MAE200B Homework #1

Jan. 14, 2006

Due 5:00pm Friday, Jan. 20, 2006

Suggested Reading:

Farlow, Lessons 1–5.

Haberman, Chapter 1, Sections 2.1-2.4.

Greenberg, Sections 17.1-17.4, 18.1-18.3.2., 18.4.

1. Solve the following IBVP for the 1D heat conduction problem over the interval $[0, L]$ by separation of varables, repeat all steps we performed in class.

$$\begin{cases} \frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, & 0 < x < L \\ u(0,t) = u(L,t) = 0 & \\ u(x,0) = \begin{cases} x & 0 < x < L/2 \\ L - x & L/2 < x < L \end{cases} & \end{cases}$$

   Consider the case when $\alpha = 1$ and $L = 1$. Use a computer program to plot (in the same figure) the exact initial condition and your series expansion of the initial condition by keeping only one, two, three, ..., terms. Determine the number of terms you need to keep so that you cannot see much difference in your figures by adding more terms. Then use the same number of terms to find your solution of $u(x, t)$ at $t = 0, 0.1, 0.2$, etc. and plot them in the same figure.

2. A box of dimensions $2L \times L \times L$ containing two square cubes of the same (solid) materials initially separated by an insulating diaphragm between the cubes. The left cube has an initial temperature of u_l, the right cube has an initial tempearture of u_r. The right face of the box is fixed at the temperature u_r. All other faces of the box are insulated. The diaphragm that seperates the two cubes is suddenly withdrawn at $t = 0$. Following what we did in class, derive the heat conduction PDE, pose the correct initial and boundary conditions, and then solve it by separation of variables. Do the same plots as in the above problem for $u_l = 0\ K$ and $u_r = 1000\ K$.

3. (Optional) Consider a fully developed incompressible flow in a two-dimensional channel of height H. The top wall of the channel moves at a speed U. The bottom wall is fixed. For fully developed flow, we assume (1) the flow velocity along the channel is a function of y only and there is no transverse flow velocity; (2) the pressure gradient along the channel $\frac{\partial p}{\partial x}$ is a function of time only. Show that the mometum conservation (Newton's Law) gives

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x}$$

   where $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the fluid density, μ is the dynamic viscosity, and as we discussed in class viscous stress is $\tau = \mu \frac{\partial u}{\partial y}$. Pose the correct boundary conditions at the channel walls $y = 0$ and $y = H$. Assume $\frac{\partial p}{\partial x}$ is a constant and the fluid is initially at rest, solve for $u(y, t)$. Hint: solve the final equilibrium solution (steady-state) $U(y)$ first and then expresse $u(y, t) = U(y) + v(y, t)$ and solve for $v(y, t)$. Plot $U(y)$ and $u(y, t)$ at various times in the figure to see how the flow develops from zero veolocity to the steady-state solution $U(y)$.