliament in 1689, warden of the Royal Mint in 1696 and Master of the mint in 1699. This made him a wealthy man.

He was elected president of the Royal Society in 1703 and reelected every year until his death. He became involved in a bitter dispute with Leibniz concerning who should get credit for inventing calculus and had what appears to behave been a nervous breakdown. His assistant Whiston wrote:

> *Newton was of the most fearful, cautious and suspicious temper that I ever knew.*

Later in life Newton devoted his time to theology, history, chronology and alchemy. He did not write "Newton's laws" in the form we see them in textbooks today but he provided the foundation for the vectorial mechanics universally presented in introductory engineering courses.

Pierre Louis Moreau de Maupertuis 1689-1759 was a French mathematician and astronomer who confirmed Newton's theory of the flattening of the poles of the earth by leading an expedition to Lapland. He misused variational principles in support of teleological and theological arguments. Voltaire satirized him as well as Leibniz.

Jean le Rond d'Alembert 1717-1783 was the illegitimate son of the chevalier Destouches and was named for the St. Jean le Rond church on whose steps he was found. He wrote a treatise on dynamics on 1743. For a time, he was co-editor with Diderot of the *Encyclopedie* but he was attacked for his unorthodox views. The *d'Alembert's principle* we use today is really due to Euler and Lagrange. d'Alembert didn't really like Newton's concept of force which he found obscure and thought should be banished. He was happier regarding the motion of systems as a whole as were Descartes and Leibniz. d'Alembert could, however, use the Newtonian force scheme to compute results.

Formula $F=ma$?

Leonard Euler 1707-1783. Euler was born in Switzerland but in 1727 he was called by Catherine I to St. Petersburg in a resarch position. Later he was mathematics professor during the period 1730-1741. Friedrich der Grosse brought him to Berlin from 1741-1766 and then he returned to St Petersburg at the urging of Catherine the Great. She provided him and his 18 dependents with a house and a cook.

Euler first wrote out the differential equations of dynamics in generality in 1747 and published them in 1749. In *Discovery of a New Principle on Mechanics* in 1752 he wrote for the first time $F_x = ma_x$, $F_y = ma_y$, $F_z = ma_z$ and he derived the famous "Euler equations" for rigid bodies. Together with the Bernoulli brothers, he gave form to the "Newtonian Mechanics" as it is presented today.

Joseph-Louis Lagrange 1736-1813 was a professor before the age of 20 at the Royal Artillery School in Turin. After recommendations from Euler and d'Alembert , Friedrich der Grosse invited him to succeed Euler as Director of Mathematics at the Berlin Academy of Science. Under Napoleon, he was a senator and count. He survived the French revolution perhaps because of his philosophy summed up in his statement:

I believe that, in general, one of the first principles of every wise man is to conform strictly to the laws of the country in which he is living, even when they are unreasonable.

He published his famous *Méchanique Analytique* in 1788. He proudly announced that this work had no need of figures. In it the well known "Lagrange's equations" were first presented. The basic mechanical principle involved was virtual work. Although it is not necessary to make elaborate figures to define constraint forces in a mechanical system when using Lagrange's technique, it is not easy to describe a complicated mechanical system without any figures at all. Lagrange was trying to make a point that using Newton's laws for a complex system often requires many figures defining internal forces and moments and careful algebraic manipulations to arrive at a set of differential equations describing the motion of the system.

William Rowan Hamilton 1805-1865. Hamilton was a Professor of Astronomy as an undergraduate at the University of Dublin. He generalized restricted variational principles and invented quaternions, which never achieved the popularity of vectors and tensors for describing mechanical systems.

Hamilton transformed the second order Lagrange equations into twice as many first order equations which , in many cases are simpler to use. He also found that principles of analytical mechanics could be applied in optics. We will generalize "Hamilton's Principle" through the use of co-energies and we will also incorporate non-conservative forces in the principle.

Ernst Mach 1838-1916. Mach was an Austrian physicist and philosopher. He taught at Graz, Prague and Vienna and studied the philosophy of science. He strove to rid science of all metaphysical and religious assumptions. He greatly influenced the development of the philosophy of "logical positivism" which aims for a pure description of natural events.

In *The Science of Mechanics* in 1893 Mach wrote about analytical mechanics thus:

No fundamental light can be expected from this branch of mechanics. On the contrary, the discovery of matters of principle must be substantially completed before we can think of framing analytical mechanics, the sole aim of which is a perfect practical mastery of problems. Whosever mistakes this situation will never comprehend Lagrange's great performance, which here too is essentially of an economical character.

This is a common view and the view of our text. It is also common to assume that variational methods and Lagrange's equations are (1) more difficult, and (2) less intuitive than vectorial methods using Newton's laws for studying mechanical dynamics. We, however, dispute these notions. Deep thinking about what forces really are has convinced many people that the forceless mechanics involving kinetic co-energy and potential energy is even philosophically better than the conventional mechanics.

Heinrich Hertz, 1857-1894, was not, as you may suppose, in the rental car business but rather was a professor of physics at the technical school at Karlsruhe and the university at Bonn. Hertz accomplished a great deal even though he did not live to see his 37th birthday. He proved that electricity can be transmitted as electromagnetic waves at the speed of light and he was the first person to broadcast and receive radio waves.

Hertz also was the inventor of a "forceless mechanics" because he felt that the concept of a force was an artifact that could not be demonstrated to have physical reality. After this course, you may come to the same conclusion.

MAE 222 ①

Thursday 9/23/10. UC Davis. Fall 2010

Concentrate on Mechanical systems.

differen between this course and lower division is that we will look at the problem in 2 different way.

F = ma is standard way. (Newton)

but for more complicated systems, F=ma become hard to work with. because system must be taken apart (free body diagrams) and some we don't want to talk about.

There is another way, we don't have to take apart. but need a principle based on variational principle, we will use Hamilton principle.

— book Variational principle of mechanics

Virtual ≡ like imaginary position
or small change from the
optimal.

<u>Virtual work</u> : imaginary work.
it is change of energy
under virtual displacement.

In Mechanics there are 3 catagories (2)
that need to be correctly formulated,
if this is done correctly, then the
Differential equations can be written.

static is easy. no motion. only need
to satisfy equilibrium.

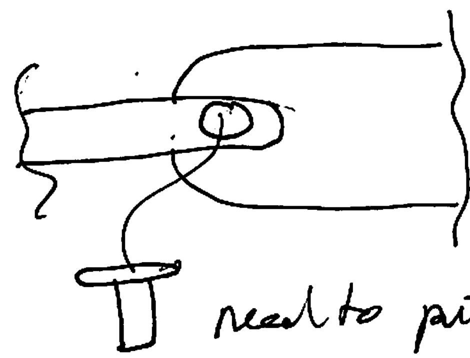
first, look for Variational principles for
~~statistics~~ statics.

Catagories

Statics
(1) Geometric constraints. (involves
velocities, deformations, positions.
(2) equilibrium ~~Cost~~ Constraints
(Forces, momentum, impulses etc.)
in statics, we need to balance the
forces. for dynamics, we need to
balance dynamics forces.

(3) Constitutive Laws
(force-deformation law, ex. spring or
elastic) or force-velocity Law
such as damper or shock observer.
or Velocity - momentum Law ($F=ma$)

Pin Connection

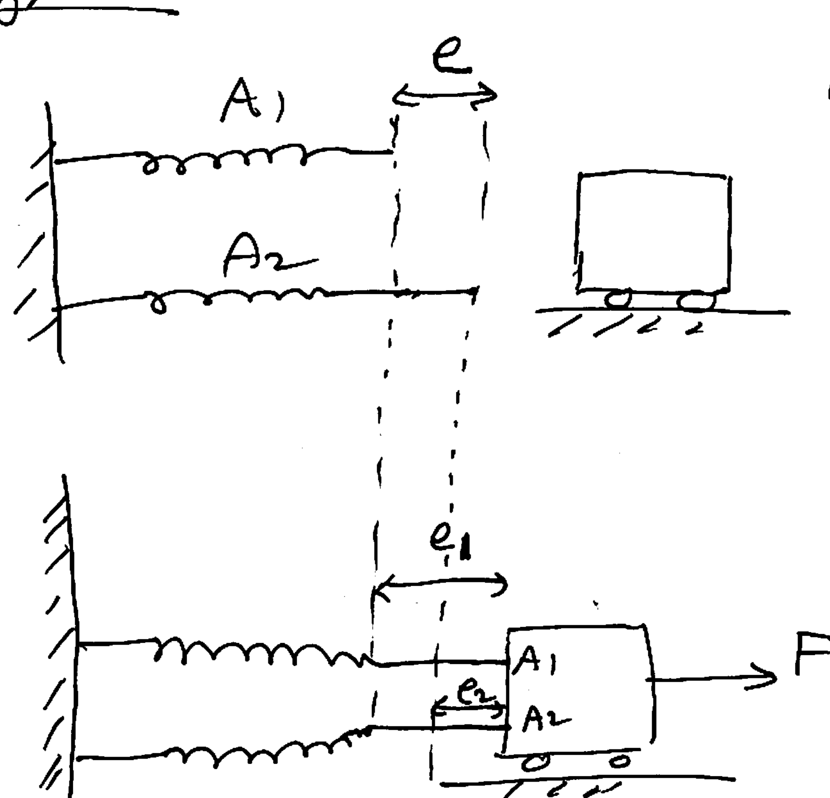


need to put pin here.

to analyse this problem, need to decide what we know, and what we do not know.

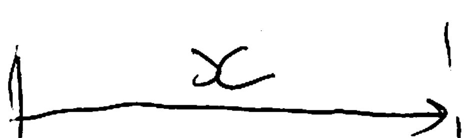
<u>system</u>

before Assembly



after assembly

Find Find position of cart.

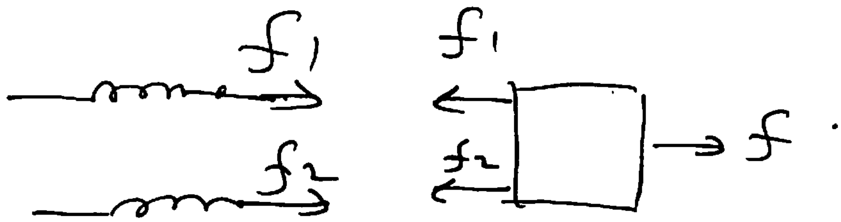


we have

- Known force
- Known initial gap e.

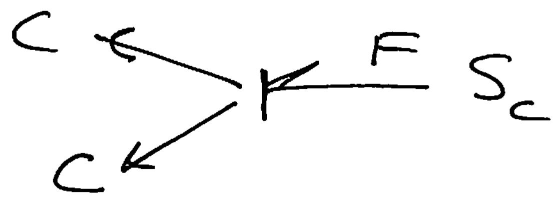
need to find x.

Free body diagram



4 unknowns, f_1, f_2, e_1, e_2.

a bond graph.



Direct method

Category 1 : geometric constraint.

$$e_1 = e + e_2$$

(written in book in different form than this but it is same)

Category 2 from free body diagram

$$f - f_1 - f_2 = 0$$

(this is F=ma)

Category 3 Constitution laws.

$f_1 = F_1(e_1)$ function of e_1

$f_2 = F_2(e_2)$ Function of e_2.

So now we 4 equations! and 4 unknowns.

<u>example</u> $f_1 = k_1 e_1$

$f_2 = k_2 e_2$

we <u>obtain</u> ~~e1~~ by solving the 4 equations.

$$e_1 = \frac{f}{k_1+k_2} + \frac{k_2 e}{k_1+k_2}$$

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now we will try to solve it in variational method:

- we accept the geometrical constraints (~~[illegible]~~ Category one).
- and accept the constitutive relations Category (3).
- and will assume the equilibrium (2) is statisfied.

we need how to talk about variations:

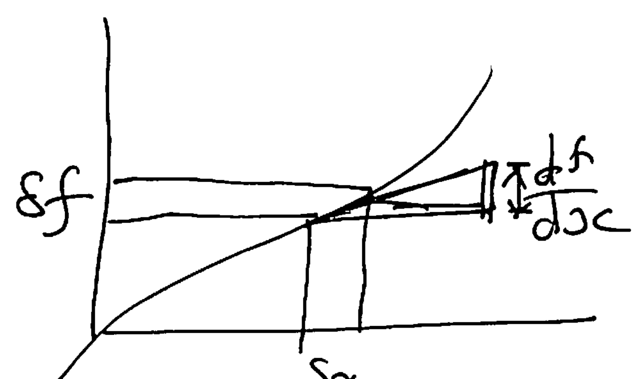
Consider hypothetical, infinitsmall (virtual) variation from a natural state in which equilibrium obtains.

From God ← i.e imagine it to be there.

beginning of Calculus of variations.

if x is a variable, then δx is its variation. (not dx, this dx is actual change) but δx is an imaginary variation.

if $f = f(x)$, then $\boxed{\delta(f) = \frac{df}{dx}\,\delta x}$

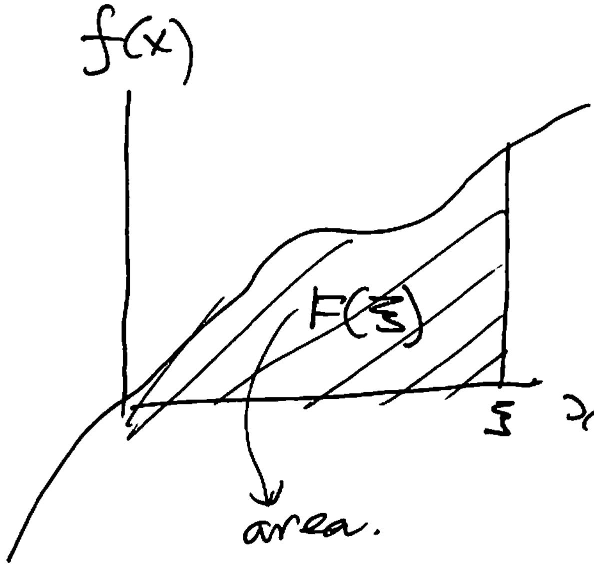


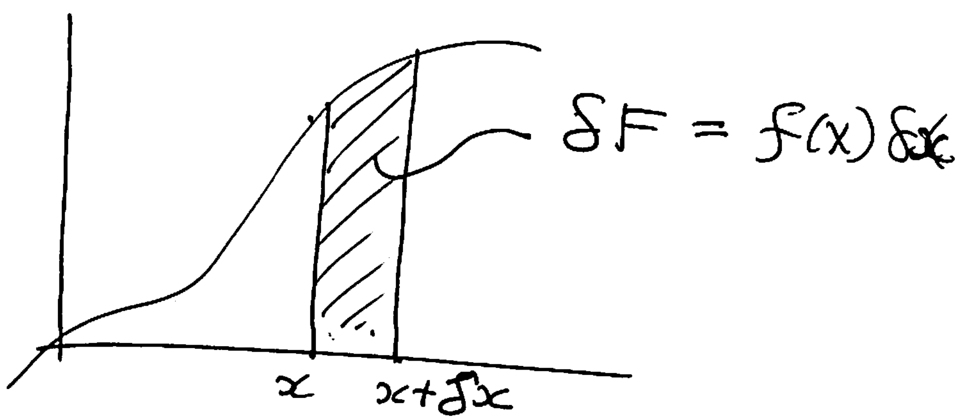
$$\delta(f) = \frac{df}{dx}\,\delta x$$

if $F(x) = \int_0^x f(\xi)\, d\xi$

will be energy

then $\delta F = f(x)\, \delta x$





now, using variational principle, we need to find principle to make $f = f_1 + f_2$ in the first example.

in the example.

$$\delta e_1 = \delta e_2 = \delta x.$$

if system in equilibrium, we ~~want~~ want

$$f - f_1 - f_2 = 0.$$

talk about work.

work done in system

$$\delta W = (f - f_1 - f_2)\,\delta x$$

δW will vanish for arbitrary δx if the system is in equilibrium

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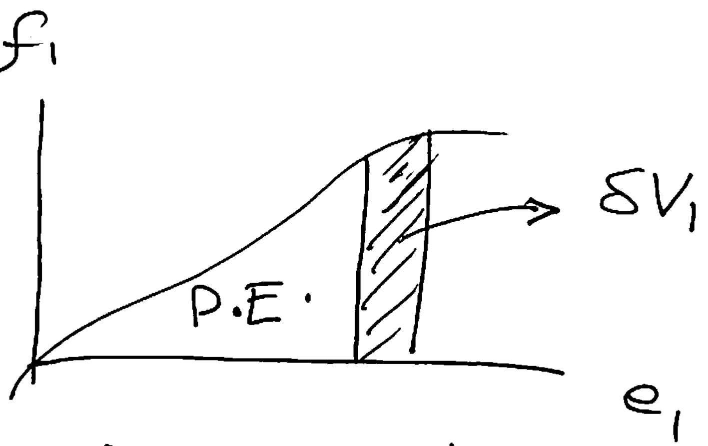
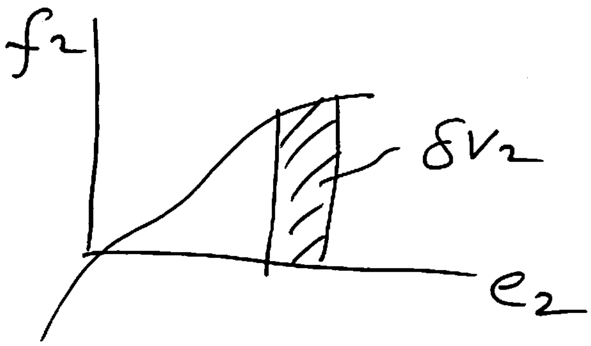
$f\,\delta x$ is virtual work.

so $\quad f\,\delta x - f_1\,\delta e_1 - f_2\,\delta e_2 \equiv$ V.I.

the variational indicator



δV_1 is change in P.E. caused by δe_1



δV_2 is change in P.E. caused by δe_2.

So $V = V_1 + V_2$

variational change in P.E. of system

so $V.I. = f\,\delta x - \delta V$

<u>note</u> V.I. must be zero if system is in equilibrium.

so V.I. will vanish (under certain conditions).

virtual work = work . virtual change in energy

<u>Example</u> Linear springs.

$$V_1 = \frac{1}{2} K_1 e_1^2 \quad \text{(energy in spring 1)}$$

$$V_2 = \frac{1}{2} K_2 e_2^2 \quad \text{(energy in spring 2)}.$$

need geometric constraints. i.e e_1, e_2 are not independent.

$$e_2 = e_1 - e$$

$$\delta x = \delta e_1$$ — Q: why did we write this? where did it come from?

so rewrite, we obtain

$$\boxed{\begin{aligned} V &= \tfrac{1}{2} K_1 e_1^2 + \tfrac{1}{2} K_2 (e_1 - e)^2 \\ \delta x &= \delta e_1 \end{aligned}}$$

$$V.I. = f\,\delta x - \delta V$$ ← import starting point <u>virtual</u> <u>energy</u>

so $$V.I. = f\,\delta e_1 - \delta V$$

$$= f\,\delta e_1 - \delta\left(\tfrac{1}{2} K_1 e_1^2 + \tfrac{1}{2} K_2 (e_1 - e)^2\right)$$

e_1 is only unknown here.

$$V.I. = \left(f - K_1 e_1 - K_2 (e_1 - e)\right)\delta e_1$$

↙ arbitrary admissible variation.

so only way for V.I. to vanish for arbitrary admissible δe_1 is that

$$\boxed{f - k_1 e_1 - k_2 (e_1 - e) = 0}$$

now we talk about generalization of this.

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the Variational Indicator V.I. is

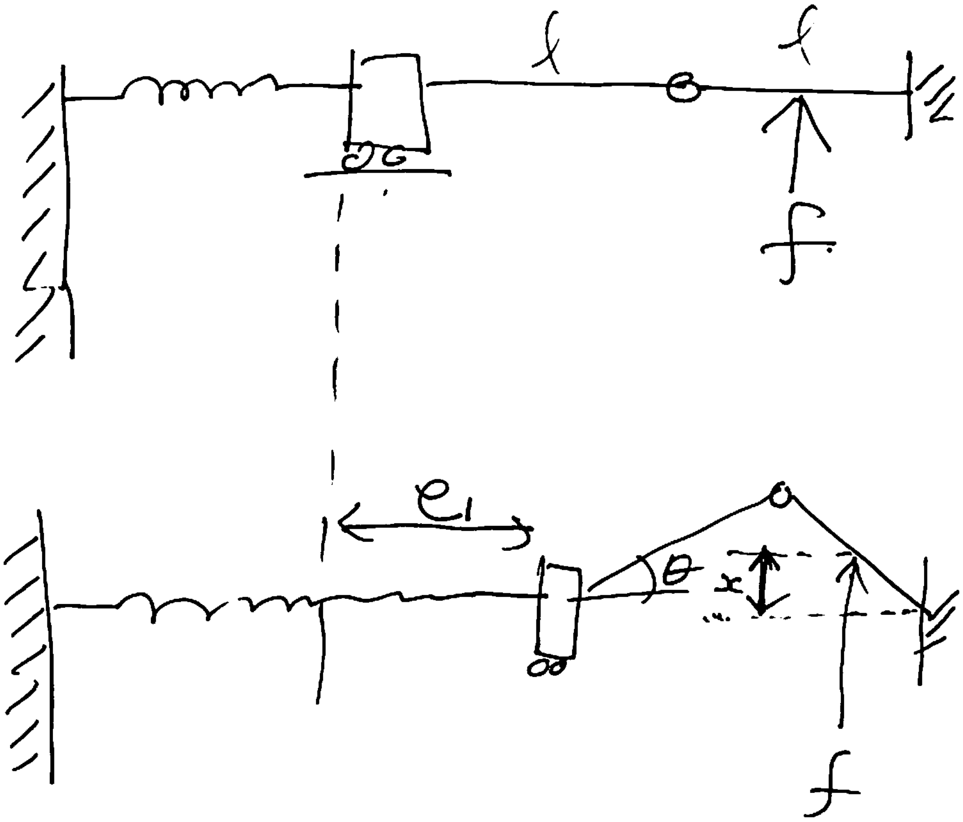
dot product.

$$\left(\sum \bar{f}_i \cdot \delta \bar{R}_i \right) - \delta V$$

allowed motion$_i$

so $\delta \bar{R}_i$ is always in the f_i

<u>direction</u> of f_i due to <u>dot</u> product.



note, only the up displacement is what causes work. side motion, no work, since f is up only.

~~$V = \frac{1}{2} k \delta e_1^2$~~
~~so $\delta V = k$~~

$V = \frac{1}{2} k e_1^2$

$\delta V = k e_1 \delta e_1$

so $V.I = f\,\delta x - \delta V$

so $V.I. = f\,\delta x - k e_1 \delta e_1$

need to express δx in terms of δe_1.
express all in terms of θ. since it relates δx to δe_1.

I set