MAE200B Homework #2

Jan. 21, 2006

Due 5:00pm Friday, Jan. 27, 2006

Suggested Reading:

Farlow, Lesson 16,19, and 20.

Haberman, Chapters 3 and 4.

Assignments:

1. Consider the finite string problem with the left end fixed. The right end is left free to move vertically without any friction. Show that the boundary condition on the right end should be $\frac{\partial u}{\partial x}(L,t) = 0$. Assume initially the string is stationary and at equlibrium and is plucked upward at $x = L/3$ by a distance of h (h is much smaller than L). Pose the PDE and its initial and boudnary conditions for this problem. Solve the problem by the method of separation of variables. (Be sure to review Haberman Chapter 4.)

   Pick the simple case where $c = \sqrt{\frac{T}{\rho}} = 4$, $L = 1$, and $h = 0.1$. What is the frequency of the fundamental mode (first mode)? Plot first 5 terms of your Fourier series solution and their sum (spatial distribution of u) at time $t = 0.0$. Do the same for $t = 0.25T_1$, $0.5T_1$, $0.75T_1$, and $1.0T_1$, where T_1 is the period of the fundamental mode. Plot in the same figure the first 5 terms of your Fourier series solutions at $x = 0.5L$ and their sum (time history of $u(0.5L,t)$). Examine your plots and try to appreciate the physical significance of your solutions. Now if you are computer savvy, make a movie of the motion of the string (optional).

2. Consider the same string problem as above. This time, however, the string is subject to an aerodynamic or hydraulic damping so that the PDE becomes

   $$\frac{\partial^2 u}{\partial t^2} + b\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

   with the same initial and boundary conditions. Solve the problem by the method of separation of variables.

   Notice that the solution in time for each modal shape may take different forms depending on some critical values of the damping coefficient b. Discuss, for instance, the cases: (1) $b < \pi c/L$; (2) $b = \pi c/L$; and (3) $b > \pi c/L$.

   Do the same plots (and optional movie) as in Problem 1 for this problem for the three cases: (1) $b = 0.5\pi c/L$; (2) $b = \pi c/L$; and (3) $b = 2\pi c/L$. Compare your results with those from Problem 1. Plot the solutions over several periods of the fundamental mode. Notice the shape and magnitude of the different modes and the total solution in a few cycles of the solution.