# HW4 solution. MAE 185. Spring 2006

All blames to Nasser Abbasi

### June 24, 2013

## 1 Problem

Integrate the following using Romberg integration

$$\int_{0}^{1} e^{x^{2}} dx, \quad \int_{0}^{1} x^{x} dx, \quad \int_{0}^{1} \sin(50 x) dx, \quad \int_{0}^{10} \ln(x) dx, \quad \int_{0}^{1} \frac{1}{\sqrt{x}} e^{x^{2}} dx, \quad \int_{1}^{5} \frac{(x-1)^{0.2}}{x^{2}+1} dx$$

## 2 Answer

An important part is to select a tolerance which can be used to stop the Romberg iteration. This is the tolerance shown in the pseudocode on page 212 with the letter  $\epsilon$ .

The source code is attached at the end, and the output.txt (in which the Romberg table is also printed for each problem).

The following is the Fortran source code to solve this HW. ??

The following is the output from running the program. ??

### 2.1 Problem 1

$$\int_{0}^{1} e^{x^2} dx$$

No preprocessing needed. Use Romberg integration on the integral as shown. The answer is 1.4626517459

### 2.2 Problem 2

$$\int_{0}^{1} x^{x} dx$$

No preprocessing needed. Use Romberg integration on the integral as shown. The answer is 0.7834305142

#### $\mathbf{2.3}$ Problem 3

$$\int_{0}^{1} \sin(50 x) dx$$

No preprocessing needed. Use Romberg integration on the integral as shown. Just make sure the tolerance is small enough. Use a multiple of machine double precision limit.

The answer is 0.0007006794

#### Problem 4 2.4

$$\int_{0}^{10} \ln\left(x\right) dx$$

To handle the singularity at x = 0 we can either integrate from some very small  $\epsilon > 0$  or use integration by parts to change the integrand to something without singularity.

Using integration by parts.  $\int u dv = uv - \int v du$  hence let  $u = \ln(x)$  and dv = dx. Hence  $du = \frac{1}{x} dx$ and v = x.

$$\int_{0}^{10} (\ln(x) \times 1) \ dx = [x \ \ln(x)]_{0}^{10} - \int_{0}^{10} x \left(\frac{1}{x} dx\right) = [10 \times \ln(10)] - \int_{0}^{10} dx$$

So the integral becomes

$$23.02585 - \int_{0}^{10} dx$$

(Clearly we do not need to do numerical integration now, since the answer is 13.02585 by just looking at the above, but we need to use Romberg integration, so our integrand here is 1)

The answer is 13.0258509299

#### 2.5Problem 5

$$\int_{0}^{1} \frac{1}{\sqrt{x}} e^{x^2} dx$$

To handle the singularity at x = 0 we can either integrate from some very small  $\epsilon > 0$  or use

integration by parts to change the integrand to something without singularity. Using integration by parts.  $\int u dv = uv - \int v du$  hence let  $u = e^{x^2}$  and  $dv = \frac{1}{\sqrt{x}} dx$ . Hence  $du = 2xe^{x^2} dx$ and  $v = \int x^{-\frac{1}{2}} dx = 2\sqrt{x}$ 

Hence

$$\int_{0}^{1} \frac{1}{\sqrt{x}} e^{x^{2}} dx = \left[ 2\sqrt{x} e^{x^{2}} \right]_{0}^{1} - \int_{0}^{1} 2\sqrt{x} \left( 2x \ e^{x^{2}} \right) dx$$
$$= \left[ 2\sqrt{x} e^{x^{2}} \right]_{0}^{1} - 4 \int_{0}^{1} x^{\frac{3}{2}} \ e^{x^{2}} dx$$

And we apply Romberg integration on the second part above. The answer is 2.5425742098

### 2.6 Problem 6

$$\int_{1}^{5} \frac{(x-1)^{0.2}}{x^2+1} dx$$

No preprocessing needed. Use Romberg integration on the integral as shown. The answer is 0.5613773544

# 3 Appendix

```
1******
!* Solve HW4, MAE 185. Spring 2006
!* UCI.
!* all blames to Nasser Abbasi
!* Compiler: g95 on windows XP
!* (GCC 4.0.2 (g95) Mar 3 2006)
PROGRAM hw4
IMPLICIT NONE
REAL*8 result,solve_problem
INTEGER I
  OPEN(UNIT=999,FILE='hw4_output.txt')
  DO I=1,6
    result=solve_problem(I)
    WRITE(UNIT=999,FMT='(A,I1,A,F14.10)') '\n result problem ',I,'=',result
    WRITE(UNIT=999,FMT='(A)') '======
  END DO
END PROGRAM
!*********
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.
|********
REAL*8 function solve_problem(I)
IMPLICIT NONE
real*8 :: romberg,TOL,xa,xb
real*8,external:: f_1,f_2,f_3,f_4,f_5,f_6
PARAMETER (TOL=1D5*EPSILON(1.0D0)) ! pick some tolerance value
integer :: RMAX,I
parameter (RMAX=12)
  SELECT CASE(I)
     CASE(1)
      xa=0.0d0
      xb=1.d0
      solve_problem = romberg(xa,xb,f_1,TOL,RMAX)
     CASE(2)
      xa=0.0d0
      xb=1.d0
      solve_problem = romberg(xa,xb,f_2,TOL,RMAX)
     CASE(3)
      xa=0.0d0
      xb=1.d0
      solve_problem = romberg(xa,xb,f_3,TOL,RMAX)
     CASE(4)
      xa=0.0d0
      xb=10.d0
      solve_problem = xb*LOG(xb) - romberg(xa,xb,f_4,TOL,RMAX)
     CASE(5)
      xa=0.0d0
      xb=1.d0
      solve_problem = 2*SQRT(xb)*EXP(xb**2) - 4*romberg(xa,xb,f_5,TOL,RMAX)
     CASE(6)
      xa=1.d0
      xb=5.d0
      solve_problem = romberg(xa,xb,f_6,TOL,RMAX)
  END SELECT
END
*******
!* ROMBERG
1*
.
|***************************
REAL*8 function romberg(xa,xb,f,EPSILON,RMAX)
IMPLICIT NONE
real*8, external:: f
real*8 :: xa,xb,dx,epsilon,x,s,dl
integer :: RMAX,r,k,m,c
real*8 :: I(0:RMAX,0:RMAX) ! the romberg table
r = 0
 dx = xb-xa
I(0,0) = dx^{*} (f(xa)+f(xb))/2
WRITE(UNIT=999,FMT='(F14.10)',advance='no') I(0,0)
DO WHILE( .TRUE. )
  r = r+1
  dx = dx/2
  k = 2**(r-1)
                                     5
  s = 0
```

DO m = 1,k x = xa +  $(2^{*}m-1)^{*}dx$ s = s + f(x)

# 4 Output

	1.5715831655	1.4757305825	1.4906788617	1.4637107604	1.4629094390		
1.4697122764	1.4627234147	1.4626575916	1.4626535940				
1.4644203101	1.4626563214	1.4626518485	1.4626517573	1.4626517501			
1.4630941026	1.4626520334	1.4626517476	1.4626517460	1.4626517459	1.4626517459		
1.4627623486	1.4626517639	1.4626517459	1.4626517459	1.4626517459	1.4626517459	1.4626517459	
result problem	n 1= 1.46265174	459					
1.0000000000	0.8535533906	0.8047378541	0.8050352528	0.7888625402	0.7878041860		
0.7898353948	0.7847687754	0.7844958578	0.7844433446				
0.7852756861	0.7837557832	0.7836882504	0.7836754312	0.7836724198			
0.7839511939	0.7835096966	0.7834932908	0.7834901962	0.7834894698	0.7834892909		
0.7835752392	0.7834499209	0.7834459359	0.7834451842	0.7834450077	0.7834449642	0.7834449534	
0.7834702854	0.7834353008	0.7834343261	0.7834341418	0.7834340985	0.7834340878	0.7834340852	0.7834340845
0.7834413454	0.7834316987	0.7834314586	0.7834314131	0.7834314024	0.7834313997	0.7834313991	0.7834313989
0.7834334411	0.7834308063	0.7834307468	0.7834307355	0.7834307329	0.7834307322	0.7834307320	0.7834307320
0.7834312986	0.7834305844	0.7834305696	0.7834305668	0.7834305661	0.7834305660	0.7834305659	0.7834305659
0.7834307215	0.7834305291	0.7834305254	0.7834305247	0.7834305246	0.7834305245	0.7834305245	0.7834305245
0.7834305668	0.7834305153	0.7834305144	0.7834305142	0.7834305142	0.7834305142	0.7834305142	0.7834305142
result problem	n 2= 0.78343053	142					
-0 1311874269	-0 1317695885	-0 1319636423	-0 1319149685	-0 1319634285	-0 1319634142		
-0.1319513035	-0.1319634151	-0.1319634143	-0.1319634143	0.1010004200	0.1010004142		
0.0000090831	0.0439958787	0.0557264982	0.0587057032	0.0594534252			
0.0005519662	0.0007329272	-0.0021512695	-0.0030699643	-0.0033122218	-0.0033735763		
0.0006646731	0.0007022421	0.0007001965	0.0007454578	0.0007604203	0.0007644013	0.0007654118	
0.0006917471	0.0007007717	0.0007006737	0.0007006813	0.0007005057	0.0007004471	0.0007004315	0.0007004275
0.0006984506	0.0007006851	0.0007006793	0.0007006794	0.0007006794	0.0007006796	0.0007006797	0.0007006797
0.0007001225	0.0007006798	0.0007006794	0.0007006794	0.0007006794	0.0007006794	0.0007006794	0.0007006794
0.0007005402	0.0007006795	0.0007006794	0.0007006794	0.0007006794	0.0007006794	0.0007006794	0.0007006794
result problem	n 3= 0.0007006	794					
10.0000000000	10 0000000000	10 0000000000					
TAPHIE DRODIER	10.00000000000	20.00000000000					
result problem	13.02585092	299					
1.3591409142	4= 13.02585092	0.7556946645	0.7715289249	0.7265198243	0.7245748349		
1.3591409142 0.7356981803	13.02585092 4= 13.02585092 0.9065562270 0.7237545987	0.7556946645 0.7235702503	0.7715289249	0.7265198243	0.7245748349		
1.3591409142 0.7356981803 0.7265665518	13.02585092 4= 13.02585092 0.9065562270 0.7237545987 0.7235226756	0.7556946645 0.7235702503 0.7235072141	0.7715289249 0.7235543046 0.7235062135	0.7265198243	0.7245748349		
1.3591409142 0.7356981803 0.7265665518 0.7242670291	0.9065562270 0.7237545987 0.7235226756 0.7235005216	0.7556946645 0.7235702503 0.7235072141 0.7234990446	0.7715289249 0.7235543046 0.7235062135 0.7234989150	0.7265198243 0.7235060249 0.7234988863	0.7245748349		
1.3591409142 0.7356981803 0.7265665518 0.7242670291 0.7236901351	0.9065562270 0.7237545987 0.7235226756 0.7235005216 0.7234978371	0.7556946645 0.7235702503 0.7235072141 0.7234990446 0.7234976581	0.7715289249 0.7235543046 0.7235062135 0.7234989150 0.7234976361	0.7265198243 0.7235060249 0.7234988863 0.7234976311	0.7245748349 0.7234988794 0.7234976299	0.7234976296	
1.3591409142 0.7356981803 0.7265665518 0.7242670291 0.7236901351 0.7235456142	0.9065562270 0.7237545987 0.7235226756 0.7235005216 0.7235005216 0.7234978371 0.7234974406	0.7556946645 0.7235702503 0.7235072141 0.7234990446 0.7234976581 0.7234974141	0.7715289249 0.7235543046 0.7235062135 0.7234989150 0.7234976361 0.7234974103	0.7265198243 0.7235060249 0.7234988863 0.7234976311 0.7234974094	0.7245748349 0.7234988794 0.7234976299 0.7234974092	0.7234976296 0.7234974091	0.7234974091
1.3591409142 0.7356981803 0.7265665518 0.7242670291 0.7236901351 0.7235456142 0.7235094351	0.9065562270 0.7237545987 0.7235226756 0.7235005216 0.7234978371 0.7234974406 0.7234973754	0.7556946645 0.7235702503 0.7235072141 0.7234990446 0.7234976581 0.7234974141 0.7234973710	0.7715289249 0.7235543046 0.7235062135 0.7234989150 0.7234976361 0.7234974103 0.7234973703	0.7265198243 0.7235060249 0.7234988863 0.7234976311 0.7234976311 0.7234974094 0.7234973702	0.7245748349 0.7234988794 0.7234976299 0.7234974092 0.7234973702	0.7234976296 0.7234974091 0.7234973701	0.7234974091 0.7234973701
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result problem 6= 0.5613773544