$f(0) = \lim_{s\to\infty} sF(s)$

$f(\infty) = \lim_{s\to 0} sF(s)$

common transforms

$\frac{1}{s+1} \to e^{-t}$

$\frac{1}{s+2} \to e^{-2t}$

$\frac{\omega}{(s+a)^2+\omega^2} \to e^{-at}\sin\omega t$

$\frac{s+a}{(s+a)^2+\omega^2} \to e^{-at}\cos\omega t$

$s \to \frac{d}{dt}\delta(t)$

$1 \to \delta(t)$

$\frac{1}{s} \to u(t)$

$\frac{1}{s^2} \to t$, $t^2 \to \frac{2}{s^3}$

$e^{-at} \to \frac{1}{s+a}$, $te^{-at} \to \frac{1}{(s+a)^2}$

remember for partial fraction. if deg numerator > denominator, divid by denominator.

$$\frac{s^2+2s+3}{(s+1)^3} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$$

⇐ start by multiplying by $(s+1)^3$ to find b_3. Then differentiate result to find b_2 and differentiate results to find b_1

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\boxed{\sin\omega t \to \frac{\omega}{s^2+\omega^2}}$ $\boxed{\cos\omega t \to \frac{s}{s^2+\omega^2}}$

properties of laplace

$\mathcal{L}(f'(t)) = sF(s) - f(0)$

$\mathcal{L}(f''(t)) = s^2F(s) - sf(0) - f'(0)$

$\mathcal{L}\int_0^t f(t) = \frac{1}{s}F(s)$

$\int_0^\infty f(t)\,dt = \lim_{s\to 0} F(s)$

$$\mathcal{L}(e^{-\alpha t} f(t)) = F(s+\alpha)$$

$$\mathcal{L}(t\, f(t)) = -\frac{dF(s)}{ds}$$

$$\mathcal{L}(t^2 f(t)) = \frac{d^2}{ds^2}F(s).$$

$F(s) = \frac{2s+12}{s^2+2s+5}$

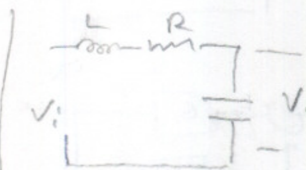
complex roots. do not do partial fractions.

$F(s) = \frac{10+2(s+1)}{(s+1)^2+2^2}$

$= 5\frac{2}{(s+1)^2+2^2} + 2\frac{s+1}{(s+1)^2+2^2}$

$= 5e^{-t}\sin 2t + 2e^{-t}\cos 2t.$

if $f(t)$ is periodic, period T, then $\mathcal{L} f(t) = \dfrac{\int_0^T f(t)e^{-st}dt}{1-e^{-Ts}}$



$\frac{V_o}{V_i} = \frac{1}{LCs^2+RCs+1}$

to convert from ODE to state space, $\ddot{y} = -\frac{b}{m}\dot{y} - \frac{k}{m}y - \frac{u(t)}{m}$

start with $\boxed{x_1 = y(t),\ x_2 = \dot{x}_1 = \dot{y}(t)}$ $\Longrightarrow$ $\boxed{\dot{x}_1 = x_2,\ \dot{x}_2 = \ddot{y} = -\frac{b}{m}x_2 - \frac{k}{m}x_1 - \frac{u}{m}}$

relation between function Transfer and State space.

from $\dot{x} = Ax + Bu$

$y = Cx + Du$

$Y(s) = CX$ ③

where $X = \underbrace{(sI-A)^{-1}}_{①}\underbrace{[X(0) + BU(s)]}_{②}$

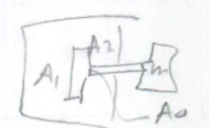
Control system performance
- steady state error
- transient response
- stability

$G(s) = \frac{1}{as+1}$ here $\boxed{\frac{1}{a}}$ is time constant τ $e^{-at} \equiv e^{-\frac{t}{\tau}}$

response to derivative of input can be found by diff the derivative of the original input

standard second order system

$$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$



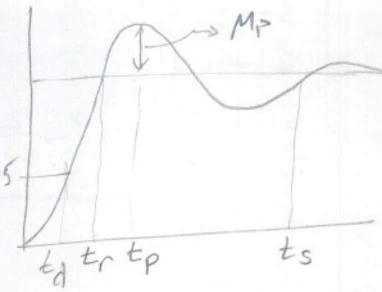
$m\ddot{x} = P_1A_1 - P_2A_2 - P_oA_o - c\dot{x}$

$m\dddot{x} = a(x)u - b(x)\dot{x} - c\ddot{x}$

$u \to \boxed{\frac{a}{ms^3+cs^2+bs}} \to X$

Power = Force × Velocity. For car, 20hp, at 60mph

$\Rightarrow$ Force $= \frac{20\text{ hp}}{60\text{ mph}} = \frac{20\times 746 \text{ watt}}{60\times\frac{1610}{3600}\text{ m/s}} = 557\text{ N}$



td = delay time

tr = rise time $\longrightarrow \dfrac{\pi-\beta}{\omega_d}$

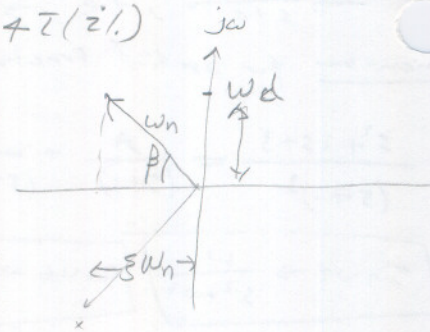
tp = peak time $\longrightarrow \dfrac{\pi}{\omega_d}$

Mp = Max overshoot $\longrightarrow e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\pi}$ $\Rightarrow$ i.e. if Mp = .254 $\rightarrow \xi = 0.4$

ts = settling time $\longrightarrow 3\tau\,(5\%)$ or $4\tau\,(2\%)$

$\boxed{\tau = \text{time constant} = \dfrac{1}{\xi\omega_n}}$

$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$ $\boxed{\xi = \cos\beta}$



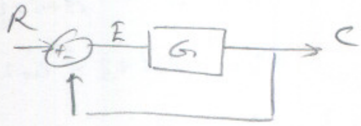
$$\boxed{\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

pole of input → form of forced response part of output

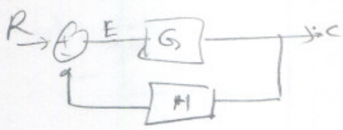
pole of G(s) → form of natural response of output.

$\boxed{\text{total response} = \text{forced} + \text{natural}}$ pole at origin of input → 1 response.

poles at $-\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$



$\boxed{E = \dfrac{R}{1+G}}$, open loop tf = G, closed loop = $\boxed{\dfrac{G}{1+G}}$



E = R − CH, open loop = GH, closed loop = $\dfrac{G}{1+GH}$

but C = EG

so E = R − EGH

E(1+GH) = R $\Rightarrow$ $\boxed{E = \dfrac{R}{1+GH}}$ $\Rightarrow e(\infty) = \lim_{s\to 0} sE(s) = \boxed{\lim_{s\to 0} s\dfrac{R}{1+GH}}$

Static error constants

K_p (position constant) $= \lim_{s\to 0} G(s)$

K_v (velocity constant) $= \lim_{s\to 0} sG(s)$

K_a (acc. constant) $= \lim_{s\to 0} s^2 G(s)$

K

integral: Controller output is ~~proportional~~ changed at rate proportional to error i.e. $\dfrac{du}{dt} = K_i e(t)$

for step input $\frac{1}{s}$, $e(\infty) = \dfrac{1}{1+K_p}$, for Ramp input, $e(\infty) = \dfrac{1}{K_v}$, for acc. input, $e(\infty) = \dfrac{1}{K_a}$

system type: number of integrators in forward path. $G(s) = \dfrac{K(s+z_1)(---)}{s^{?}(s+p_1)(s+p_2)---}$ ↳ type.

$e(\infty) = \dfrac{1}{1+\lim_{s\to 0} G(s)}$ for step input $\dfrac{1}{1+K_p}$

$e(\infty) = \dfrac{1}{\lim_{s\to 0} sG(s)}$ for Ramp input $\dfrac{1}{K_v}$

$e(\infty) = \dfrac{1}{\lim_{s\to 0} s^2G(s)}$ for acc. $\dfrac{1}{K_a}$

Proportional $u(t) = K_p e(t)$

integral $u(t) = K_i \int^t e(t) \Rightarrow \dfrac{U}{E} = \dfrac{K_i}{s}$

PI $\Rightarrow u(t) = K_p e(t) + \dfrac{K_p}{T_i}\int^t e(t)$ or $\dfrac{U}{E} = K_p\left(1+\dfrac{1}{T_i s}\right)$

PId $\Rightarrow K_p e(t) + \dfrac{K_p}{T_i}\int e(t) + K_p T_d \dfrac{d}{dt} e(t)$

or $K_p\left(1+\dfrac{1}{T_i s} + sT_d\right)$

	Step	ramp	Acc $= \frac{1}{2}t^2$
type 0	$\frac{1}{1+K}$	∞	∞
type 1	0	$\frac{1}{K}$	∞
type 2	0	0	$\frac{1}{K}$

ss(∞)

Velocity error means input and output move at same velocity but have position error.