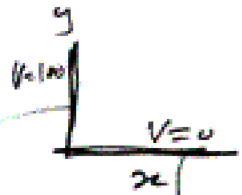
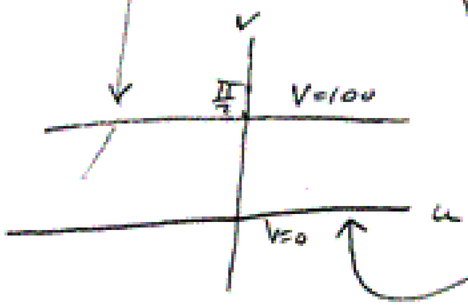
problem 5, section 10, chapter 14
Nasser Abbasi

this is the capacitor



to solve, transform to W plan using $w = \ln(z)$ transformation. this gives



now we solve the problem in the w plane. need to solve for V in shaded area.

the solution is linear. so at $v=0$, $V=0$

at $v=\frac{\pi}{2}$, $V=100$.

So need $\boxed{V = \frac{100}{\pi/2} v}$

now convert back to x-y plane.

since $v = \arg(z)$, then $v = \arctan\frac{y}{x}$

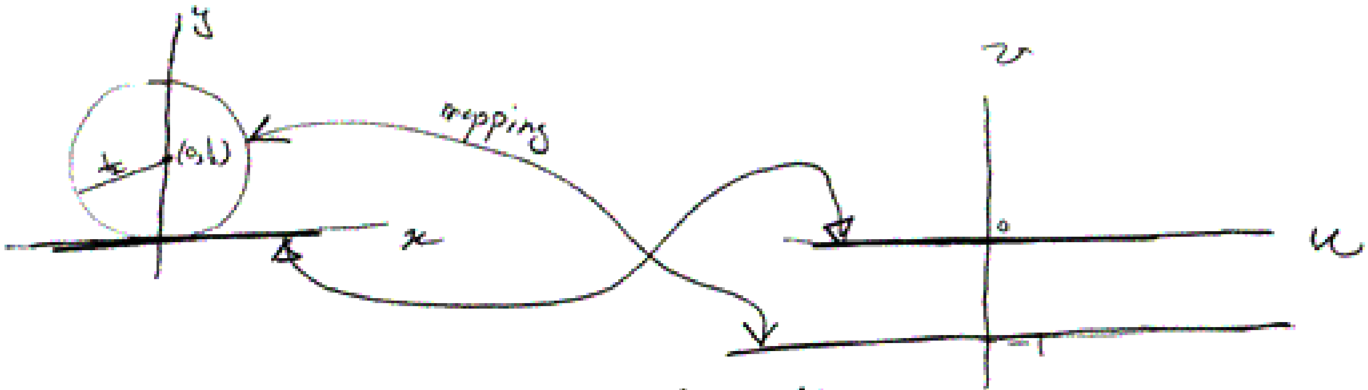
so solution is $\boxed{V = \frac{100}{\pi/2} \arctan\frac{y}{x}}$

Section 10, problem 6, chapter 14.

Name Abbas:

$\boxed{\omega = \frac{1}{z}}$

Z W



the mapping $\omega = \frac{1}{z}$ is found as this:

let $z = x + iy$.

so $\frac{1}{z} = \frac{1}{x+iy} \frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2}$.

so $w = \frac{1}{z} = u + iv$.

hence $u = \frac{x}{x^2+y^2}$, $v = \frac{-y}{x^2+y^2}$.

for $v=-1$, we get $-1 = \frac{-y}{x^2+y^2}$ or $x^2+y^2-y=0$. (1)

a circle equation is $(x-a)^2 + (y-b)^2 = r^2$ (2)

to convert (1) to (2)

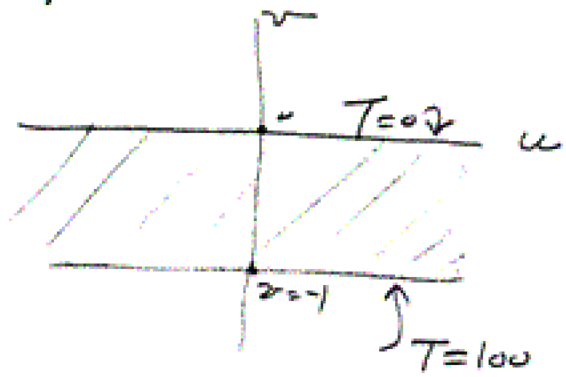
$(x-0)^2 + (y-\frac{1}{2})^2 - \frac{1}{4} = 0 \Rightarrow \sqrt{(x-0)^2 + (y-\frac{1}{2})^2} = \frac{1}{4}$.

so the line $\boxed{v=-1 \text{ maps to circle with center } (0,\tfrac{1}{2}) \text{ and radius} = \tfrac{1}{2} \text{ in } Z \text{ plane}}$

and line $v=0$ (u axis) maps to $y=0$ or x-axis

hence area outside cylinder $\Longrightarrow$ area between lines $v=-1$ and $v=0$ in W plane $\longrightarrow$

here in W plane we have this



So $T = 100$ at $v=-1$

$T = 0$ at $v=0$. ✓

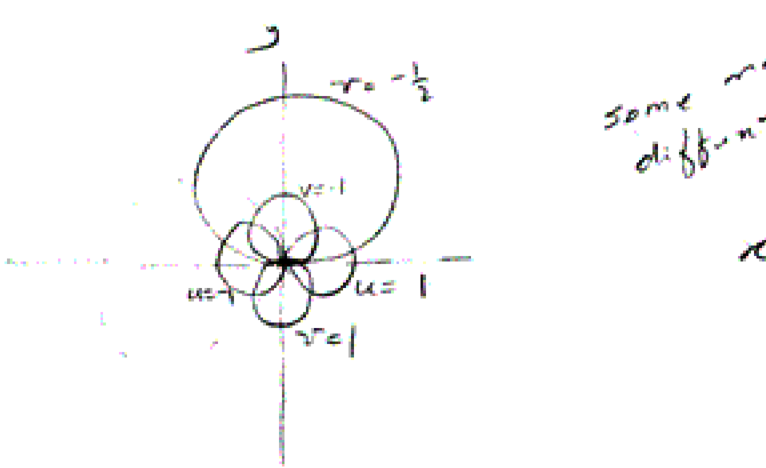
so since T is linear $\Rightarrow$ $\boxed{T = -100\, v}$

now to get the solution in the x-y, replace v by

$$\frac{-y}{x^2+y^2}.$$

hence $T = -100\left(\frac{-y}{x^2+y^2}\right)$

$$\boxed{T = 100\,\frac{y}{x^2+y^2}}$$ ✓



some mapping for different u and v values

problem 11, section 10, chapter 14
Nasser Abbasi

for $w=\ln[(z+1)/(z-1)]$ show that the images of $u=$ const and $v=$ const are two orthogonal sets of circles. Find centers and radii of 5 or six circles of each set and sketch them.

let $z=x+iy$.

then $w=\ln\left(\frac{x+iy+1}{x+iy-1}\right)=\ln\left(\frac{(x+1)+iy}{(x-1)+iy}\ \frac{(x-1)-iy}{(x-1)-iy}\right)$

$$w=\ln\left(\frac{x^2+y^2-1-2yi}{(x-1)^2+y^2}\right)$$

so $e^{w}=\frac{(x^2+y^2-1)+i(-2y)}{(x-1)^2+y^2}$.

but $w=u+iv$.

so $e^{u}e^{iv}=\frac{(x^2+y^2-1)+i(-2y)}{(x-1)^2+y^2}$ (0)

$e^{u}(\cos v+i\sin v)=$ " (1)

$e^{u}\cos v+ie^{u}\sin v=$ " (2)

so $\boxed{e^{u}\cos v=\frac{(x^2+y^2-1)}{(x-1)^2+y^2}\ ;\quad e^{u}\sin v=\frac{-2y}{(x-1)^2+y^2}}$

$\longrightarrow$

so divide one equation by another, we get

$$\frac{e^u \sin v}{e^u \cos v} = \boxed{\tan v = \frac{-2y}{x^2+y^2-1}}$$

for v = const, this mean $\tan v$ = const. i.e.

$$\frac{-2y}{x^2+y^2-1} = \text{constant} = \frac{1}{k} \text{ (say)}$$

so equation is $\quad -2yk = x^2+y^2-1$

or $\quad x^2+y^2-1+2yk = 0$

or $\quad x^2+y^2+2yk = 1$

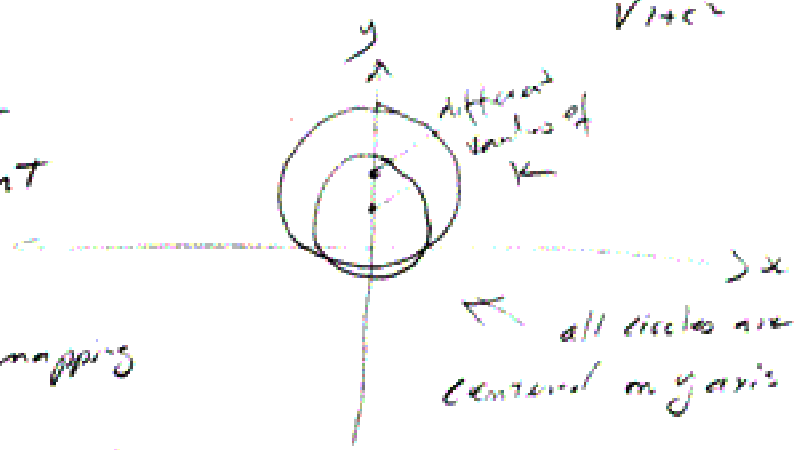
this is equation of circle

$$(x-0)^2 + (y+k)^2 - k^2 = 1$$

$$(x-0)^2 + (y+k)^2 = 1+k^2$$

so this is a circle with radius at $(0,k)$ and radius $\sqrt{1+k^2}$

mapping of v = constant



now I need to find mapping of u = constant $\longrightarrow$

for u = constant, we get, by taking Abs of

$$e^u e^{iv} \Rightarrow |e^u e^{iv}| = e^u$$

$$= \left| \frac{(x^2+y^2-1) + i(-2y)}{(x-1)^2+y^2} \right|$$

so $$e^{2u} = \frac{(x^2+y^2-1)^2 + (-2y)^2}{((x-1)^2+y^2)^2} = \frac{(x+1)^2+y^2}{(x-1)^2+y^2}$$

for u = constant, then e^{2u} = const = k say.

so $k((x-1)^2+y^2) = (x+1)^2+y^2$

or $k(x^2-2x+1+y^2) = x^2+2x+1+y^2$

or $kx^2-2kx+k+ky^2 = x^2+2x+1+y^2$

$$x^2(k-1)+y^2(k-1)+x(-2k-2)+k-1=0$$

let $k-1 = A$, a new constant.

so $Ax^2+Ay^2+2Ax = -A$ ——— (3)

or $x^2+y^2+2x = M$ new constant

this is equation of circle.

$$(x-a)^2+(y-b)^2 = r^2$$

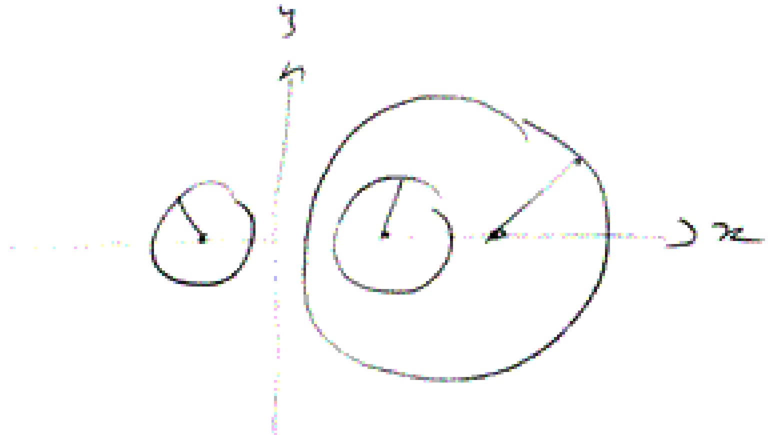
i.e. $x^2-2ax+a^2+y^2-2yb+b^2 = r^2$ ——— (4)

Compare (3), (4) we get $\boxed{(x+1)^2+(y-0)^2 = \text{Constant}}$ $\longrightarrow$

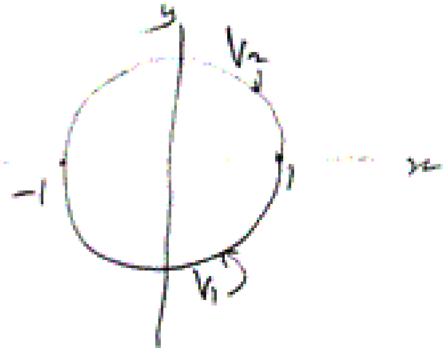
So mappings of $u =$ constant are

circles with center on x-axis.

different circles
$u =$ constant
for different
constants.



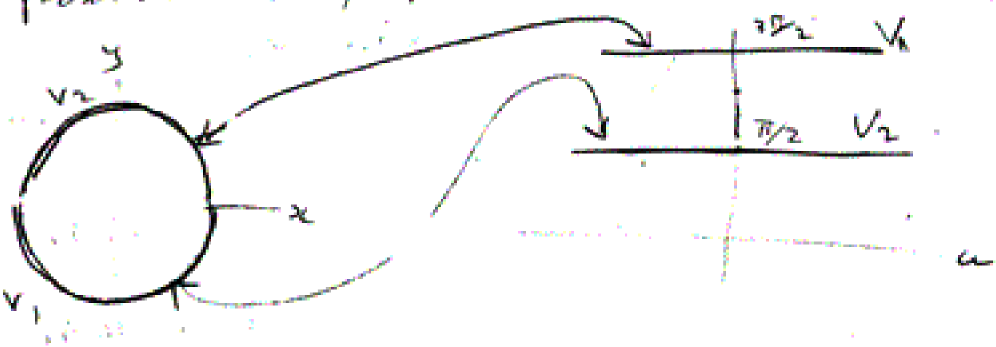
problem 13, section 10, chapter 14
Nami Abbasi



Find $V(x,y)$ between plates i.e. inside circle.

let $z = re^{i\theta}$.
use mapping $w = \ln\left(\frac{z+1}{z-1}\right)$ ✓

from result of problem 11, is mapping we want.



hence V at $\frac{3\pi}{2} = V_1$
V at $\frac{\pi}{2} = V_2$. ✓

so solution in W plane is

$$V = V_2 + \left(v - \frac{\pi}{2}\right) \cdot \left(\frac{V_1 - V_2}{\pi}\right)$$

$$= V_2 + \frac{vV_1}{\pi} - \frac{vV_2}{\pi} - \frac{V_1}{2} + \frac{V_2}{2} = \frac{3}{2}V_2 - \frac{V_1}{2} + \frac{v}{\pi}(V_1 - V_2)$$

$$\boxed{V = V_2\left(\frac{3}{2} - \frac{v}{\pi}\right) + V_1\left(\frac{v}{\pi} - \frac{1}{2}\right)}$$

now change back to x-y plane →

$$v = \text{angle of } \frac{x^2+y^2-1-2iy}{(x-1)^2+y^2}$$ (from problem 11)

so this substitute in (1) below gives

V in x-y plane

✓