HW 9, Math 121 A
Spring, 2004
UC BERKELEY

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Contents

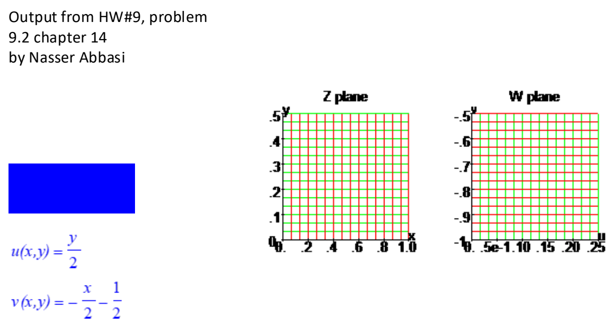
1 chapter 14, problem 9.2

For following function $w = f(z) = u + iv$ find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = const$ and $v = const$ for several values of u and several values of v where $w = \frac{z+1}{2i}$

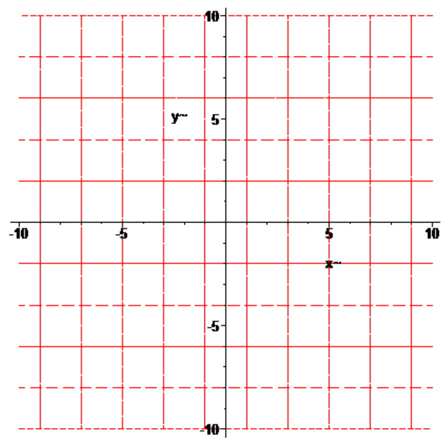
Answer let $z = x + iy$, hence $w = \frac{z+1}{2i} = \frac{x+iy+1}{2i} = -i\frac{x+iy+1}{2} = \frac{-ix+y-i}{2} = \frac{y}{2} + i\left(\frac{-1-x}{2}\right)$. So, since $w = u + iv$ then $u = \frac{y}{2}$ and $v = \left(\frac{-1-x}{2}\right)$ Then $u = C$, where C is a constant, gives the equation $\frac{y}{2} = C$. Which is the equation of a straight line $y = C$.

v =constant, gives the equation $\left(\frac{-1-x}{2}\right) = C$, gives the equation of the straight line $x = C - 1$.

These two equations are plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of u=const and the image of v=const back into the xy plane.



Mapping of f(z) from the Z plane to the W plane.



Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

2 chapter 14, problem 9.3

For following function $w = f(z) = u + iv$ find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = C$ and $v = C$, where C is constant, for several values of u and several values of v. $w = \frac{1}{z}$

Answer let $z = x + iy$, hence $w = \frac{1}{z} = \frac{1}{x+iy} = \frac{1}{x+iy}\frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} + \frac{-iy}{x^2+y^2}$. Hence, since $w = u + iv$ then $u = \frac{x}{x^2+y^2}$ and $v = \frac{-y}{x^2+y^2}$. Then $u = C$ gives the equation $\frac{x}{x^2+y^2} = C$.

$v = C$ gives the equation $\frac{-y}{x^2+y^2} = C$.

These 2 equations were plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of $u = C$ and the image of $v = C$ back into the xy plane.

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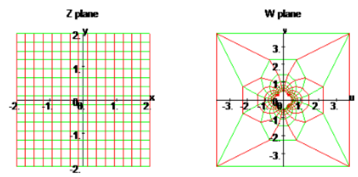
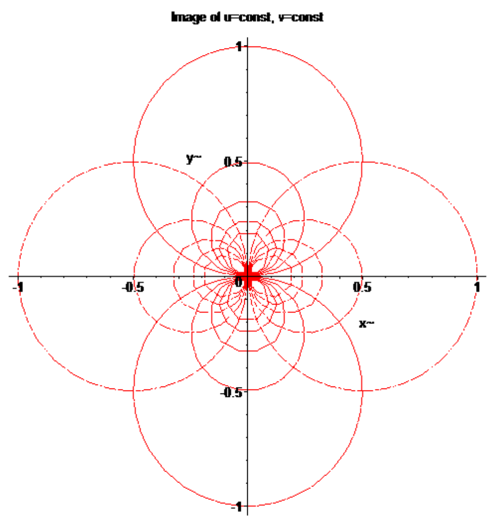
$f := z \to \frac{1}{z}$

$u(x,y) = \frac{x}{x^2+y^2}$

$v(x,y) = -\frac{y}{x^2+y^2}$

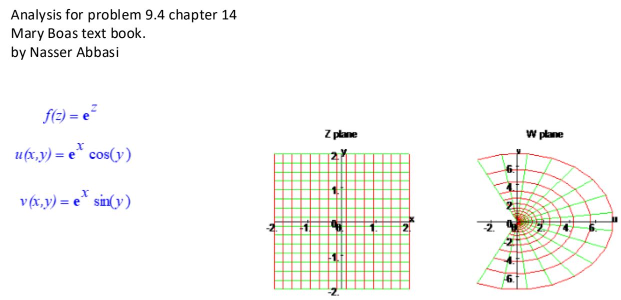


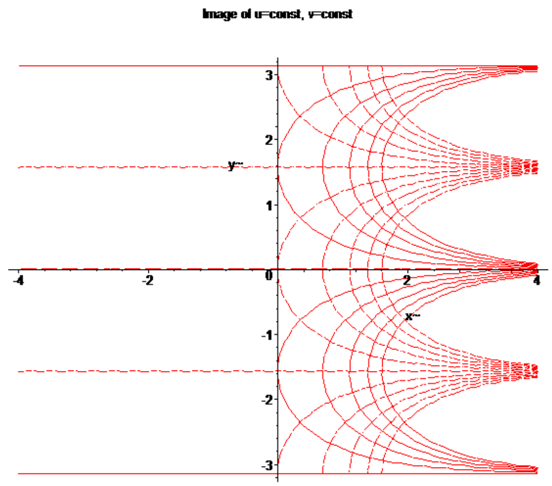


Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

3 chapter 14, problem 9.4

For the function $w = f(z) = u + iv$ shown below, find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = const$ and $v = const$ for several values of u and several values of v. $w = e^z$
Answer let $z = x + iy$, hence $w = e^z = e^{x+iy} = e^x e^{iy} = e^x (\cos y + i \sin y) = e^x \cos(y) + i\, e^x \sin(y)$. Therefore, since $w = u + iv$ then $u = e^x \cos(y)$ and $v = e^x \sin(y)$. Then $u = C$ gives the equation $e^x \cos(y) = C$ and $v = C$ gives the equation $e^x \sin(y) = C$
These 2 equations are plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of u=const and the image of v=const back into the xy plane.





Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

4 chapter 14, problem 9.7

For the function $w = f(z) = u + iv$ shown below, find u and v as functions of x and y. Sketch the graphs in the (x, y) plane of the images of $u = C$ and $v = C$ for several values of u and several values of v. use $w = \sin(z)$

<u>Answer</u> let $z = x + iy$, hence

$$\begin{aligned} w &= \sin(z) = \sin(x + iy) \\ &= \sin(x)\cos(iy) + \cos(x)\sin(iy) \end{aligned}$$

But $\cos(iy) = \cosh(y)$ and $\sin(iy) = i\sinh(y)$, therefore

$$w = \sin(x)\cosh(y) + \cos(x)\, i \sinh(y)$$

Since $w = u + iv$ then $u = \sin(x)\cosh(y)$ and $v = \cos(x)\sinh(y)$. Then $u = C$ gives the equation

$$\sin(x)\cosh(y) = C$$

And $v = C$ gives the equation

$$\cos(x)\sinh(y) = C$$

These 2 equations are plotted for few points. The following shows the plots generated for the mapping from the z-plane to the w-plane, and then the image of u=const and the image of v=const back into the xy plane.

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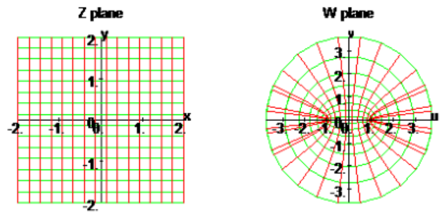
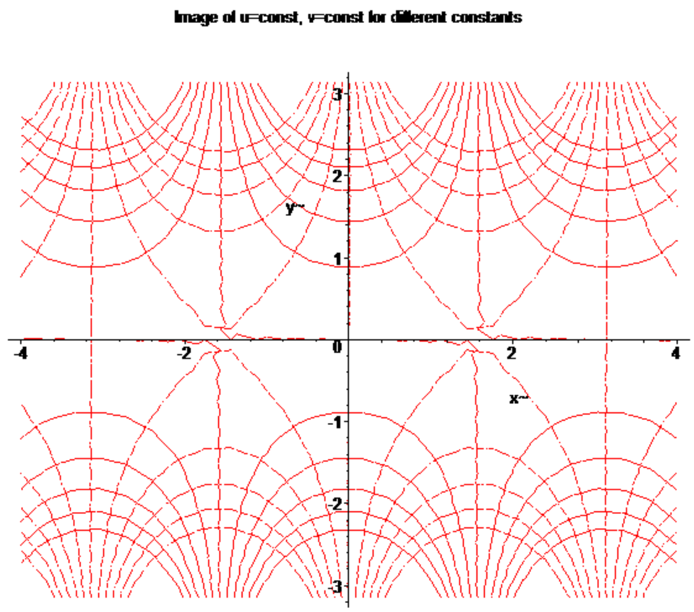
$f(z) = \sin(z)$

$u(x,y) = \sin(x)\cosh(y)$

$v(x,y) = \cos(x)\sinh(y)$



Mapping of f(z) from the Z plane to the W plane.



Showing the images of U and V on the Z plane (the X,Y plane). The DASHED lines are the image of the equation U=const, and the solid lines are the image of the V=const equation.

5 chapter 7, problem 3.4

Draw a graph over a whole period for $f(t) = \cos(2\pi t) + \cos(4\pi t) + \frac{1}{2}\cos(6\pi t)$
Answer First, find the period of the above function. A function is periodic with period p if $f(t+p) = f(t)$ for all t. We know that $\cos(nt)$ has the same period as $\cos(nt + 2n\pi)$ for n integer, since the function $\cos(x)$ has a period of 2π. So a period of $f(t)$ is 2π since it is the sum of $\cos(x)$ functions. To plot this function, we plot each of its components over the same period of 2π and then sum them together.
This plot below shows the result

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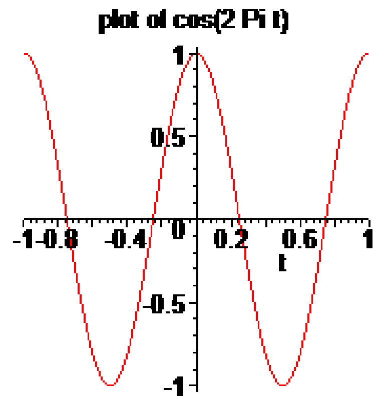

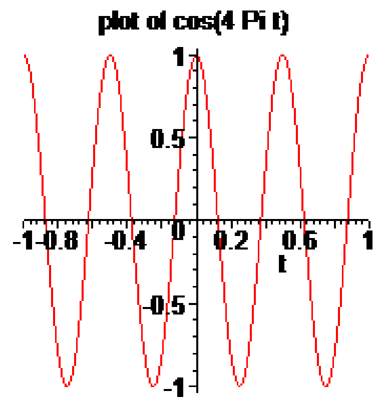

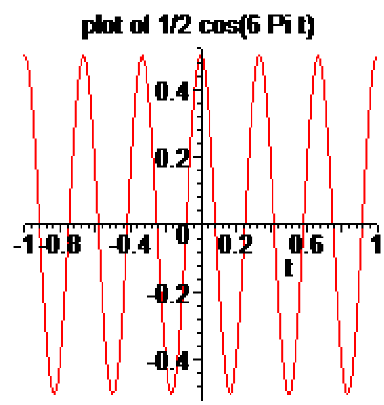

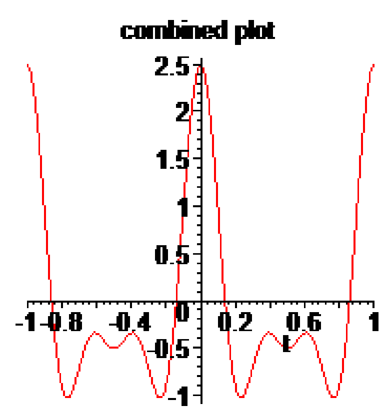

6 chapter 7, problem 3.6

Draw a graph of $f(x) = \sin(2x) + \sin 2\left(x + \frac{\pi}{3}\right)$. what are the period and amplitude? Write as a single harmonic.

Answer

Since $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ then

$$\sin 2\left(x + \frac{\pi}{3}\right) = \sin\left(2x + \frac{2}{3}\pi\right) = \sin(2x)\cos\left(\frac{2}{3}\pi\right) + \cos(2x)\sin\left(\frac{2}{3}\pi\right)$$

Hence

$$f(x) = \sin(2x) + \sin 2\left(x + \frac{\pi}{3}\right) = \sin(2x) + \sin(2x)\cos\left(\frac{2}{3}\pi\right) + \cos(2x)\sin\left(\frac{2}{3}\pi\right)$$

Now $\cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$ and $\sin\left(\frac{2}{3}\pi\right) = \frac{\sqrt{3}}{2}$ so above can be written as

$$\begin{aligned} f(x) &= \sin(2x) - \frac{1}{2}\sin(2x) + \frac{\sqrt{3}}{2}\cos(2x) \\ f(x) &= \frac{1}{2}\sin(2x) + \frac{\sqrt{3}}{2}\cos(2x) \end{aligned}$$

But $\sin(2x) = \cos(\frac{\pi}{2} - 2x)$

$$f(x) = \frac{1}{2}\cos(\frac{\pi}{2} - 2x) + \frac{\sqrt{3}}{2}\cos(2x)$$

Now this is in term of a single harmonic function. Hence, we see that $f(x)$ is the sum of harmonics of the same periods (the cos function have period of 2π).hence the period of $f(x)$ is 2π. To find Max amplitude, this is a problem of finding a maximum of a function.

$$\begin{aligned} \frac{d}{dx}f(x) &= -\frac{1}{2}\sin(\frac{\pi}{2} - 2x)\ (-2) - \frac{\sqrt{3}}{2}\sin(2x)(2) \\ &= \sin(\frac{\pi}{2} - 2x) - \sqrt{3}\sin(2x) \\ &= \cos(2x) - \sqrt{3}\sin(2x) \end{aligned}$$

Hence for a maximum, $\cos(2x) - \sqrt{3}\sin(2x) = 0$. A root for this equation is found at $x = 0.261799$ so I use this value in $f(x)$ to find the amplitude.

$f(0.261799) = 1$. This is the maximum value, or the amplitude.The following is a plot of this function

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