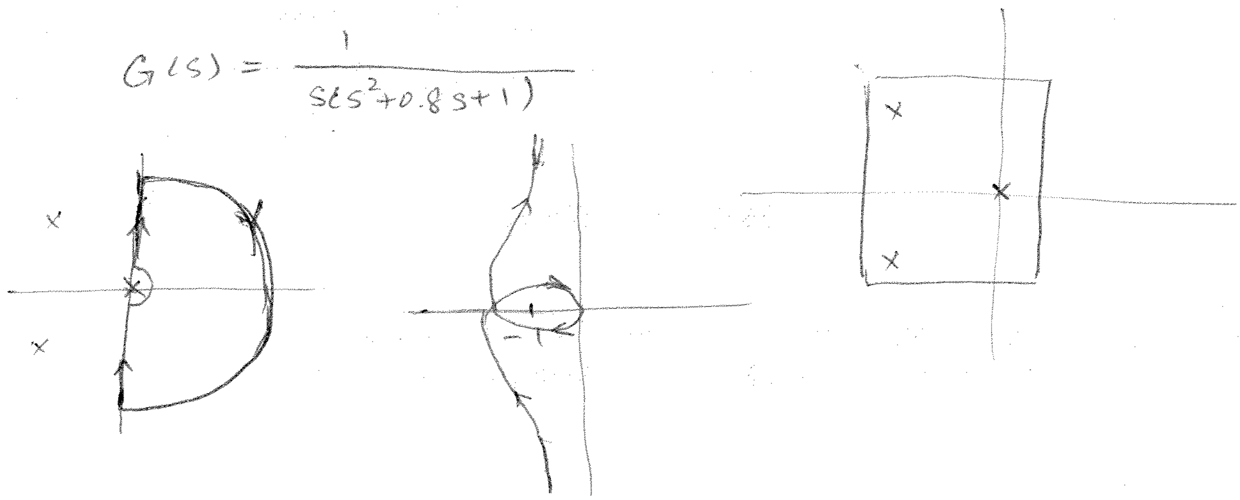
B - 8 - 14

HW 8 solutions.

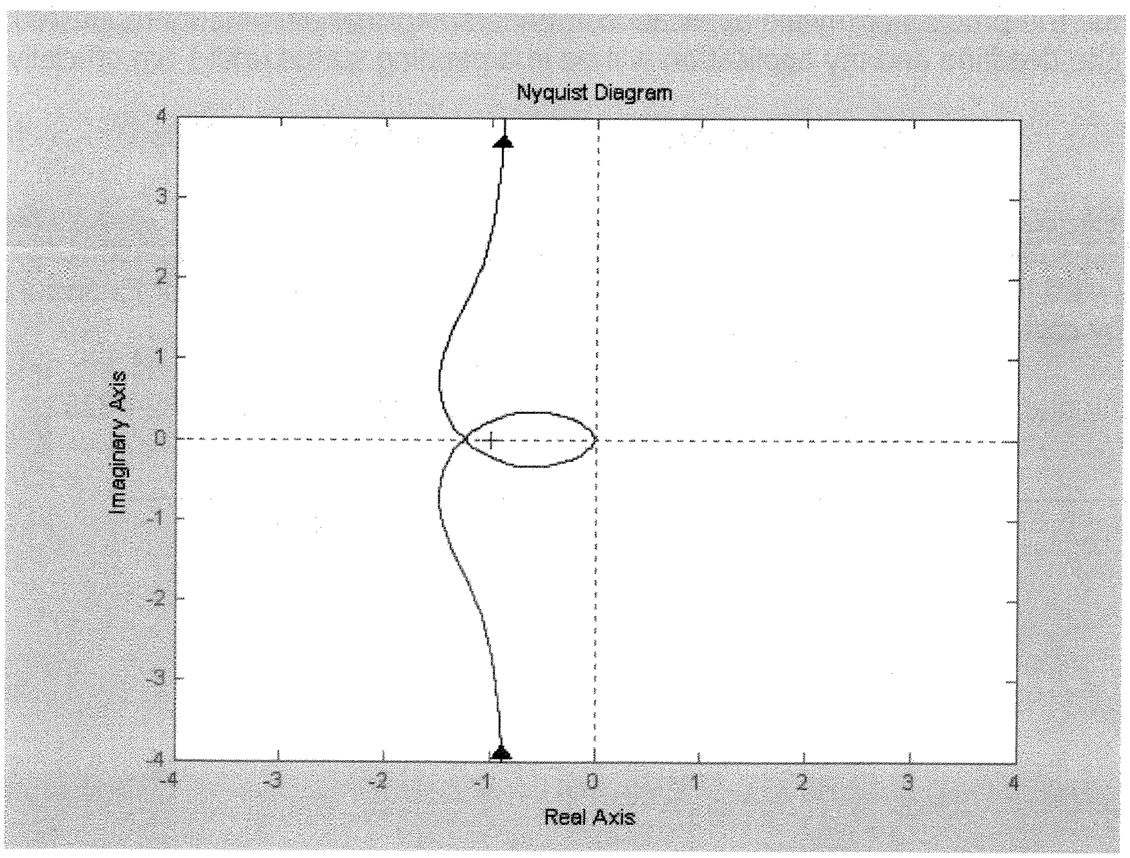
$$G(s) = \frac{1}{s(s^2+0.8s+1)}$$



$P = 0$ $\quad N = 2 \Rightarrow Z = 2$ unstable.

nyquist([0 0 0 1], [1 0.8 1 0])

axis([-4 4 -4 4])



8 - 26

$$G_{OL}(s) = \frac{as+1}{s^2}$$

$$|G_{OL}(j\omega)| = \sqrt{\frac{a^2\omega^2+1}{\omega^4}} = \frac{\sqrt{a^2\omega^2+1}}{\omega^2}$$

$$\angle G_{OL}(j\omega) = \tan^{-1} a\omega - \tan^{-1}\left(\frac{0}{-\omega^2}\right) = \tan^{-1} a\omega - 180°$$

$\phi_m = 45°$ given

$$\phi_m = 180° + \angle G_{OL}(j\omega_1)$$

$$\Rightarrow \tan^{-1} a\omega_1 = 45° \Rightarrow \underline{a\omega_1 = 1}$$

$$|G(j\omega_1)| = 1 = \sqrt{\frac{(a\omega_1)^2+1}{\omega_1^2}}$$

$$\frac{\sqrt{1^2+1}}{\omega_1^2} = \frac{\sqrt{2}}{\omega_1^2} = 1 \Rightarrow \omega_1 = 2^{\frac{1}{4}}$$

$$a = \frac{1}{\omega_1} = \frac{1}{2^{\frac{1}{4}}} = \boxed{0.841}$$

8-27

$$G_{OL}(s) = \frac{25}{s(s+1)(s+10)} = \frac{25}{s^3+11s^2+10s}$$

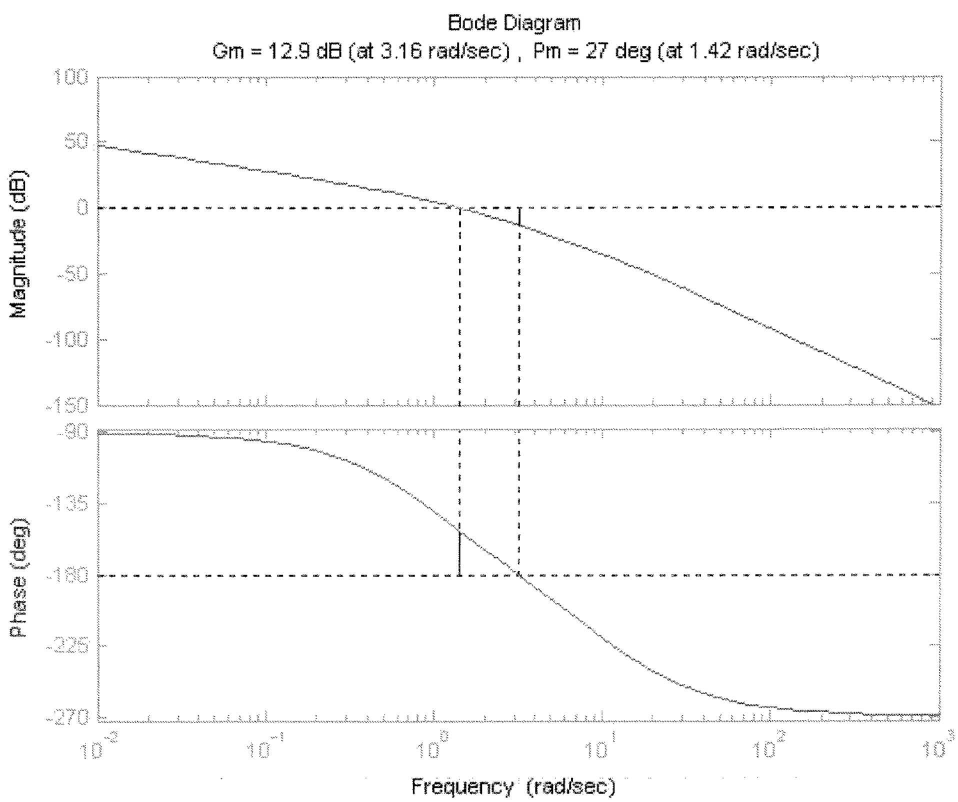
```
num = [0  0  0  25];
den = [1  11  10  0];

[gm, pm, wcp, wcg] = margin(num, den)
```

gm = 4.4 $\longrightarrow$ gm_dB = 20 log(gm) = 12.9 dB

pm = 26.9973°

wcp = 3.1623 rad/sec

wcg = 1.4230 rad/sec.



8 - 29

$$G_{OL}(s) = \frac{K}{s(s^2 + s + 4)}$$

$$\phi_m = 180° + \angle G(j\omega_1)$$

$$50° = 180° + 0° - 90° - \angle -\omega_1^2 + j\omega_1 + 4$$

$$-40° = -\tan^{-1}\frac{\omega_1}{4-\omega_1^2}$$

$$\omega_1 = (4-\omega_1^2)(0.8391)$$

$$+0.8391\,\omega_1^2 + \omega_1 - 4(0.8391) = 0$$

$$\Rightarrow \underline{\omega_1 = 1.491 \text{ rad/sec}}$$

$$|G(j\omega_1)| = 1 = \left|\frac{K}{-j\omega_1^3 - \omega_1^2 + j4\omega_1}\right| = \frac{K}{3.46}$$

$$\Rightarrow \boxed{K = 3.46}$$

phase crossover frequency

$$\angle G(j\omega_{cp}) = -\cancel{\angle j\omega_{cp}}^{\,90°} - \angle -\omega_{cp}^2 + j\omega_{cp} + 4 = -180°$$

$$\tan^{-1}\frac{\omega_{cp}}{4-\omega_{cp}^2} = 90°$$

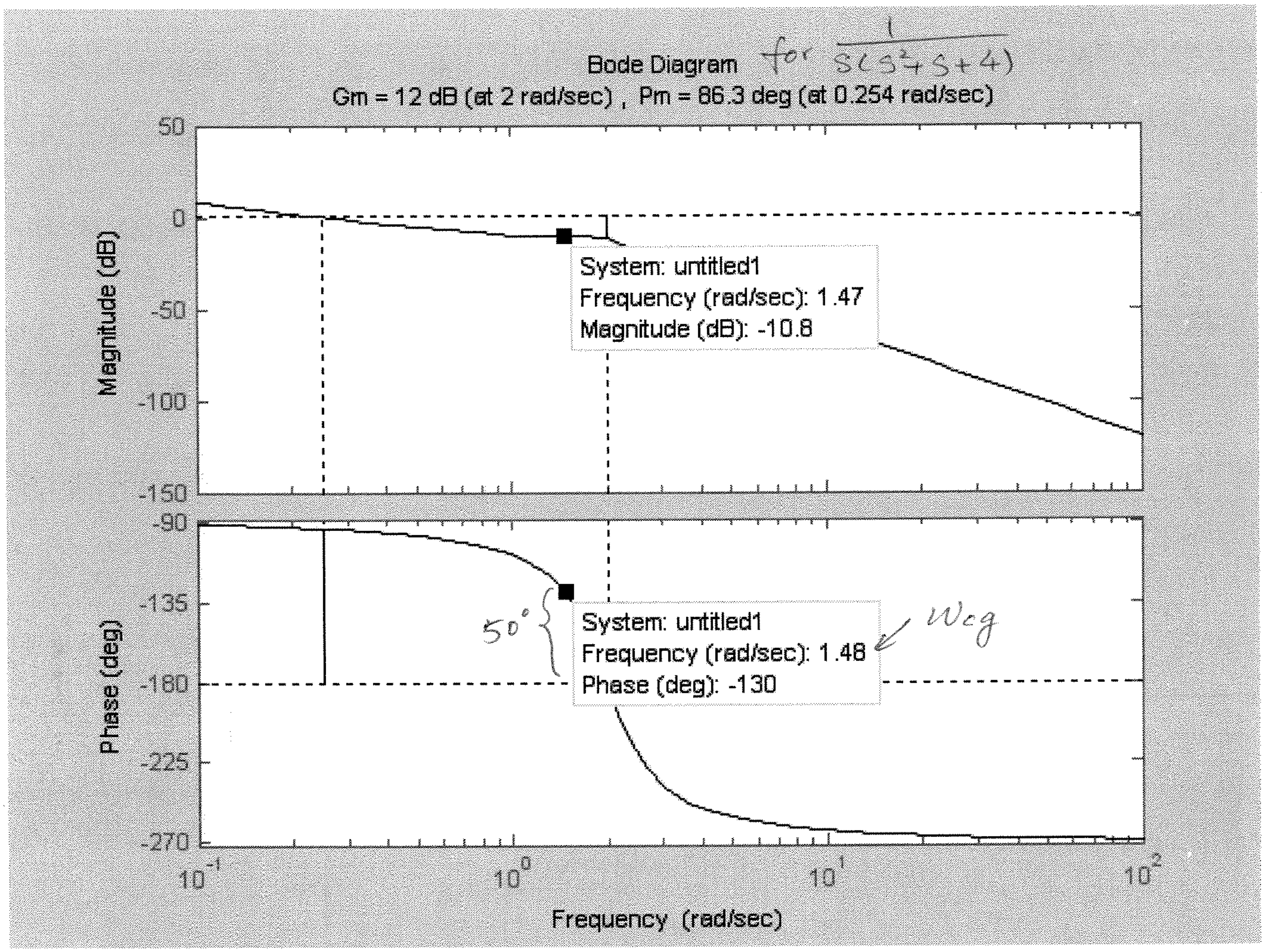
$$\Rightarrow \frac{\omega_{cp}}{4-\omega_{cp}^2} = \infty \quad \Rightarrow \quad \underline{\omega_{cp} = 2 \text{ rad/sec}}$$

$$gm = |G(j2)| = \left|\frac{3.46}{(j2)(-4 + j2 + 4)}\right| = 0.865$$

$$|gm\_dB| = |20\log(0.865)| = |-1.26\,dB| = \boxed{1.26\,dB}$$

gain margin

Method 2 (B-8-29)



W_{cg} for $50° = \phi_m$ is 1.48 rad/sec

$$|G(jW_{cg})| = |G(j1.48)| = 10.8\ dB$$

$$K = 10^{\frac{10.8}{20}} = \boxed{3.467}$$ same as method 1

8-30

$$G_{OL}(s) = \frac{K(s+0.1)10}{s(s+0.5)(s+1)} = \frac{10K(s+0.1)}{s^3+1.5s^2+0.5s}$$

We will solve this problem graphically. First we let $10K=1$, and plot ~~the~~ bode diagram for $sys = \frac{s+0.1}{s^3+1.5s^2+0.5s}$. Since we require the phase margine to be 50°, ω is found to be at 1.44 rad/sec according to our bode plot. Since the diagram indicates that $|G(j1.44)| = 8.56\,dB$, we need to choose $10K = 8.56\,dB$

$$\therefore\ 8.56\,dB = 20\log(10K) \Rightarrow \boxed{K = 0.268}$$

Since the phase curve lies above the -180° line for all ω, the gain margin is ∞ dB.