HW # 8

MAE 170

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$\frac{45}{50}$

problem B-8-14

$$G(s) = \frac{1}{s(s^2+0.8s+1)}$$

solution. let $s=j\omega$,

$$G(j\omega) = \frac{1}{j\omega(-\omega^2+0.8j\omega+1)} \quad \Rightarrow \quad |G(j\omega)| = \frac{1}{\omega\sqrt{(1-\omega^2)^2+(0.8\omega)^2}}$$

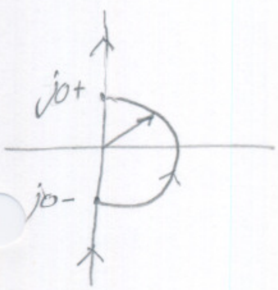
For $j0^+$, $\angle G(j\omega) = 0 - \left[90^\circ + \tan^{-1}\frac{0.8\omega}{1-\omega^2}\right] = -90^\circ$

for $j0^-$, $\angle G(j\omega) = 0 - \left[-90^\circ + \tan^{-1}\frac{0.8\omega}{1-\omega^2}\right] = +90^\circ$.

For both cases, $|G(j\omega)| = \infty$.

so the point $j0^+$ maps to $-j\infty$ and point $j0^-$ maps to $+j\infty$

now consider the small semicircle around the origion.



let point on this semicircle be $s = \varepsilon e^{j\theta}$ and change θ from -90° to $+90^\circ$.

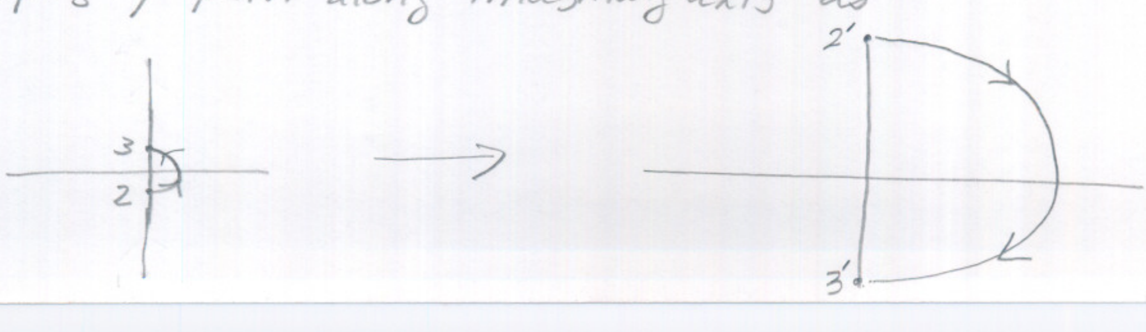
write
$$G(s) = \frac{1}{\varepsilon e^{j\theta}((\varepsilon e^{j\theta})^2+0.8\varepsilon e^{j\theta}+1)} = \frac{1}{\varepsilon e^{j\theta}(\varepsilon^2 e^{2j\theta}+0.8\varepsilon e^{j\theta}+1)}$$

$$= \frac{1}{\varepsilon^3 e^{j3\theta}+0.8\varepsilon^2 e^{j2\theta}+\varepsilon e^{j\theta}}$$

since $\varepsilon \ll 1$, then $\varepsilon^3 \to 0$, $\varepsilon^2 \to 0$ and above can be approximated as

$$G(s) \simeq \frac{1}{\varepsilon e^{j\theta}} = \frac{1}{\varepsilon}e^{-j\theta} = \boxed{\frac{1}{\varepsilon}\angle -\theta}$$

now change θ from -90° to $+90^\circ$. we see that this maps to infinite Large circle from $+90^\circ$ to -90°. so now can draw the mapings of path along imaginary axis as



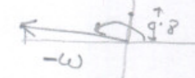
For points $+j\infty$, and $-j\infty$:

From $|G(j\omega)|$ we see that $|G(j\omega)| \to 0$ in both Cases.

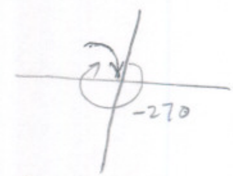
For phase, at point $+j\infty$, $\angle G(j\omega) = 0 - \left[90^\circ + \tan^{-1}\frac{0.8\omega}{1-\omega^2}\right]$

$$= -\left[90^\circ + \tan^{-1}\frac{0.8}{\underbrace{\frac{1}{\omega}}_{=0}-\omega}\right] = -\left[90^\circ + \tan^{-1}\frac{0.8}{-\omega}\right]$$

so $\angle G(j\omega)\Big|_{\omega\to+\infty} = -\left[90^\circ + 180\right] = -270^\circ$



so point $+j\infty$ maps to origion, at phase -270°



For point $-j\infty$, $\angle G(j\omega) = 0 - \left[-90^\circ + \tan^{-1}\frac{0.8}{-\omega}\right] = -\left[-90^\circ + 0\right] = +90^\circ$

Now For large semi circle.
let $s = Re^{j\theta}$.

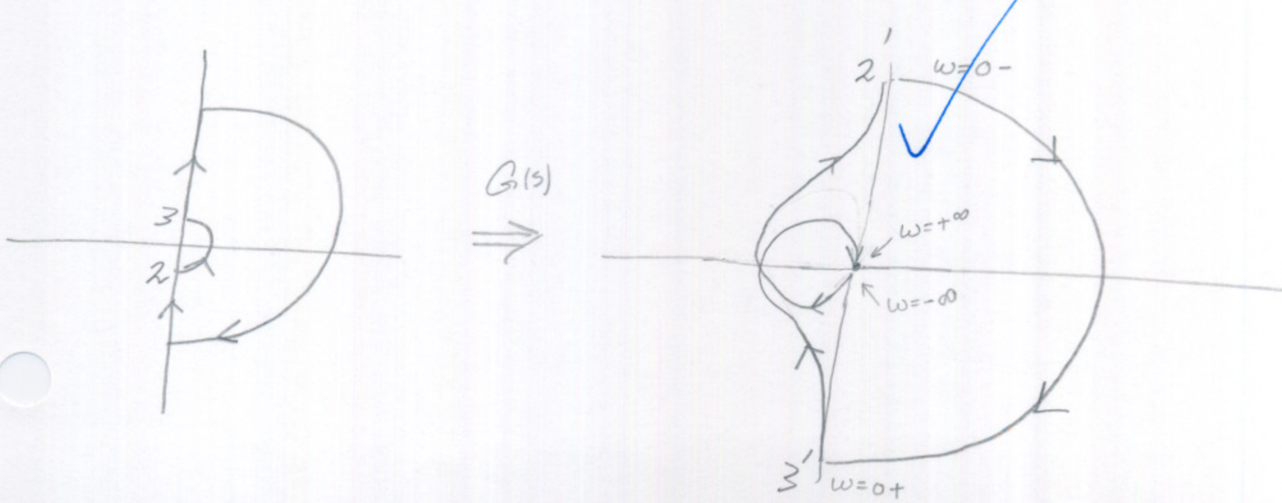
so $G(s) = \dfrac{1}{R^3 e^{j3\theta} + 0.8R^2 e^{j2\theta} + Re^{j\theta}}$

so $|G(s)| = \dfrac{1}{R^3 + R^2 + R} \to 0$ as $R \to \infty$.

$\angle G(s) = 0 - [3\theta + 2\theta + \theta] = -6\theta$.

so the whole Large Circle maps to orision.

hence Nyqrist plot is



HW # 8
Problem B-8-26

open loop $G(s) = \frac{as+1}{s^2}$. Find value of a so that phase margin = 45°

Solution

$$G(j\omega) = \frac{aj\omega+1}{(j\omega)^2}$$

$$\angle G(j\omega) = \angle aj\omega+1 - 2\angle j\omega$$

$$\angle G(j\omega) = \tan^{-1} a\omega - 2(90^\circ) = \tan^{-1} a\omega - 180^\circ$$

we want $45^\circ = 180 + \angle G(j\omega_c)$ where ω_c is cross over freq.

so

$$\tan^{-1} a\omega_c - 180^\circ = -135^\circ$$

$$\tan^{-1} a\omega_c = 45^\circ$$

$$\boxed{a\omega_c = 1} \quad \text{--- } 1$$

Now need another equation to solve for a.

Since at $\omega = \omega_c$, $|G(j\omega)| = 1$

Then $1 = \frac{\sqrt{(a\omega_c)^2+1}}{\omega_c} \Rightarrow \omega_c = \sqrt{(a\omega_c)^2+1}$

so $\omega_c^2 - a^2\omega_c^2 - 1 = 0$

$$\omega_c^2(1-a^2) = 1$$

$$\omega_c^2 = \frac{1}{1-a^2} \Rightarrow \boxed{\omega_c = \frac{1}{\sqrt{1-a^2}}} \quad \text{--- } 2$$

plug 2 into 1 we get

$$a\frac{1}{\sqrt{1-a^2}} = 1 \Rightarrow a^2 = 1-a^2 \Rightarrow 2a^2 = 1 \Rightarrow \boxed{a = \sqrt{\tfrac{1}{2}}}$$

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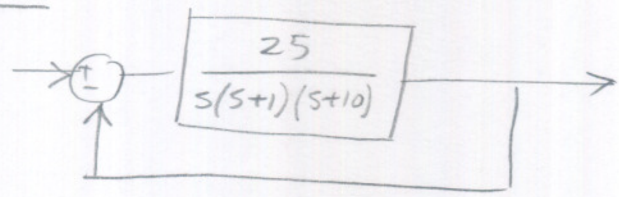
X $\boxed{a = 0.707}$

This is the value of a such as phase margin = 45°

HW#8

Problem B-8-27

Consider



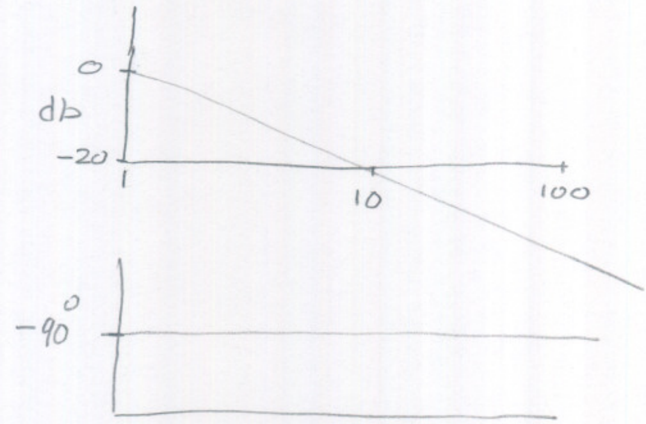
Draw Bode plot of openloop. Determine phase and gain margin.

Solution

For $\frac{1}{s}$ bode plot is

$\phi = -90^\circ$

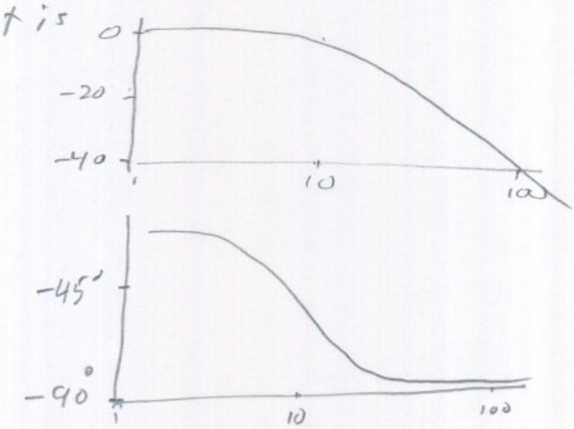
$|G(j\omega)| = \frac{1}{\omega}$



for $\frac{1}{s+1}$, bode plot is

$\phi = -\tan^{-1}\omega$

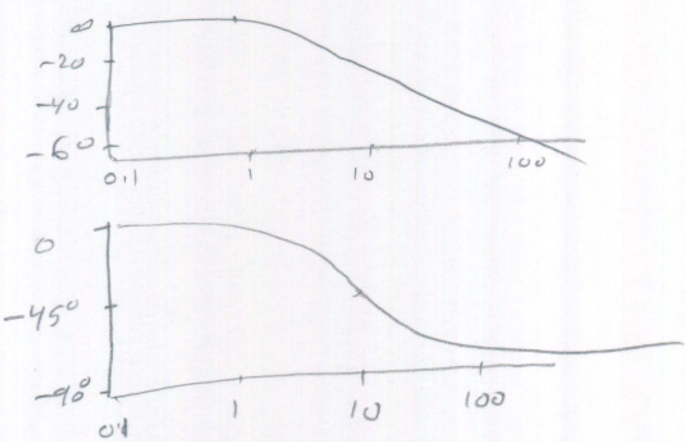
$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}$



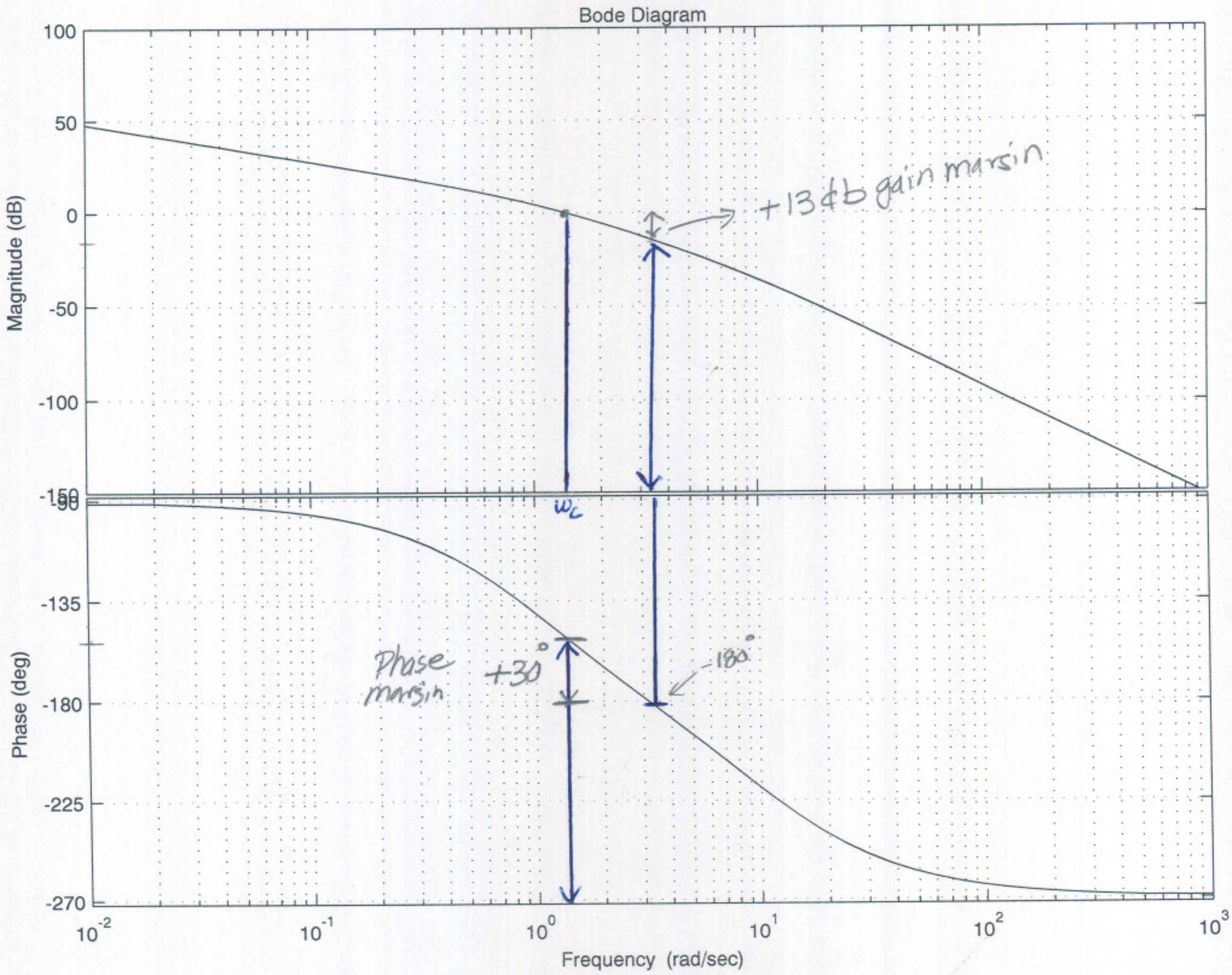
for $\frac{1}{s+10}$

$\phi = -\tan^{-1}\frac{\omega}{10}$

$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+100}}$



adding all these together gives bode plot. to get accurate phase and gain Margin I did the above on top of Matlab output ⇒



From diagram, when $\omega = \omega_c \cong 1.03$ rad/sec, is Cross over freq.

at this frequency, $\angle G(j\omega_c) \cong -150^\circ$

so $\phi_m = 180 - 150 = \boxed{30^\circ}$ a positive phase margin.

when $\angle G(j\omega) = -180^\circ$, $|G(j\omega)| \cong \boxed{+13\ db}$. (since below the 0db line)

HW #8

problem B-8-29

open Loop $G(s)=\frac{K}{s(s^2+s+4)}$

determine K such as phase margin = 50°. what is gain margin with this gain K?

Solution

$$\angle G(j\omega) = \angle \frac{K}{j\omega(-\omega^2+j\omega+4)} = -\left[90^\circ + \tan^{-1}\frac{\omega}{4-\omega^2}\right]$$

$\phi_m = 180 + \angle G(j\omega_c)$ where ω_c is cross over frequency.

so $50 = 180 + \angle G(j\omega_c)$

$\angle G(j\omega_c) = -130^\circ$

so $-\left[90^\circ + \tan^{-1}\frac{\omega_c}{4-\omega_c^2}\right] = -130^\circ$

$$-\tan^{-1}\frac{\omega_c}{4-\omega_c^2} = -40^\circ$$

$$\tan^{-1}\frac{\omega_c}{4-\omega_c^2} = 40^\circ \Rightarrow \frac{\omega_c}{4-\omega_c^2} = 0.839$$

$$\omega_c = 3.3564 - 0.839\,\omega_c^2 \Rightarrow 0.839\,\omega_c^2 + \omega_c - 3.3564 = 0$$

$$\omega_c = \frac{-1 \pm \sqrt{1-4(-3.356)(0.839)}}{2\times 0.839} = \frac{-1\pm 3.5}{1.678} = \boxed{1.4898} \text{ rad/sec (taking +ve } \omega)$$

at this frequency $|G(j\omega_c)| = 1$.

so $\left|\frac{K}{j\omega(-\omega^2+j\omega+4)}\right| = 1 \Rightarrow \frac{K}{\omega_c\sqrt{\omega^2+(4-\omega^2)^2}} = 1$

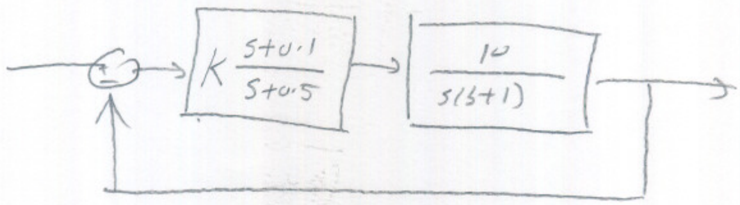
$$K = 1.4898\sqrt{1.4898^2 + (4-1.4898^2)^2} = \boxed{3.458}$$

so gain margin = 20 log 3.458 = +10.776 db

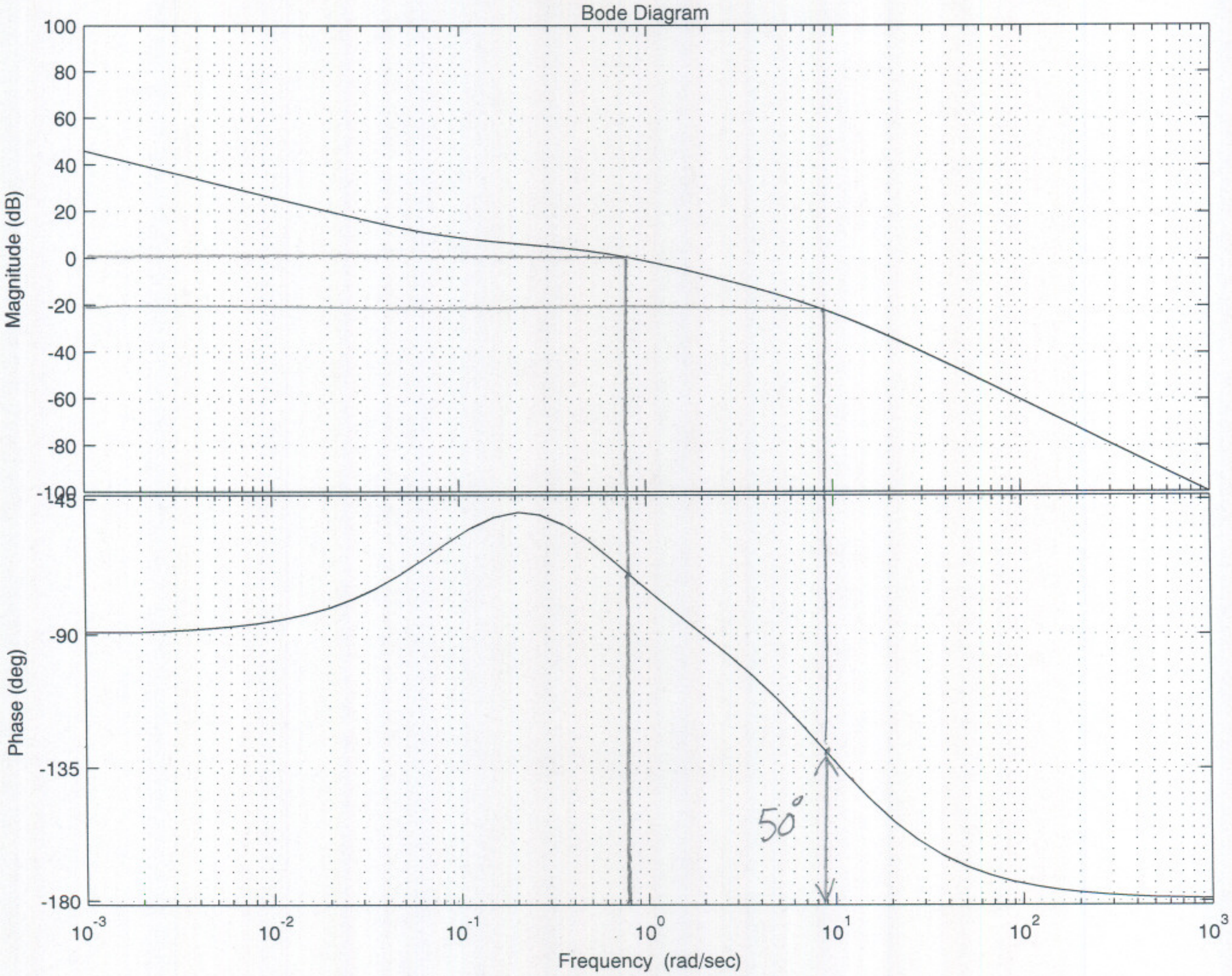
HW#8

B-8-30

Consider



Draw Bode plot for open loop, determine K such as phase margin $= 50^\circ$. what is the gain margin of this system with this gain K?



we see that at 50° phase margin $|G(j\omega)| = 0.1$ (i.e -20 db).

so $\frac{10K\,|s+0.1|}{|s+0.5|\,|s|\,|s+1|} = 0.1$ from plot we see $\omega \cong 9$ rad/sec.

so $\frac{10K\sqrt{\omega^2+0.1^2}}{\sqrt{\omega^2+0.5^2}\;\omega\;\sqrt{\omega^2+1}} = 0.1 \Rightarrow K = \frac{(0.1)\sqrt{81+0.5^2}\;(9)\sqrt{81+1}}{10\sqrt{81+0.1^2}} = \frac{73.46}{90} = \boxed{0.81}$

so gain margin with this gain is $20 \log 0.81 = \boxed{-1.76}$ db