chapter 13

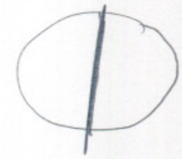
6.1

will use $\boxed{z = J_n (k_{mn} r) \cos n\theta \cos k_{mn} v t}$ to

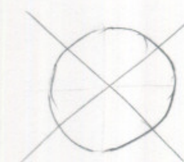
Complete Figure 6.1.

k_{mn} is the m^{th} zero of J_n.

Circle is divided into as many sectors as $2n$. for example, when $n=1$, we will set



when $n=2$, we will set



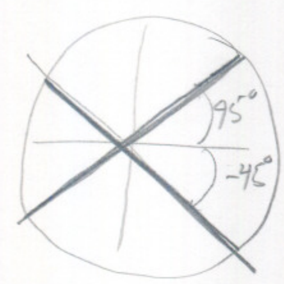
this is because we want a solution

$$\cos n\theta = \pm \frac{\pi}{2}$$

which means $\theta = \pm \frac{\pi}{2n}$

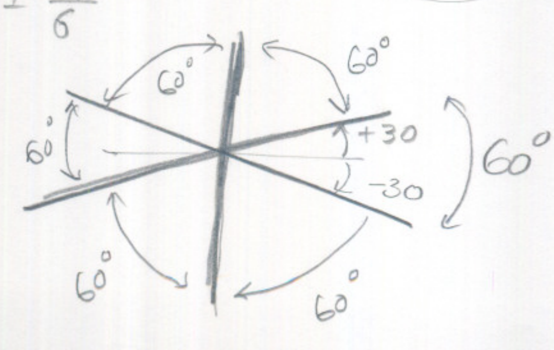
so for $n=1$, $\theta = +\frac{\pi}{2}, -\frac{\pi}{2}$

$n=2$ $\quad 2\theta = \pm\frac{\pi}{2}$ or $\theta = \pm\frac{\pi}{4}$



for $n=3$ $\quad 3\theta = \pm\frac{\pi}{2}$ or $\theta = \pm\frac{\pi}{6}$

so



etc...

now, for changing of the m.

from $z = J_n(k_{mn} r) \cos n\theta \cos k_{mn} v t$

we want $J_n(k_{mn} r)$ to zero.

as m increases, $k_{m_2 n} > k_{m_1 n}$ when $m_2 > m_1$

so r becomes smaller for each m increasing.

if original radius of drum is 1, then

r for $m = 1 \Rightarrow 1$

r for $m = 2 \Rightarrow \frac{k_{10}}{k_{20}} < 1$

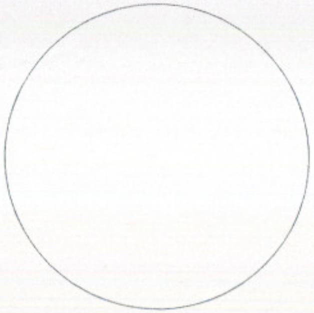
r for $m = 3 \Rightarrow \frac{k_{10}}{k_{30}} < \frac{k_{10}}{k_{20}}$

So each time m increases by 1 we add a smaller radius inside. so now I show the complete figure for $n = 0, 1, 2$, $m = 1, 2, 3$.

$\longrightarrow$

M=1

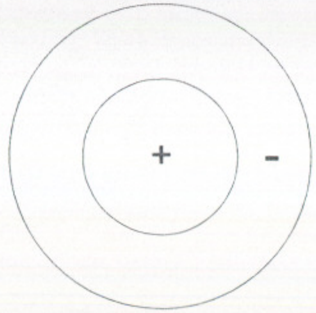
N=0



$$z = J_0(k_{10}r)\cos(k_{10}vt)$$

M=2

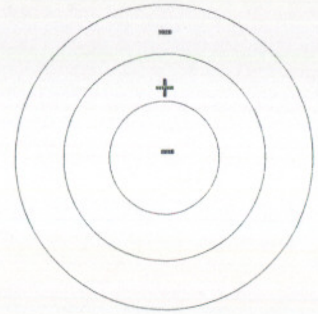
N=0



$$z = J_0(k_{20}r)\cos(k_{20}vt)$$

M=3

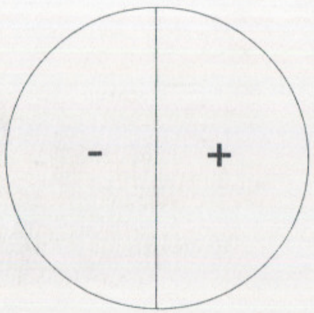
N=0



$$z = J_0(k_{30}r)\cos(k_{30}vt)$$

M=1

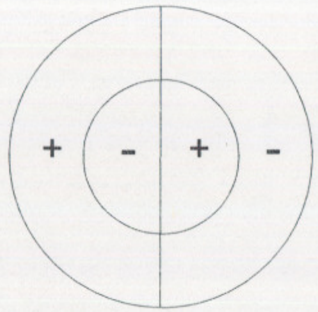
N=1



$$z = J_1(k_{11}r)\cos\theta\cos(k_{11}vt)$$

M=2

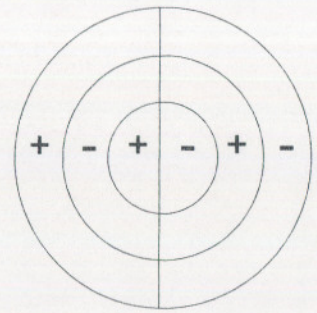
N=1



$$z = J_1(k_{21}r)\cos\theta\cos(k_{21}vt)$$

M=3

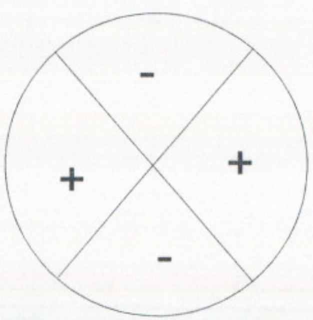
N=1



$$z = J_1(k_{31}r)\cos\theta\cos(k_{31}vt)$$

M=1

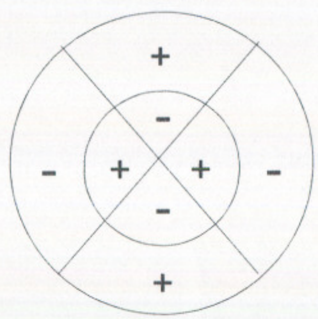
N=2



$$z = J_2(k_{12}r)\cos 2\theta\cos(k_{12}vt)$$

M=2

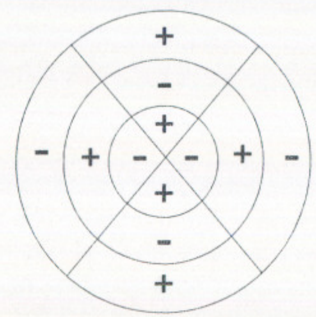
N=2



$$z = J_2(k_{22}r)\cos 2\theta\cos(k_{22}vt)$$

M=3

N=2



$$z = J_2(k_{32}r)\cos 2\theta\cos(k_{32}vt)$$

chapter 13, 6.2

look up first 3 zeros of k_{mn} of each Bessel function J_0, J_1, J_2, J_3. find the first 6 frequencies of a vibrating circular membrane as multiples of fundemental frequency.

Solution

frequency is given by $\omega_{mn} = k_{mn} v$

Fundemental frequency is $\omega_{10}\ v$

First zero of J_0

so ratio of frequency ω_{mn} to Fundemental is

$$\boxed{\frac{\omega_{mn}}{\omega_{10}} = \frac{k_{mn}}{k_{10}}}$$

First zero of J_0.

so $$\boxed{\omega_{mn} = \omega_{10} \frac{k_{mn}}{k_{10}}}$$

using this I can find frequencies as multiple of Fundemental freq.

$\longrightarrow$

First 3 Zeros of Bessel fuctions. using Handbook.

J_0

		Zero
m=1	$\longrightarrow$	2.4
m=2	$\longrightarrow$	5.65
m=3	$\longrightarrow$	8.55

J_1

m=1	$\longrightarrow$	3.75
m=2	$\longrightarrow$	7.25
m=3	$\longrightarrow$	10.05

J_2

m=1	$\longrightarrow$	5.05
m=2	$\longrightarrow$	8.45
m=3	$\longrightarrow$	11.55

J_3

m=1	$\longrightarrow$	6.3
m=2	$\longrightarrow$	13.1
m=3	$\longrightarrow$	16.3

So now I can find the first 6 frequencies.

First $= \omega_{10}$ this is Fundemental Frequencies.

Second $= \omega_{11} = \omega_{10} \frac{K_{11}}{K_{10}} = \omega_0 \frac{3.75}{2.4} = \boxed{1.56\, \omega_0}$

Third $= \omega_{12} = \omega_{10} \frac{K_{12}}{K_{10}} = \omega_{10} \frac{5.05}{2.4} = \boxed{2.1\, \omega_0}$

Fouth $= \omega_{20} = \omega_{10} \frac{K_{20}}{K_{10}} = \omega_{10} \frac{5.65}{2.4} = \boxed{2.35\, \omega_0}$

Fifth $= \omega_{21} = \omega_{10} \frac{K_{21}}{K_{10}} = \omega_{10} \frac{7.25}{2.4} = \boxed{3.01\, \omega_0}$

Sixth $= \omega_{22} = \omega_{10} \frac{K_{22}}{K_{10}} = \omega_{10} \frac{8.45}{2.4} = \boxed{3.5\, \omega_0}$