HW 9, Math 121 A
Spring, 2004
UC BERKELEY

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Contents

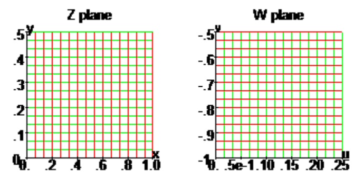
1 chapter 14, problem 9.2

For following function $w = f(z) = u + iv$ find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = const$ and $v = const$ for several values of u and several values of v where $w = \frac{z+1}{2i}$
Answer let $z = x + iy$, hence $w = \frac{z+1}{2i} = \frac{x+iy+1}{2i} = -i\frac{x+iy+1}{2} = \frac{-ix+y-i}{2} = \frac{y}{2} + i\left(\frac{-1-x}{2}\right)$. So, since $w = u + iv$ then $u = \frac{y}{2}$ and $v = \left(\frac{-1-x}{2}\right)$ Then $u = C$, where C is a constant, gives the equation $\frac{y}{2} = C$. Which is the equation of a straight line $y = C$.
v =constant, gives the equation $\left(\frac{-1-x}{2}\right) = C$, gives the equation of the straight line $x = C - 1$.
These two equations are plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of u=const and the image of v=const back into the xy plane.

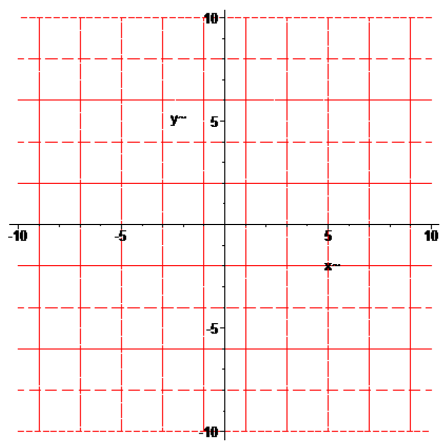
Output from HW#9, problem
9.2 chapter 14
by Nasser Abbasi

$u(x,y) = \frac{y}{2}$

$v(x,y) = -\frac{x}{2} - \frac{1}{2}$



Mapping of f(z) from the Z plane to the W plane.



Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

2 chapter 14, problem 9.3

For following function $w = f(z) = u + iv$ find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = C$ and $v = C$, where C is constant, for several values of u and several values of v. $w = \frac{1}{z}$

Answer let $z = x + iy$, hence $w = \frac{1}{z} = \frac{1}{x+iy} = \frac{1}{x+iy}\frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} + \frac{-iy}{x^2+y^2}$. Hence, since $w = u + iv$ then $u = \frac{x}{x^2+y^2}$ and $v = \frac{-y}{x^2+y^2}$. Then $u = C$ gives the equation $\frac{x}{x^2+y^2} = C$.

$v = C$ gives the equation $\frac{-y}{x^2+y^2} = C$.

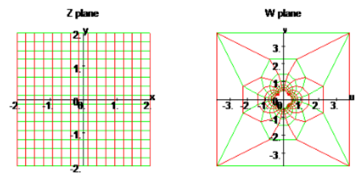
These 2 equations were plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of $u = C$ and the image of $v = C$ back into the xy plane.

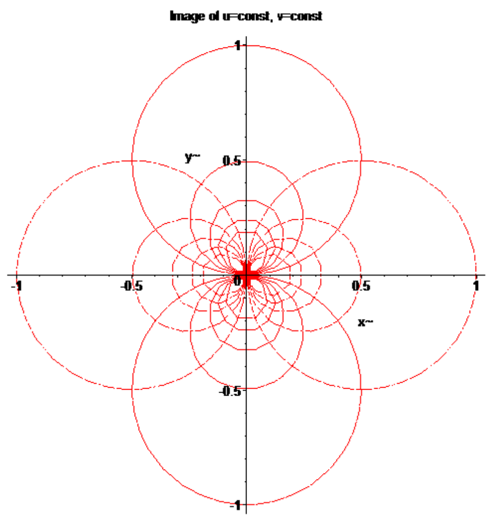
Analysis for problem 9.3 chapter 14
Mary Boas text book.
by Nasser Abbasi

$f := z \to \frac{1}{z}$

$u(x,y) = \frac{x}{x^2+y^2}$

$v(x,y) = -\frac{y}{x^2+y^2}$





Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

3 chapter 14, problem 9.4

For the function $w = f(z) = u + iv$ shown below, find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = const$ and $v = const$ for several values of u and several values of v. $w = e^z$

Answer let $z = x + iy$, hence $w = e^z = e^{x+iy} = e^x e^{iy} = e^x (\cos y + i \sin y) = e^x \cos(y) + i\, e^x \sin(y)$. Therefore, since $w = u + iv$ then $u = e^x \cos(y)$ and $v = e^x \sin(y)$. Then $u = C$ gives the equation $e^x \cos(y) = C$ and $v = C$ gives the equation $e^x \sin(y) = C$

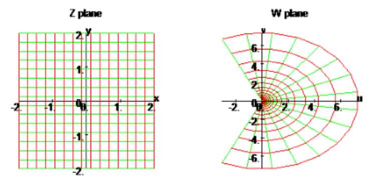
These 2 equations are plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of u=const and the image of v=const back into the xy plane.

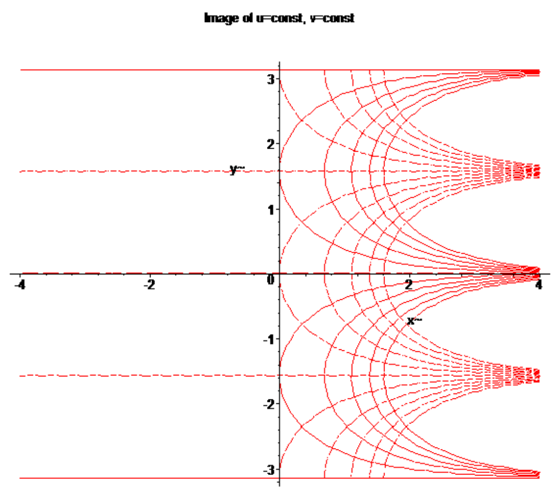
Analysis for problem 9.4 chapter 14
Mary Boas text book.
by Nasser Abbasi

$f(z) = e^z$

$u(x,y) = e^x \cos(y)$

$v(x,y) = e^x \sin(y)$





Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

4 chapter 14, problem 9.7

For the function $w = f(z) = u + iv$ shown below, find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = C$ and $v = C$ for several values of u and several values of v. use $w = \sin(z)$

Answer let $z = x + iy$, hence

$$\begin{aligned} w &= \sin(z) = \sin(x + iy) \\ &= \sin(x)\cos(iy) + \cos(x)\sin(iy) \end{aligned}$$

But $\cos(iy) = \cosh(y)$ and $\sin(iy) = i\sinh(y)$, therefore

$$w = \sin(x)\cosh(y) + \cos(x)\, i \sinh(y)$$

Since $w = u + iv$ then $u = \sin(x)\cosh(y)$ and $v = \cos(x)\sinh(y)$. Then $u = C$ gives the equation

$$\sin(x)\cosh(y) = C$$

And $v = C$ gives the equation

$$\cos(x)\sinh(y) = C$$

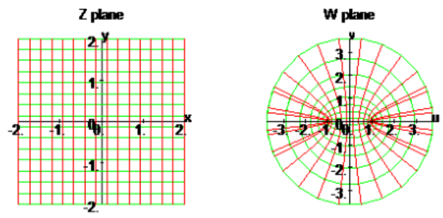
These 2 equations are plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of u=const and the image of v=const back into the xy plane.

Analysis for problem 9.7 chapter 14
Mary Boas text book.
by Nasser Abbasi

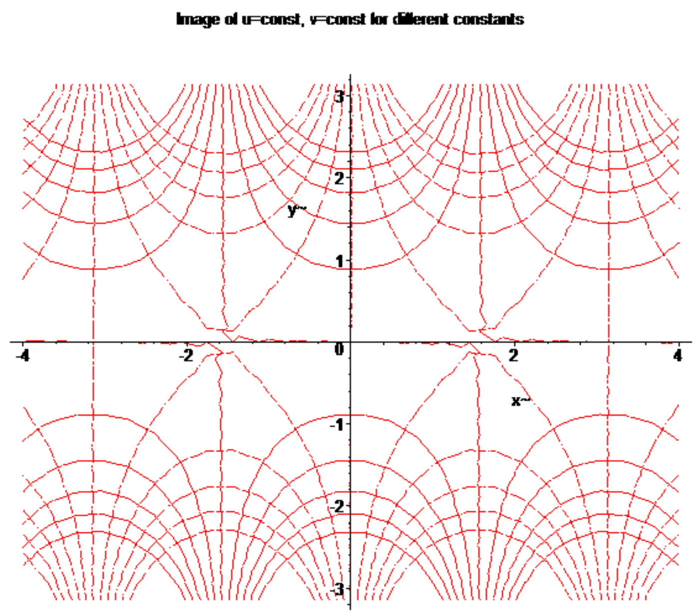
$f(z) = \sin(z)$

$u(x,y) = \sin(x)\cosh(y)$

$v(x,y) = \cos(x)\sinh(y)$



Mapping of f(z) from the Z plane to the W plane.

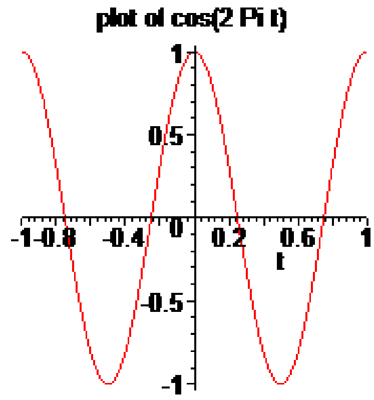


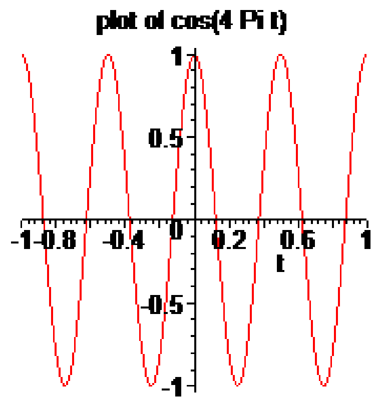
Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

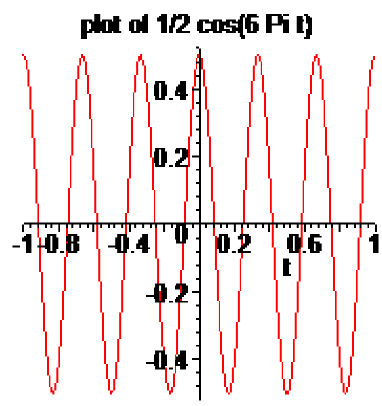
5 chapter 7, problem 3.4

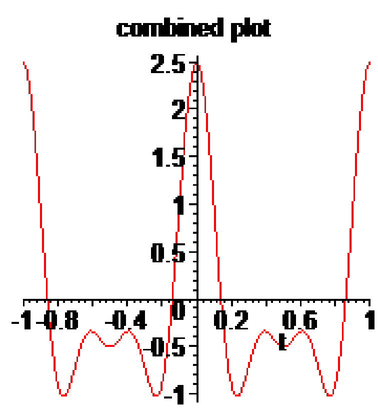
Draw a graph over a whole period for $f(t) = \cos(2\pi t) + \cos(4\pi t) + \frac{1}{2}\cos(6\pi t)$
Answer First, find the period of the above function. A function is periodic with period p if $f(t+p) = f(t)$ for all t. We know that $\cos(nt)$ has the same period as $\cos(nt + 2n\pi)$ for n integer, since the function $\cos(x)$ has a period of 2π. So a period of $f(t)$ is 2π since it is the sum of $\cos(x)$ functions. To plot this function, we plot each of its components over the same period of 2π and then sum them together.
This plot below shows the result

Analysis for problem 3.4 chapter 7
Mary Boas text book.
by Nasser Abbasi









6 chapter 7, problem 3.6

Draw a graph of $f(x) = \sin(2x) + \sin 2\left(x + \frac{\pi}{3}\right)$. what are the period and amplitude? Write as a single harmonic.

<u>Answer</u>

Since $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ then

$$\sin 2\left(x + \frac{\pi}{3}\right) = \sin\left(2x + \frac{2}{3}\pi\right) = \sin(2x)\cos\left(\frac{2}{3}\pi\right) + \cos(2x)\sin\left(\frac{2}{3}\pi\right)$$

Hence

$$f(x) = \sin(2x) + \sin 2\left(x + \frac{\pi}{3}\right) = \sin(2x) + \sin(2x)\cos\left(\frac{2}{3}\pi\right) + \cos(2x)\sin\left(\frac{2}{3}\pi\right)$$

Now $\cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$ and $\sin\left(\frac{2}{3}\pi\right) = \frac{\sqrt{3}}{2}$ so above can be written as

$$f(x) = \sin(2x) - \frac{1}{2}\sin(2x) + \frac{\sqrt{3}}{2}\cos(2x)$$
$$f(x) = \frac{1}{2}\sin(2x) + \frac{\sqrt{3}}{2}\cos(2x)$$

But $\sin(2x) = \cos(\frac{\pi}{2} - 2x)$

$$f(x) = \frac{1}{2}\cos(\frac{\pi}{2} - 2x) + \frac{\sqrt{3}}{2}\cos(2x)$$

Now this is in term of a single harmonic function. Hence, we see that $f(x)$ is the sum of harmonics of the same periods (the cos function have period of 2π).hence the period of $f(x)$ is 2π. To find Max amplitude, this is a problem of finding a maximum of a function.

$$\begin{aligned}\frac{d}{dx}f(x) &= -\frac{1}{2}\sin(\frac{\pi}{2} - 2x)\ (-2) - \frac{\sqrt{3}}{2}\sin(2x)\,(2)\\ &= \sin(\frac{\pi}{2} - 2x) - \sqrt{3}\sin(2x)\\ &= \cos(2x) - \sqrt{3}\sin(2x)\end{aligned}$$

Hence for a maximum, $\cos(2x) - \sqrt{3}\sin(2x) = 0$. A root for this equation is found at $x = 0.261799$ so I use this value in $f(x)$ to find the amplitude.
$f(0.261799) = 1$. This is the maximum value, or the amplitude.The following is a plot of this function