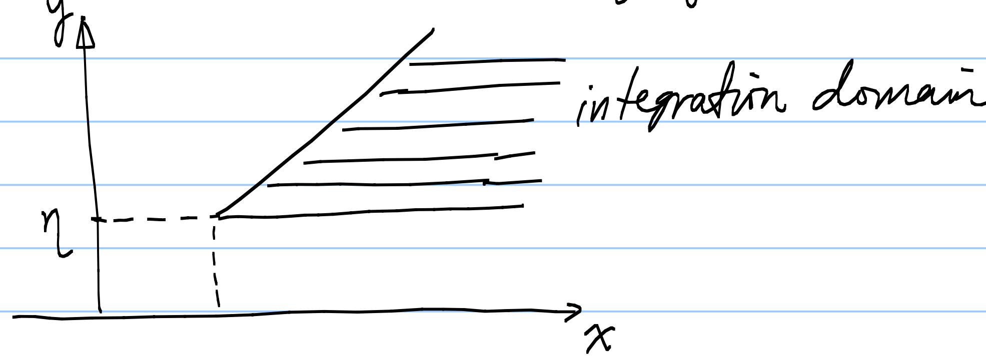
$$g = \operatorname{erfc}(\eta) = \frac{2}{\sqrt{\pi}} \int_\eta^\infty e^{-x^2} dx$$

To maintain $f'(+\infty) = 0$, we integrate

$$f'(\eta) = -\int_\eta^\infty g(y)\, dy = -\frac{2}{\sqrt{\pi}} \int_\eta^{+\infty} \left[\int_y^{+\infty} e^{-x^2} dx \right] dy$$



Exchange integration variable we get

$$f'(\eta) = -\frac{2}{\sqrt{\pi}} \int_\eta^{+\infty} \left[\int_\eta^{x} e^{-x^2} dy \right] dx$$

$$= -\frac{2}{\sqrt{\pi}} \int_\eta^{+\infty} (x-\eta)\, e^{-x^2} dx$$

$$= -\frac{2}{\sqrt{\pi}} \int_\eta^{+\infty} x e^{-x^2} dx + \eta \frac{2}{\sqrt{\pi}} \int_\eta^{+\infty} e^{-x^2} dx$$

$$= -\frac{1}{\sqrt{\pi}} e^{-\eta^2} + \eta \operatorname{erfc}(\eta)$$

Integrate again

$$f(\eta) = -\int_{\eta}^{+\infty} f'(y)\, dy = \frac{1}{\sqrt{\pi}} \int_{\eta}^{+\infty} e^{-y^2} dy - \int_{\eta}^{+\infty} y\,\mathrm{erfc}(y)\, dy$$

$$= \frac{1}{2}\,\mathrm{erfc}(\eta) - \int_{\eta}^{\infty} y\, f''(y)\, dy$$

$$= \frac{1}{2}\,\mathrm{erfc}(\eta) - y f'(y)\Big|_{\eta}^{\infty} + \int_{\eta}^{\infty} f'(y)\, dy$$

The last term is $-f(\eta)$ by definition

$$\therefore \quad f(\eta) = \frac{1}{2}\left[\frac{1}{2}\,\mathrm{erfc}(\eta) + \eta f'(\eta)\right]$$

$$= \frac{1}{4}\left[\mathrm{erfc}(\eta) + 2\left(-\frac{1}{\sqrt{\pi}}\eta e^{-\eta^2} + \eta^2\,\mathrm{erfc}(\eta)\right)\right]$$

$$= \frac{1}{4}\left[(1+2\eta^2)\,\mathrm{erfc}(\eta) - \frac{2}{\sqrt{\pi}}\,\eta e^{-\eta^2}\right]$$

Notice that the equation for $f(\eta)$ is homogeneous, therefore we are allowed an arbitrary constant coefficient. Applying the condition $f(\eta)=1$ gives

$$f(\eta) = (1+2\eta^2)\,\mathrm{erfc}(\eta) - \frac{2}{\sqrt{\pi}}\,\eta e^{-\eta^2}$$