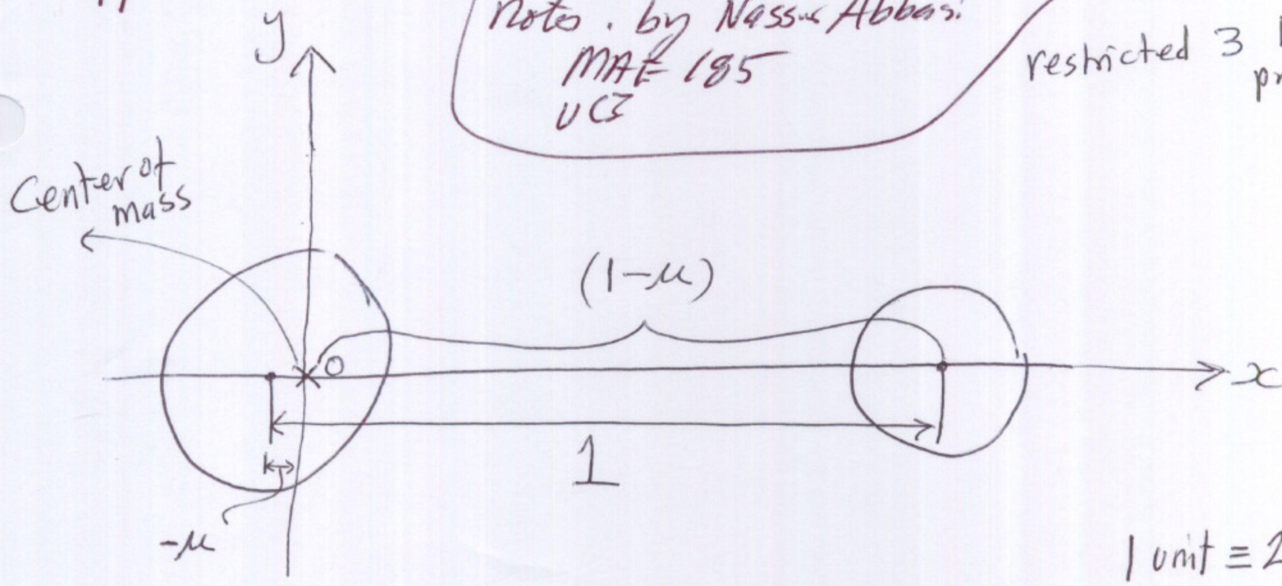
HW#2

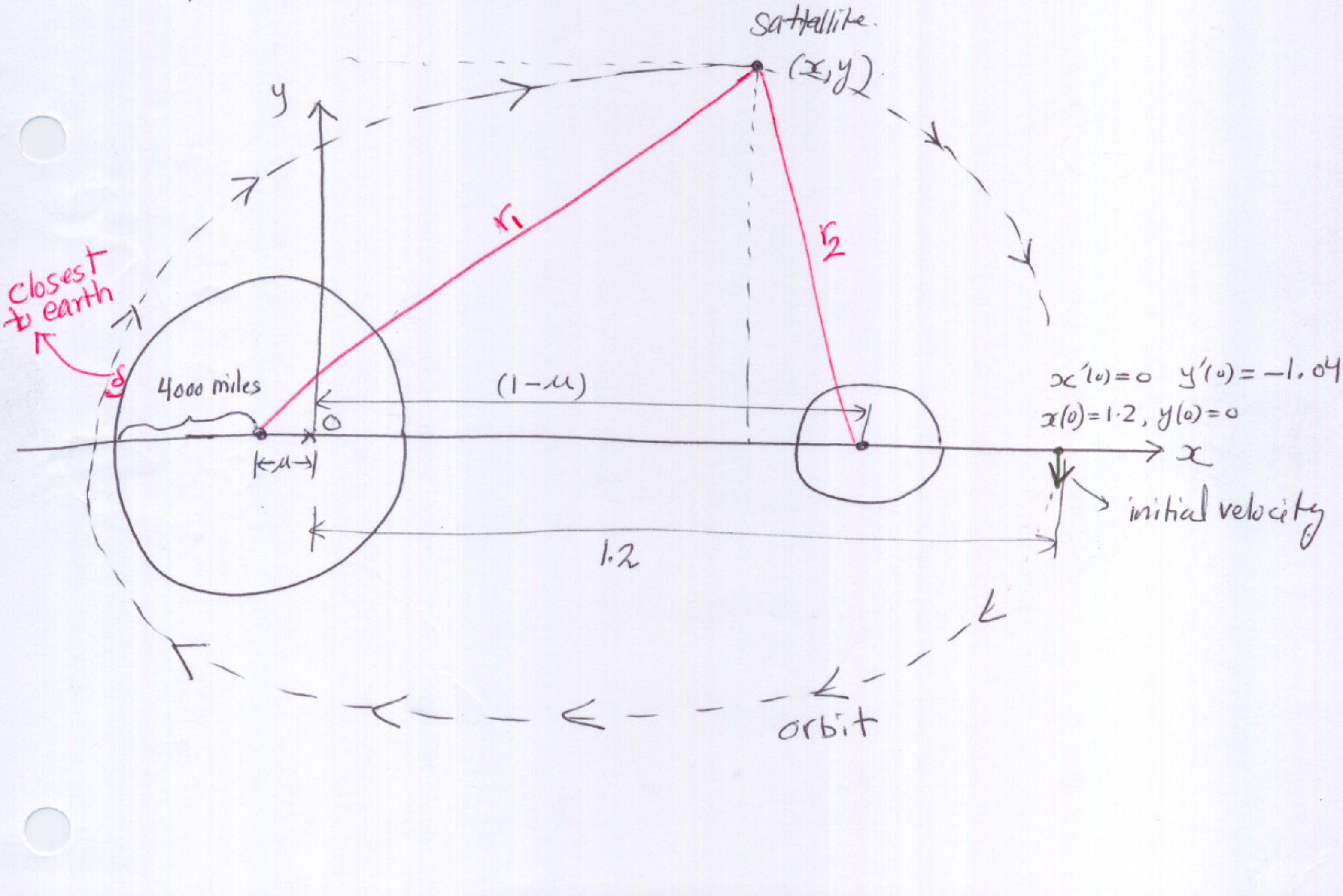
Friday April 28 discussion
notes. by Nasser Abbasi
MAE 185
UCI

restricted 3 body problem.



1 unit ≡ 238,000 miles

$$\mu = \frac{\text{Mass Moon}}{\text{Mass earth}}$$



<u>Step 1</u> Convert 2 Second order ODE's to 4 first order ODE's.

Introduce 4 new state variables.

x_1, x_2, x_3, x_4.

we want the following

$$\dot{x}_1 = M1(t, x_1, x_2, x_3, x_4)$$
$$\dot{x}_2 = M2(t, x_1, x_2, x_3, x_4)$$
$$\dot{x}_3 = M3(t, x_1, x_2, x_3, x_4)$$
$$\dot{x}_4 = M_4(t, x_1, x_2, x_3, x_4).$$

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given
$$x'' = 2y' + x - \frac{\tilde{\mu}(x+\mu)}{r_1^3} - \frac{\mu(x-\tilde{\mu})}{r_2^3} - fx'$$
$$y'' = -2x' + y - \frac{\tilde{\mu} y}{r_1^3} - \frac{\mu y}{r_2^3} - fy'$$

let
$$\boxed{\begin{aligned} x_1 &= x \\ x_2 &= x' \\ x_3 &= y \\ x_4 &= y' \end{aligned}} \xrightarrow[\text{derivation}]{\text{take}}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x'' = 2y' + x - \frac{\tilde{\mu}(x+\mu)}{r_1^3} - \frac{\mu(x-\tilde{\mu})}{r_2^3} - fx'$$
$$= 2x_4 + x_1 - \frac{\tilde{\mu}(x_1+\mu)}{r_1^3} - \frac{\mu(x_1-\tilde{\mu})}{r_2^3} - fx_2$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = y'' = -2x' + y - \frac{\tilde{\mu} y}{r_1^3} - \frac{\mu y}{r_2^3} - fy'$$
$$= -2x_2 + x_3 - \frac{\tilde{\mu} x_3}{r_1^3} - \frac{\mu x_3}{r_2^3} - fx_4$$

$\longrightarrow$

where $r_1^3 = \left((x+\mu)^2 + y^2 \right)^{3/2}$ (3)

$$\Downarrow$$

$$\boxed{r_1^3 = \left((x_1+\mu)^2 + x_3^2 \right)^{3/2}}$$

and $r_2^3 = \left((x-\tilde{\mu})^2 + y^2 \right)^{3/2}$

$$\Downarrow$$

$$\boxed{r_2^3 = \left((x_1-\tilde{\mu})^2 + x_3^2 \right)^{3/2}}$$

hence finally we write

$$\dot{x}_1 = x_2 \qquad (1)$$

$$\dot{x}_2 = 2x_4 + x_1 - \frac{\tilde{\mu}(x_1+\mu)}{\left((x_1+\mu)^2+x_3^2\right)^{3/2}} - \frac{\mu(x_1-\tilde{\mu})}{\left((x_1-\tilde{\mu})^2+x_3^2\right)^{3/2}} - f x_2 \qquad (2)$$

$$\dot{x}_3 = x_4 \qquad (3)$$

$$\dot{x}_4 = -2x_2 + x_3 - \frac{\tilde{\mu}\, x_3}{\left((x_1+\mu)^2+x_3^2\right)^{3/2}} - \frac{\mu\, x_3}{\left((x_1-\tilde{\mu})^2+x_3^2\right)^{3/2}} - f x_4 \qquad (4)$$

these are the 4 1^{st} order ODE (Coupled) to solve.

$\longrightarrow$

note that x_1 represents the x-coordinates of appollo, x_3 represents the y-coordinates of appollo.
So these are the coordinates to use to monitor position of Satallite to see how close to earth it gets.

Solving using Ruga-Kutta order 4. (RK4)

<u>example 1</u> this examples shows how to use RK4 to solve one 1st order ODE.

$$\boxed{\frac{dx}{dt} = \cancel{f(x,t)}\ f(t,x) \qquad x = x_0 \text{ at } t = 0}$$

$x = x_0$; $t = 0$, $\Delta t = 0.1$, NSTEPS = 100 (say).

DO ~~FOR~~ I = 1, NSTEPS

$$K_1 = f(t, x)$$

$$K_2 = f\left(t + \frac{\Delta t}{2}, x + \frac{\Delta t}{2} K_1\right)$$

$$K_3 = f\left(t + \frac{\Delta t}{2}, x + \frac{\Delta t}{2} K_2\right)$$

$$K_4 = f\left(t + \Delta t, x + \Delta t K_3\right)$$

$$x = x + \frac{\Delta t}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)$$

$$t = t + \Delta t$$

END DO WRITE(*, *) t, x

Example 2 Use RK4 to solve 2 1^{st} order coupled ODE's.

$$\frac{dx}{dt} = f(t, x, y) \qquad x = x_0 \quad t = 0$$

$$\frac{dy}{dt} = g(t, x, y) \qquad y = y_0 \quad t = 0$$

since these are coupled, we must determine the k's in <u>locked-step</u> fashion. as follows:

$x = x_0$
$y = y_0$
$t = 0$
$NSTEPS = 100$
$\Delta t = 0.1$

DO I=1, NSTEPS

$$K_{1,x} = f(t, x, y)$$
$$K_{1,y} = g(t, x, y)$$

$$K_{2,x} = f\left(t + \frac{\Delta t}{2}, x + \frac{\Delta t}{2} K_{1,x}, y + \frac{\Delta t}{2} K_{1,y}\right)$$
$$K_{2,y} = g\left(t + \frac{\Delta t}{2}, x + \frac{\Delta t}{2} K_{1,x}, y + \frac{\Delta t}{2} K_{1,y}\right)$$

$$K_{3,x} = f\left(t + \frac{\Delta t}{2}, x + \frac{\Delta t}{2} K_{2,x}, y + \frac{\Delta t}{2} K_{2,y}\right)$$
$$K_{3,y} = g\left(t + \frac{\Delta t}{2}, x + \frac{\Delta t}{2} K_{2,x}, y + \frac{\Delta t}{2} K_{2,y}\right)$$

$$K_{4,x} = f(t + \Delta t, x + \Delta t K_{3,x}, y + \Delta t K_{3,y})$$
$$K_{4,y} = g(t + \Delta t, x + \Delta t K_{3,x} + y + \Delta t K_{3,y})$$

$t = t + \Delta t$

$$x = x + \frac{\Delta t}{6}(K_{1,x} + 2K_{2,x} + 2K_{3,x} + K_{4,x})$$
$$y = y + \frac{\Delta t}{6}(K_{1,y} + 2K_{2,y} + 2K_{3,y} + K_{4,y})$$

So in our problem we do the same except we have 4 1st order ODE's.

$$\boxed{\begin{aligned} \frac{dx_1}{dt} &= M1(t, x_1, x_2, x_3, x_4) \\ \frac{dx_2}{dt} &= M2(t, x_1, \ldots) \\ &\vdots \\ \frac{dx_4}{dt} &= M4(\ldots) \end{aligned}}$$

Where $M1(t, x_1, x_2, x_3, x_4)$ is RHS function $\rightarrow x_2$ in this case.

$M2(t, x_1, \ldots)$ is from before see eq (2)

M3 see eq (3)

M4 see eq (4)

I will show how to find K_1 and K_2 for this problem.

~~$K_1 = M_1$~~

$t=0$

~~$x_1 = 0$; $x_2 = 0$, x_3~~

$x_1 = 1.2$; $x_2 = 0$, $x_3 = 0$, $x_4 = -1.04935751$

$\Delta t = 0.1$

$NSTEPS = \frac{1.5 * T}{\Delta t}$ ✓ Rounded. or $\frac{2 * T}{\Delta t}$ rounded

(i.e pick enough NSTEPS to see more than one complete period)

$\longrightarrow$

DO I=1, NSTEPS

$K_{1,x_1} = M1(t, x_1, x_2, x_3, x_4)$

$K_{1,x_1} = M2(\text{-----})$

$\vdots$

$K_{1,x_4} = M4(\text{-----})$

$$K_{2,x_1} = M1\left(t+\frac{\Delta t}{2}, x_1+\frac{\Delta t}{2}K_{1,x_1}, x_2+\frac{\Delta t}{2}K_{1,x_2}, x_3+\frac{\Delta t}{2}K_{1,x_3}, x_4+\frac{\Delta t}{2}K_{1,x_4}\right)$$

$$K_{2,x_2} = M2\left(t+\frac{\Delta t}{2}, x_1+\frac{\Delta t}{2}K_{1,x_1}, x_2+\frac{\Delta t}{2}K_{1,x_2}, \cdots\right)$$

$\vdots$

$$K_{2,x_4} = M4\left(t+\frac{\Delta t}{2}, x_1+\frac{\Delta t}{2}K_{1,x_1}, x_2+\frac{\Delta t}{2}K_{1,x_2}, \cdots\right)$$

$$K_{3,x_1} = M1\left(t+\frac{\Delta t}{2}, x_1+\frac{\Delta t}{2}K_{2,x_1}, x_2+\frac{\Delta t}{2}K_{2,x_2}, \cdots\right.$$

$\vdots$

$$x_1 = x_1 + \frac{\Delta t}{6}\left(K_{1,x_1} + 2K_{2,x_1} + 2K_{3,x_1} + K_{4,x_1}\right)$$

$\vdots$

$$x_4 = x_4 + \frac{\Delta t}{6}\left(K_{1,x_4} + 2K_{2,x_4} + 2K_{3,x_4} + K_{4,x_4}\right)$$

$t = t + \Delta t$

$\longleftarrow$ here print t, x_1, x_3

END DO

x_1: x coordinate; x_3: y coordinates

So you need to define 4 functions. ⑧

```
REAL FUNCTION  M1(t, x1, x2, x3, x4)
      M1 = x2
      RETURN
      END
```

```
mu=1/82.45
REAL FUNCTION  M2(t, x1, x2, x3, x4)
  mubar = 1 - mu ; f=0
M2 = 2*x4 + x1 - mubar*(x1+mu)/((x1+mu)**2+x3**2)**(3/2
```

— ..... etc.. see eq. (2)

```
RETURN
END
```

Same for M3, M4 Functions.

change $f=0$ or $f=1$ or $f=0.1$

to do Case (a), (b), (c).

Note about units

it is better to use the normalized units given.

i.e $1 \equiv$ distance from earth center to moon center.

we are told this distance is 238,000 miles.

hence radius of earth is $\frac{\cancel{238,000}}{\cancel{4000}}$ $= 0.0168067$

in Normalized units $\boxed{\frac{4,000}{238,000}}$

To determine apollo distance from earth:

as you enter the loop, at the end of each loop iteration you'll get a new (x, y) coordinates for apollo. from there you can calculate $\boxed{r_1}$ which is distance of apollo to center of earth.

hence distance of apollo from surface of earth is

$$\boxed{r_1 - d}$$

where $d = 0.0168067$