# Discussion notes. July 3, 2006. ENGR 80 (Dynamics)

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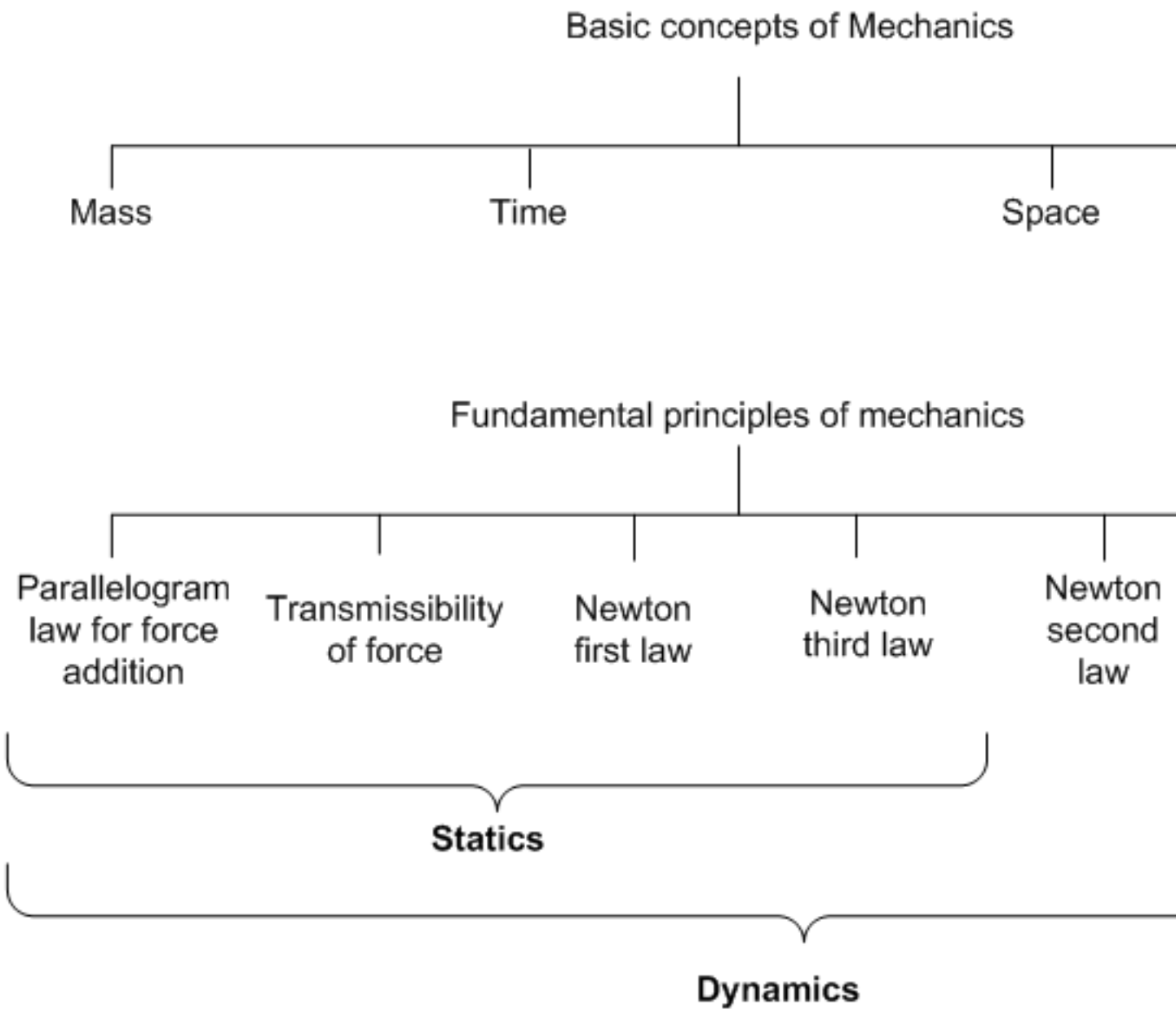
## 1 Review of basic mechanics




Newton first law: $\sum F = 0 \Rightarrow$ the body is at rest or will remain at

(object is i

Newton second law: $\sum F = ma$

Newton third law: For every action, there is a reaction, equal and opposite

## 1.1 Miscellaneous Items to remember

1. Difference between effect of forces on a particle and on a rigid body: When a force is applied on a particle, the particle will be displaced and move in a straight line. However, a force applied on a rigid body can cause a moment to be generated about a point. The moment is given by $\mathbf{M} = \mathbf{r} \times \mathbf{F}$

2. A force can always be replaced by a couple and a displaced force.

3. A particle is in equilibrium if is is at rest or if it is moving in a straigh line at constant speed.

4. Any system of forces acting on a rigid body can be reduced to a single force and a moment.

5. two system of focrce are equiveleant if $\sum \mathbf{F} = \sum \mathbf{F}'$ and $\sum \mathbf{M} = \sum \mathbf{M}'$

6. laws of sins for a triangle. The ratio of the length of each triangle side to the sine of the angle opposit to it is the same.

7. Learn how to cross multiply and dot multiply vectors. How to use the determinant to find cross products of vectors.

# 2 Derivation of the kinematics equations for motion on a line. Constant acceleration case

Kinematics: The word is derived from a Greek word meaning 'motion'. Kinematics is the study of motion of objects without direct reference to the forces causing the motion. Kinematics is also the study of the geometry of motion.

In kinematics, we study of the relationship between the displacement, velocity, and acceleration.

In Dynamics, Forces are added. We study the motion of objects, and the affect and interaction of forces on this motion.

The object whose motion we study can be a particle, or a large body. Its motion can be in a straight line, or it can be rotational.

Notations: $v_0$ is initial velocity. $v_f$ is final velocity. $\Delta t$ is elapsed time. $a$ is acceleration.

Now we derive the compete set of equations of motion to describe motion of a particle in a straight line when the acceleration is constant.

Since we assume that acceleration is constant, then the instantaneous acceleration is the same as the average acceleration and is given by

$$a = \frac{v_f - v_o}{\Delta t} \qquad (1) \qquad (1)$$

And since the acceleration is constant, then the average velocity is

$$v_{av} = \frac{v_f + v_o}{2} \qquad (2) \qquad (2)$$

From the above 2 equations, we can now derive the 4 kinematics equations for motion on a straight line with constant acceleration as follows

We obtain the first kinematic equation from (1) by solving for $v_f$

$$v_f = v_0 + a\ \Delta t \qquad (1A) \qquad (1A)$$

Now, suppose we are given $v_f, v_o, \Delta t$, how can we find $d$, the displacement?
By definition,

$$d = v_{av}\ \Delta t$$

Now substitute (2) in the above we obtain the second kinematic equation

$$d = \left(\frac{v_f + v_o}{2}\right)\ \Delta t \qquad (2A) \qquad \text{(2A)}$$

Now, suppose we want to find displacement, but are not given the final velocity $v_f$?
Substitute (1A) in (2A) we obtain the third kinematic equation

$$d = v_0\ \Delta t + \frac{1}{2}a\,(\Delta t)^2 \qquad (3A) \qquad \text{(3A)}$$

Now, suppose we want to find final velocity $v_f$ but are not given $\Delta t$?
From (1) we solve for $\Delta t$

$$\Delta t = \frac{v_f - v_o}{a}$$

Substitute the above in (2A) we obtain

$$\begin{aligned} d &= \left(\frac{v_f + v_o}{2}\right)\ \Delta t \\ &= \left(\frac{v_f + v_o}{2}\right)\left(\frac{v_f - v_o}{a}\right) \\ &= \frac{1}{2a}\left(v_f^2 - v_0^2\right) \end{aligned}$$

Solve the above for $v_f$

$$v_f^2 = 2ad + v_0^2 \qquad (4A) \qquad \text{(4A)}$$

This completes the derivation of the 4 kinematic equations for linear motion with constant acceleration. The equations are (1A), (2A), (3A), and (4A)