MAE200B Homework #3

Jan. 28, 2006
Due 5:00pm Friday, Feb. 3, 2006

Suggested Reading:

Farlow, Lesson 6, 7, 8, and 9.
Haberman, Chapter 5, Chapter 8 Section 8.1-8.3.

Assignments:

1. Haberman, Chapter 5, Problem 5.3.2
2. Haberman, Chapter 5, Problem 5.3.9
3. Consider the periodic Sturm-Liouville problem

$$X''(x) = -\omega^2 X(x), \quad -L < x < L$$

$$X(-L) = X(L), \quad X'(-L) = X'(L)$$

(a) Show by direct substitution that

$$\phi_n^{(-)} = e^{-i\omega_n x}$$

and

$$\phi_n^{(+)} = e^{i\omega_n x}$$

are two eigen functions for the eigenvalues $\omega_n = \frac{n\pi}{L}$. The eigen functions $\cos \omega_n x$ and $\sin \omega_n x$, can be expressed as linear combinations of the above two eigenfunctions (with complex coefficients).

(b) show that the above two eigenfunctions, and eigenfunctions for different eigenvalues are orthogonal with the following definition of inner product

$$(f, g) = \int_{-L}^{L} f(x)\overline{g(x)}dx$$

where the overbar denotes complex conjugate.

(c) Given a function $f(x)$, expand it in terms of the above basis functions (from Sturm-Liouville theorem, we know they form a complete basis).

$$f(x) = \sum_{n=-\infty}^{+\infty} C_n e^{i\frac{n\pi}{L}x}$$

Determine the formulae for the coefficients C_n.

(d) Show if $f(x)$ is a real function, $C_{-n} = \overline{C_n}$

4. **Forced vibration of a finite string** Consider the same string problem of Item 1 in Homework Assignment #2. This time, however, the string is subject to a periodic forcing

$$f(x,t) = A \sin \omega t$$

so that the PDE becomes

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + A \sin \omega t$$

with the same initial and boundary conditions. A is a non-zero constant. Solve the problem when

   (a) ω is not one of the eigen-frequencies, i.e. when $\omega \neq \omega_n$, where $\omega_n = \frac{(2n-1)\pi c}{2L}$;

   (b) $\omega = \omega_2$

   Optional: Do the same plots and movie as in Homework Assignment #2. For plotting purposes, you may choose $A = 1.0$.

5. **Forced vibration of a finite string with damping** Consider the same string problem as above but with the same damping as in Problem 2 of Homework Assignment #2.

$$\frac{\partial^2 u}{\partial t^2} + b\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} + A \sin \omega t$$

with the same initial and boundary conditions. Solve the problem by the method of separation of variables and eigen-function expansions (see Haberman Section 8.1-8.3). Again, consider the cases: (1) $b < \pi c/L$; (2) $b = \pi c/L$; and (3) $b > \pi c/L$.

   Optional: Do the same plots and movie as in Homework Assignment #2.

   Optional: Consider the long-time behavior of your solutions, plot the amplitudes(normalized by A) of the first 5 modes of the 'steady-state' solution and their sum vs. the forcing frequency ω from 0 to ω_5.