HW # 7

MAE 170

Winter 2005

49.5/50

by Nasser Abbasi

HW#7

Problem B-8-2

Consider system whose closed loop TF is $\frac{C(s)}{R(s)} = \frac{K(T_2 s+1)}{T_1 s+1}$

Obtain steady state output of the system when it is subjected to input $r(t) = R\sin\omega t$.

solution

let $G(s) = \frac{C(s)}{R(s)}$

since input is sinosidal, and system is LTI, then output is $R\left|G(s)\right|_{s=j\omega} \sin(\omega t + \phi)$

where $\phi = \angle G(s)\big|_{s=j\omega}$

$$G(s)\big|_{s=j\omega} = \frac{K(T_2 j\omega+1)}{T_1 j\omega+1} = \frac{j(KT_2\omega)+K}{j(T_1\omega)+1}$$

$$\text{so } \left|G(s)\right|_{s=j\omega} = \frac{\sqrt{(KT_2\omega)^2+K^2}}{\sqrt{T_1^2\omega^2+1}} = \frac{K\sqrt{T_2^2\omega^2+1}}{\sqrt{T_1^2\omega^2+1}}$$

$$\angle G(j\omega) = \tan^{-1}\frac{KT_2\omega}{K} - \tan^{-1}T_1\omega$$

$$= \tan^{-1}T_2\omega - \tan^{-1}T_1\omega.$$

$$\text{so } \boxed{c(\infty) = R\,\frac{K\sqrt{T_2^2\omega^2+1}}{\sqrt{T_1^2\omega^2+1}}\sin\left(\omega t + \left(\tan^{-1}T_2\omega - \tan^{-1}T_1\omega\right)\right)}$$

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The above is the answer.

side question why can't I seem to be able to solve above using Final value theorm? what Am I doing wrong below?

$C(s) = X(s)\,G(s)$. but $X(s) = R\,\frac{\omega}{s^2+\omega^2}$.

hence $C(s) = \frac{R\omega}{(s^2+\omega^2)}\cdot\frac{K(T_2 s+1)}{(T_1 s+1)} \Rightarrow C(\infty) = \lim_{s\to 0} s\,C(s)$

i.e. $C(\infty) = \lim_{s\to 0} s.\frac{R\omega\,K(T_2 s+1)}{(s^2+\omega^2)(T_1 s+1)} = \lim_{s\to 0}\frac{s^2 R\omega K T_2 + sR\omega K}{T_1 s^3 + s^2 + T_1\omega^2 s + \omega^2}$

$= \lim_{s\to 0}\frac{R\omega K T_2 + \frac{R\omega K}{s}}{T_1 s + 1 + T_1\omega^2/s + \omega^2/s} \longrightarrow \frac{R\omega K T_2}{\infty} = 0$?? how is this equivalent to (no oscillation)?

see pg. 25
FVT only applies when f(t) settles down to a definite value for $t\to\infty$

```
close all;
clear all;
%problem B-8-3, by Nasser Abbasi

s=tf('s');
sys=(1+s)/(1+2*s);
bode(sys);
title('Bode plot for (1+s)/(1+2 s)');

sys=(1-s)/(1+2*s);
figure;
bode(sys);
title('Bode plot for (1-s)/(1+2 s)');
```

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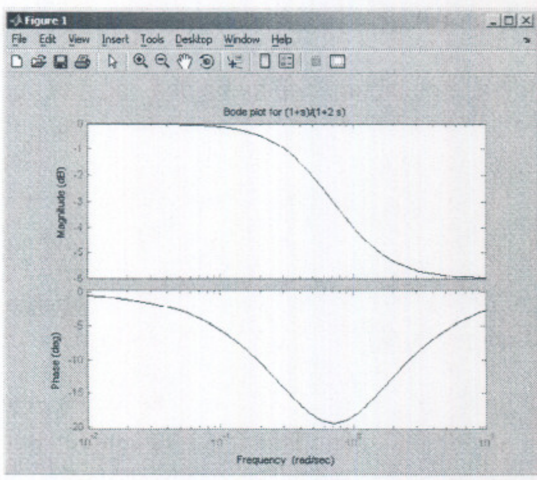
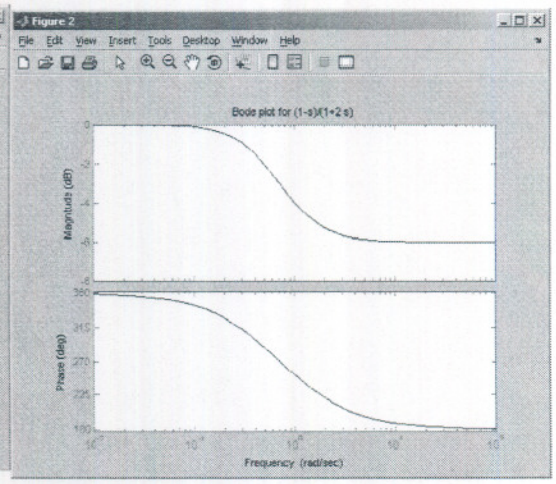




$$\frac{1+s}{1+2s} \qquad\qquad \frac{1-s}{1+2s}$$

HW# 7

Problem B-8-4

Sketch Bode diagram of

a) $G(s) = \dfrac{T_1 s+1}{T_2 s+1}$ $\quad T_1 > T_2 > 0$

b) $G(s) = \dfrac{T_1 s-1}{T_2 s+1}$ $\quad T_1 > T_2 > 0$

c) $G(s) = \dfrac{-T_1 s+1}{T_2 s+1}$ $\quad T_1 > T_2 > 0$

note: after doing this I see that $|G(s)|$ for parts a, b, c is the same only phase diagrams changed. I should have taken advantage of this to reduce work needed.

yup

9.5

Answer

a) let $G(s) = G_1(s)\, G_2(s)$ where $G_1(s) = T_1 s+1$, $G_2(s) = \dfrac{1}{T_2 s+1}$

now plot bode plot for G_1 and G_2 separalty and combine plots.

G_1 $\quad T_1 s+1$, then $G_1(j\omega) = T_1 j\omega + 1$, $|G_1(j\omega)| = \sqrt{(T_1\omega)^2+1}$

so $\log|G_1(j\omega)| = \frac{1}{2}\log\left(1+(T_1\omega)^2\right)$

For small ω, $\log|G_1| \rightarrow \frac{1}{2}\log(1) = 0$

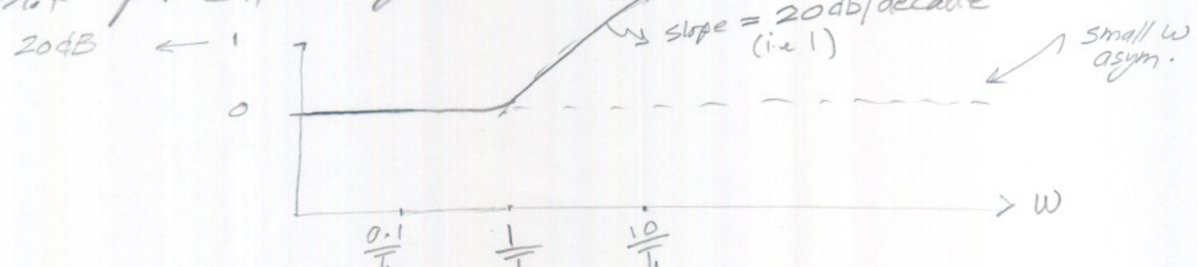
for Larg ω, $\log|G_1| \rightarrow \frac{1}{2}\log(T\omega)^2 = \log T\omega$

$\angle G_1 = \tan^{-1} T_1\omega$

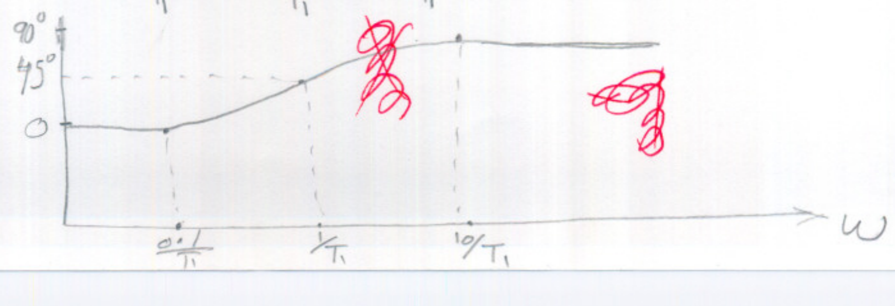
For small ω, $\angle G_1 = 0°$

for Large ω, $\angle G_1 = 90°$

so Bode plot for G_1 only is



next I do bode for G_2



$\underline{G_2}$ $\quad \dfrac{1}{T_2 s+1}$ so $G_2(j\omega) = \dfrac{1}{T_2 j\omega + 1}$

$$|G_2| = \frac{1}{\sqrt{1+(T_2\omega)^2}} \rightarrow \log|G_2| = \log(1) - \log\sqrt{1+(T_2\omega)^2}$$

$$= 0 - \tfrac{1}{2}\log\left(1+(T_2\omega)^2\right)$$

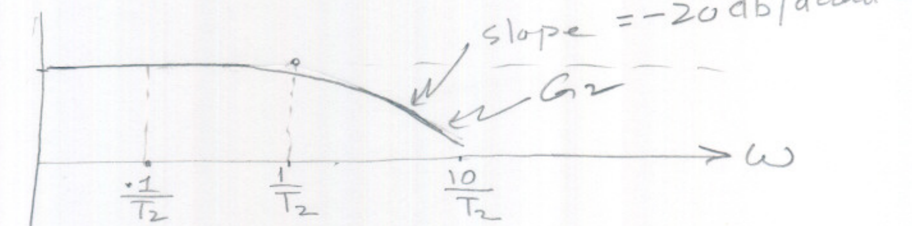
for small ω, $\log|G_2| \rightarrow -\frac{1}{2}\log(1) = 0$

for Large ω, $\log|G_2| \rightarrow -\log T_2\omega$

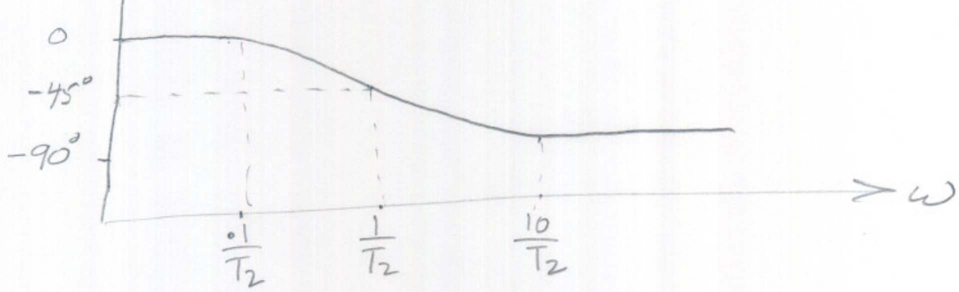
$\angle G_2(j\omega) = 0 - \tan^{-1} T_2\omega$

for small ω, $\angle G_2 = 0 - \tan^{-1} 0 = 0$

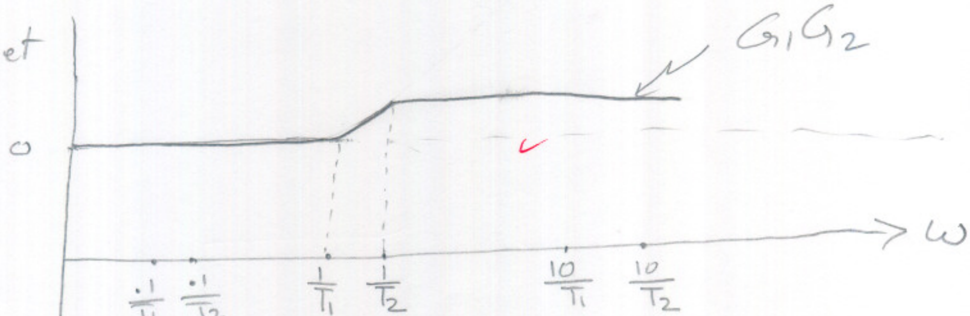
for Large ω, $\angle G_2 = 0 - \tan^{-1}\infty = -90^\circ$

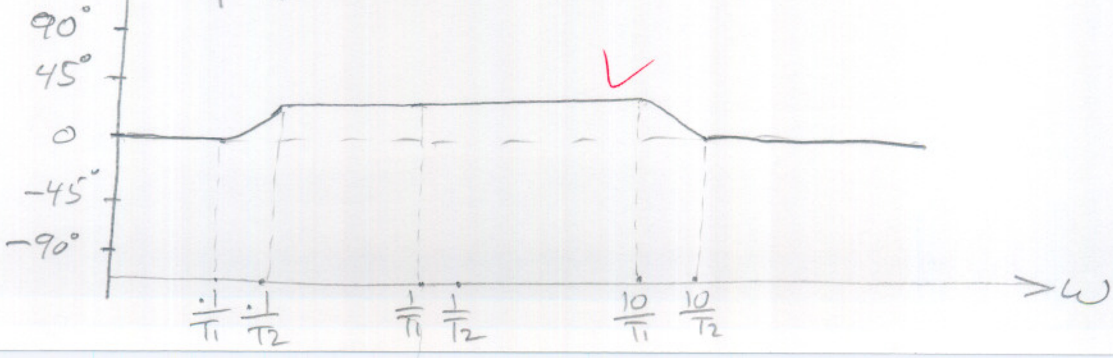


G_2



so now add G_1, G_2 we get





Part b

$$G(s) = \frac{T_1 s - 1}{T_2 s + 1} \qquad T_1 > T_2 > 0$$

let $G(s) = G_1 G_2$ where $G_1 = T_1 s - 1$, $G_2 = \frac{1}{T_2 s + 1}$

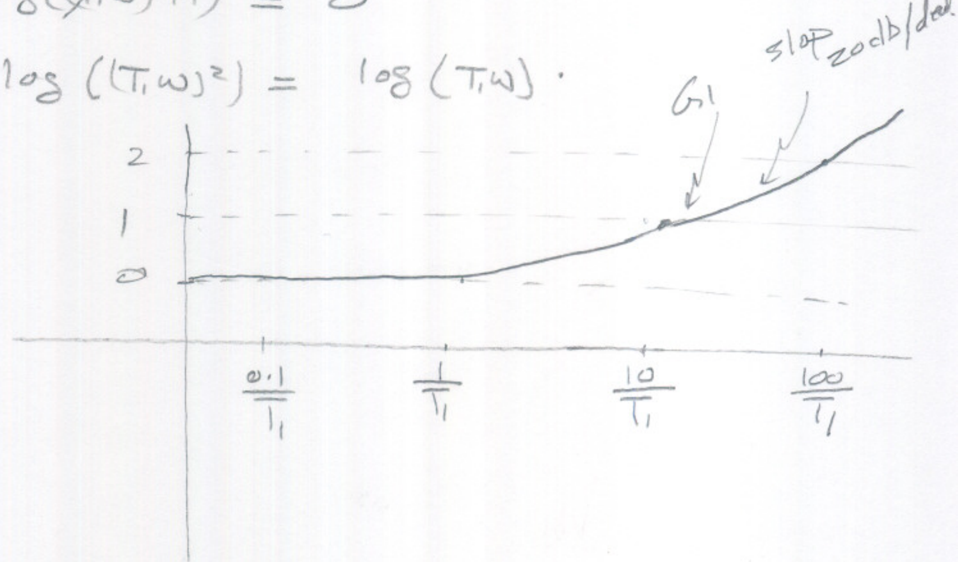
G_1

$$G_1(j\omega) = T_1 j\omega - 1$$

$$|G_1(j\omega)| = \sqrt{(T_1\omega)^2 + 1}, \quad \log|G(j\omega)| = \frac{1}{2}\log\left((T_1\omega)^2 + 1\right).$$

small ω, $\log|G(j\omega)| \to \frac{1}{2}\log((\cancel{T_1\omega})^2 + 1) = 0$

large ω, $\log|G_1(j\omega)| \to \frac{1}{2}\log((T_1\omega)^2) = \log(T_1\omega)$.

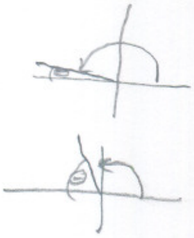


To Find Phase:

$$\angle G_1 = \tan^{-1}\frac{T_1\omega}{-1}$$

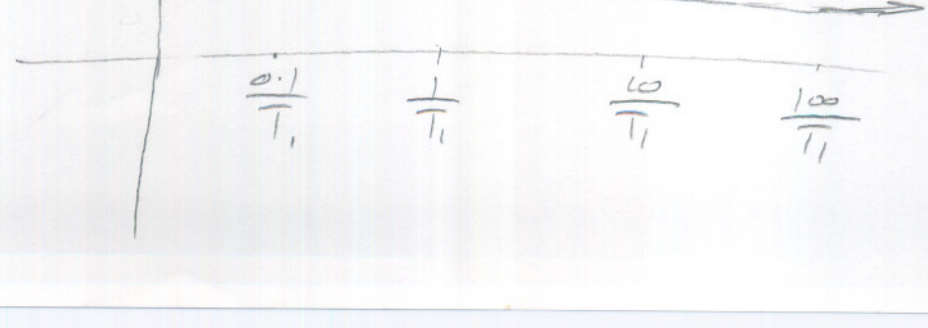
so for small ω, we see that $\angle G_1$ goes to $180°$

and for Large ω, we see that $\angle G_1$ goes to $90°$



when $\omega = \frac{1}{T_1}$, $\angle G_1 = \tan^{-1} -1$ i.e $135°$

When $\omega = \frac{10}{T_1}$, $\angle G_1 = \tan^{-1} -10$, etc.



next I do G_2 $\longrightarrow$

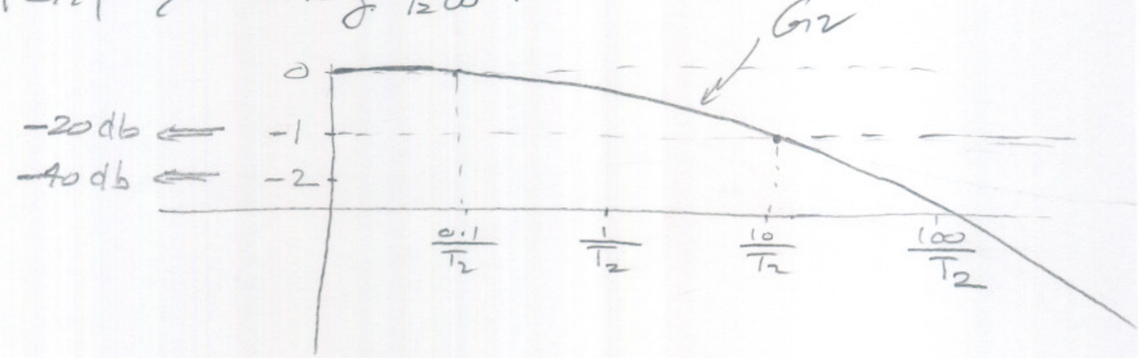
$$G_2 = \frac{1}{T_2 s + 1}$$

$$G_2(j\omega) = \frac{1}{T_2 j\omega + 1} \qquad |G_2(j\omega)| = \frac{1}{\sqrt{(T_2\omega)^2 + 1}}$$

$$\log|G_2| = 0 - \log\sqrt{(T_2\omega)^2+1} = -\frac{1}{2}\log\left((T_2\omega)^2+1\right)$$

For small ω, $\log|G_2| \to 0$

For Large ω, $\log|G_2| \to -\log T_2\omega$.

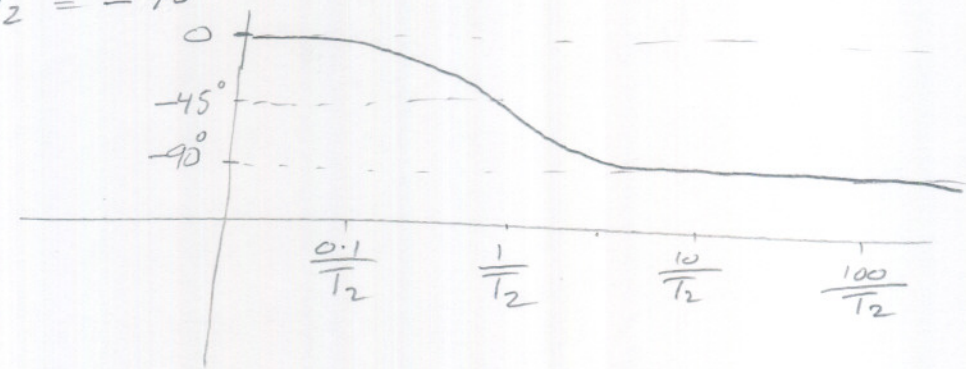


For phase

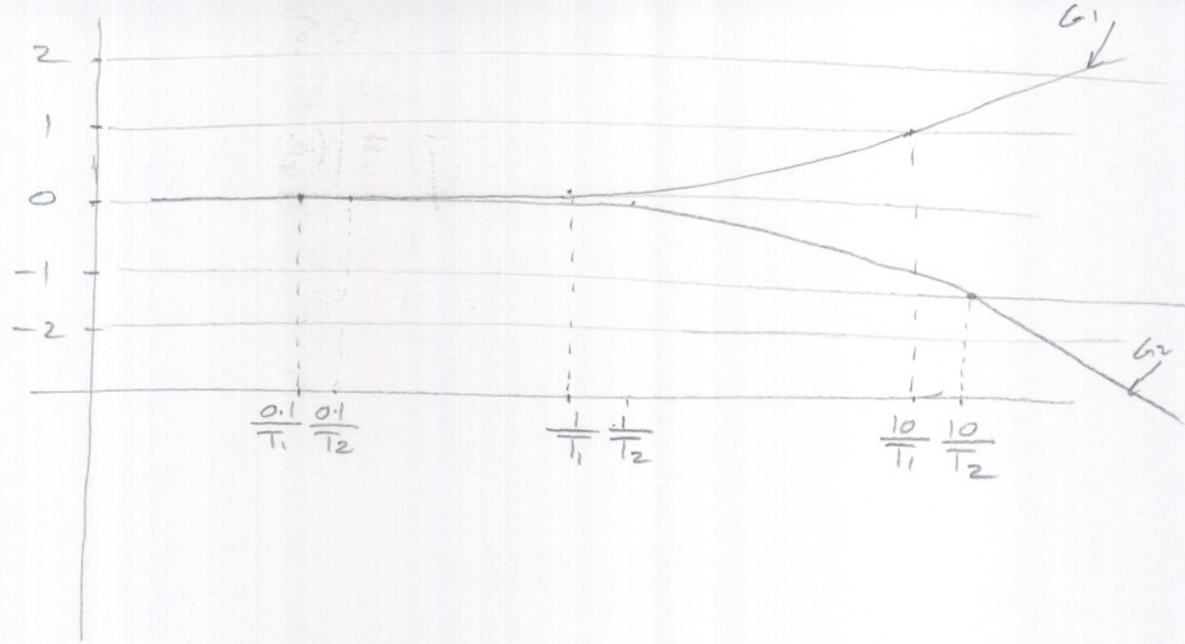
$$\angle G_2 = 0 - \tan^{-1}(T_2\omega) = -\tan^{-1} T_2\omega.$$

For small ω, $\angle G_2 = 0$

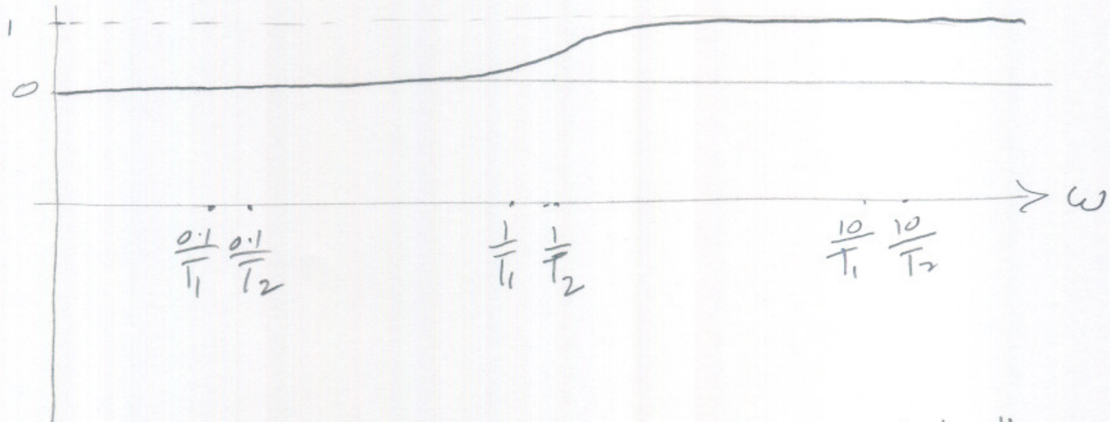
for Large ω, $\angle G_2 = -90^\circ$



so now I put G_1, G_2 together $\Rightarrow$



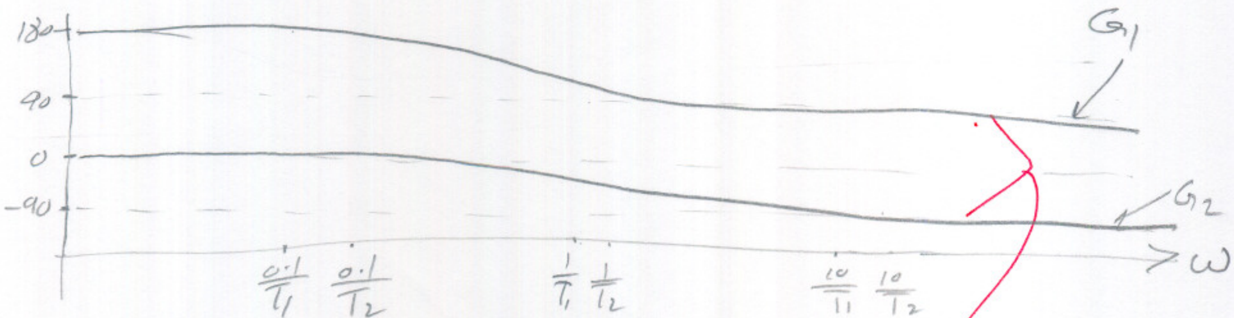
Combine ⇒



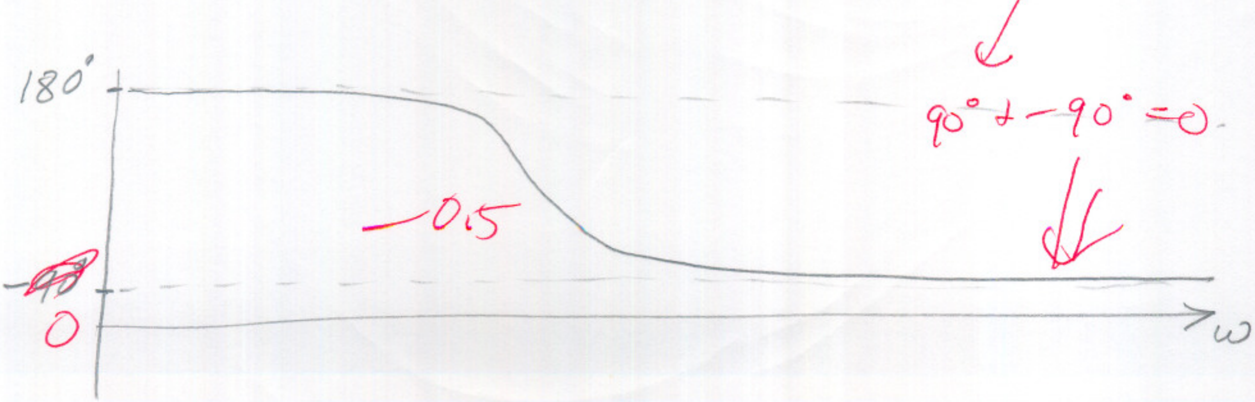
at large ω, G_1 slope cancell G_2, we get straight line.

at low ω, G_1 rises befor G_2 since $T_1 > T_2$.

for phase:



Combine



part c

$$G(s) = \frac{-T_1 s+1}{T_2 s+1} \qquad T_1 > T_2 > 0$$

let $G_1(s) = -T_1 s + 1$, $\quad G_2(s) = \dfrac{1}{T_2 s+1}$

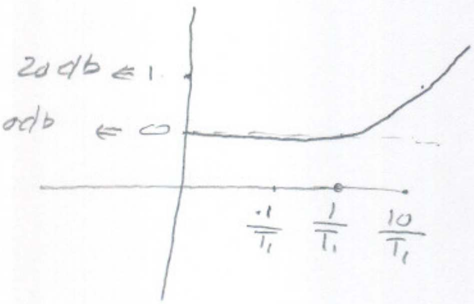
now do Bode plot for each G separatly and combine results.

G_1

$G_1(j\omega) = -T_1 j\omega + 1$

$|G_1| = \sqrt{(T_1\omega)^2+1}$

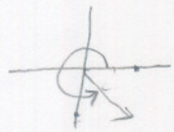
$\log|G_1| = \frac{1}{2}\log[(T_1\omega)^2+1]$



for Large ω, $\quad \log|G_1| \longrightarrow \log(T_1\omega)$

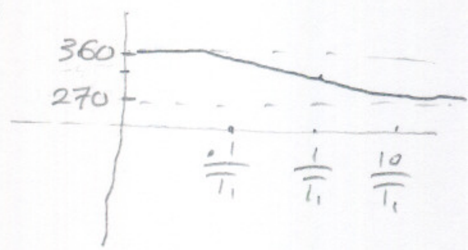
for small ω, $\quad \log|G_1| \longrightarrow 0$

$\angle G_1 = \tan^{-1} -T_1\omega$



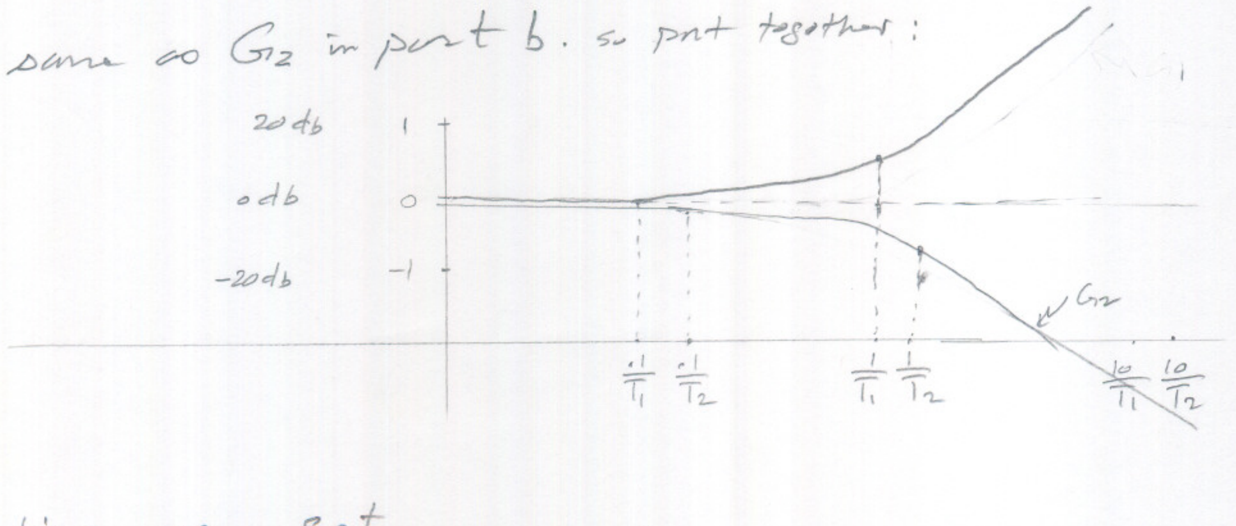
so for small ω, $\angle G_1 = 360°$ (could also say 0)

for Large ω, $\angle G_1 = 270°$ (could also say $-90°$)



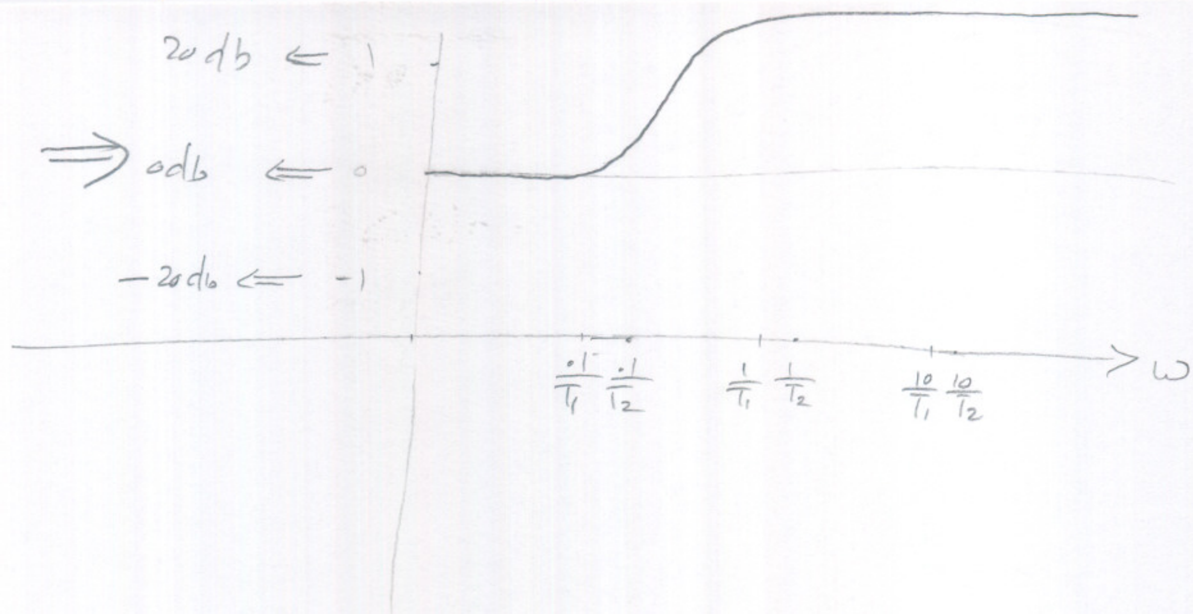
Now do G_2

This is the same as G_2 in part b. so put together:



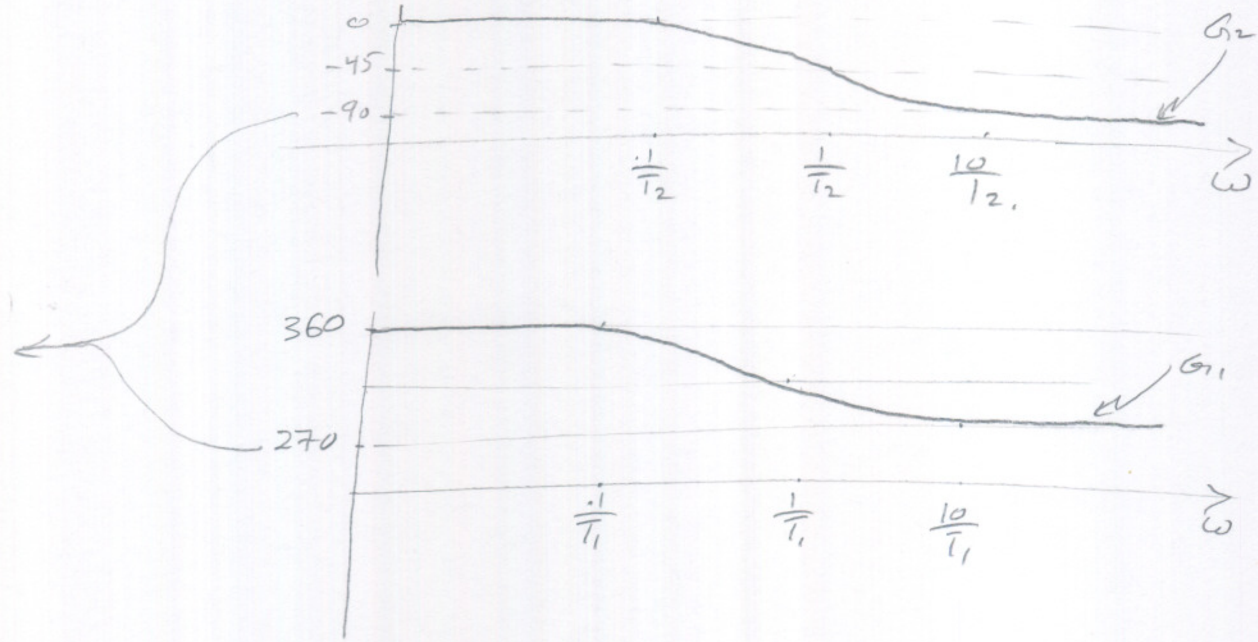
Combine, we get $\longrightarrow$

$$G(s) = \frac{-T_1 s + 1}{T_2 s + 1} \Rightarrow$$



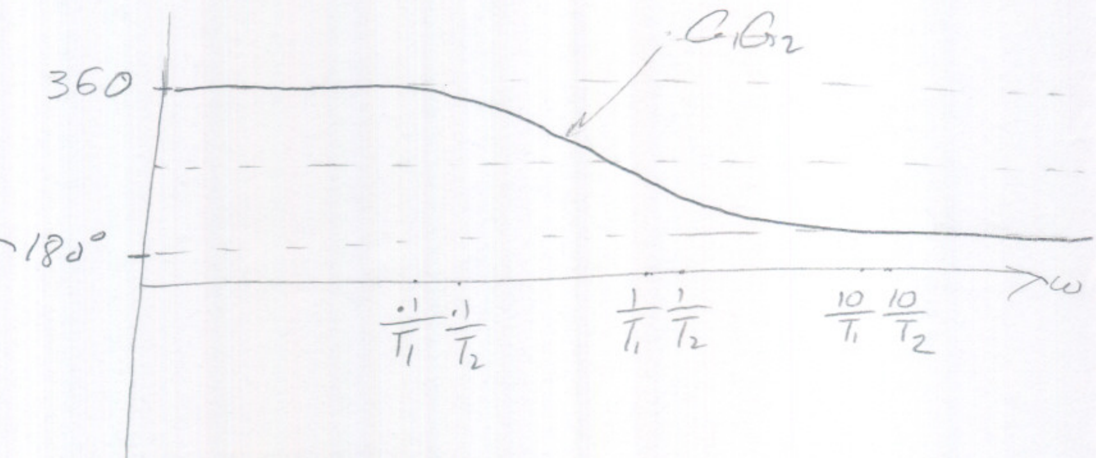
Now do phase

∠ for G_2 is same as part (b) G_2 which is



combine :

This (270-90) ←



HW#7

Problem B-8-6

Show that $|G(j\omega_n)| = \frac{1}{2\zeta}$

when $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$.

Solution

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2\zeta\omega_n j\omega + \omega_n^2}.$$

$$|G(j\omega)| = \frac{\omega_n^2}{\sqrt{(-\omega^2+\omega_n^2)^2 + (2\zeta\omega_n\omega)^2}}$$

let $\omega = \omega_n$ as required

$$|G(j\omega_n)| = \frac{\omega_n^2}{2\zeta\omega_n^2} = \boxed{\frac{1}{2\zeta}}$$

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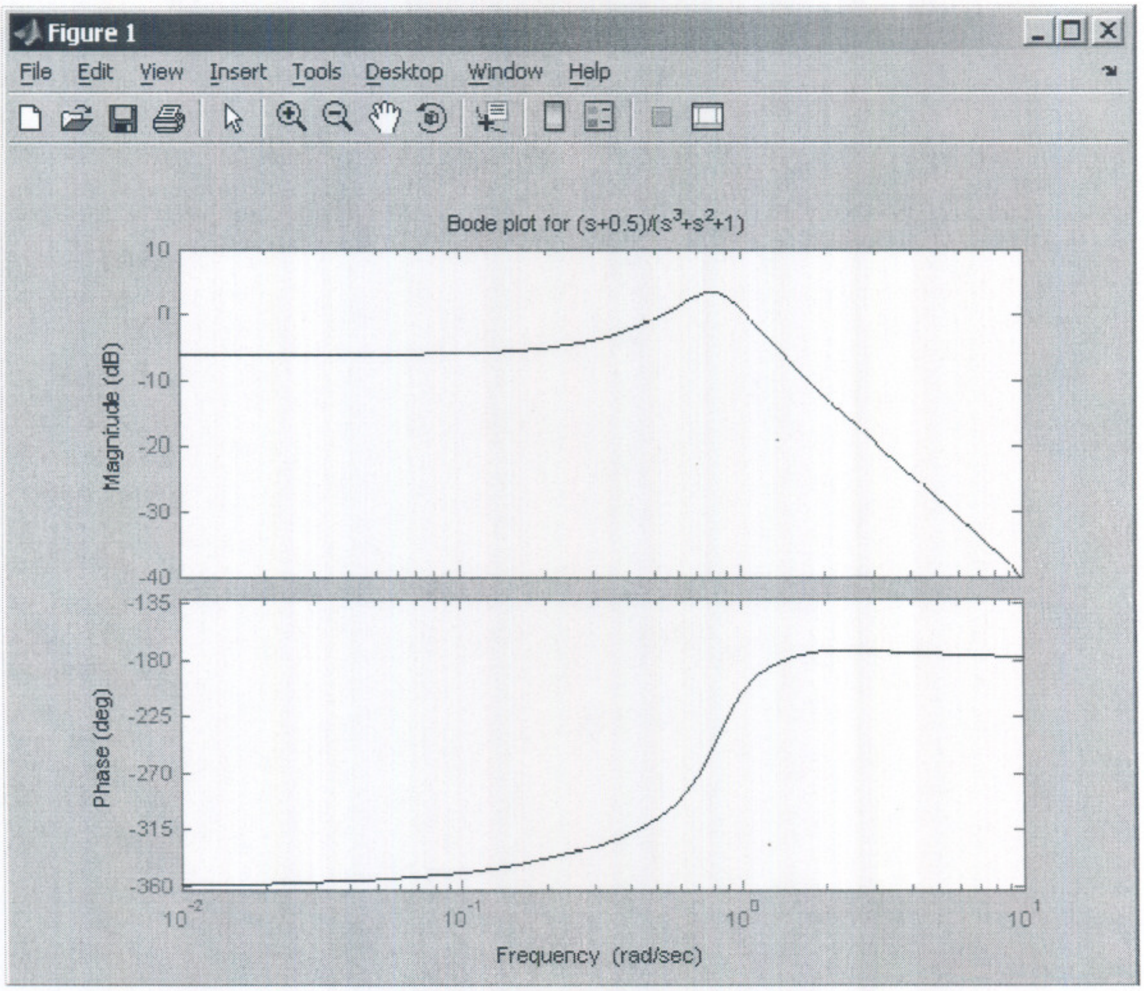
```
close all;
clear all;
%problem B-8-7, by Nasser Abbasi

s=tf('s');
sys=(s+0.5)/(s^3+s^2+1);
bode(sys);
title('Bode plot for (s+0.5)/(s^3+s^2+1)');
```



(1)

now need to explain phase diagram.

Phase as shown by Matlab starts from -360° and ends at -180°. This is the same as starting from 0° and ending at 180° as per problem statment.

$$G(j\omega) = \frac{j\omega+1}{-j\omega^3-\omega^2+1}$$

so

$$\angle G(j\omega) = \angle j\omega+1 \;-\; \angle\; -j\omega^3-\omega^2+1$$

$$= \tan^{-1}\omega \;-\; \tan^{-1}\left(\frac{-\omega^3}{-\omega^2+1}\right).$$

For small ω, $\dfrac{-\omega^3}{-\omega^2+1} \longrightarrow \dfrac{-\omega}{1} \longrightarrow 0$

So we get in the limit

$$\angle G(j\omega)_{\omega\to 0} = \tan^{-1}\omega - \tan -\omega = 0 - 360^\circ.$$
$$= -360^\circ$$

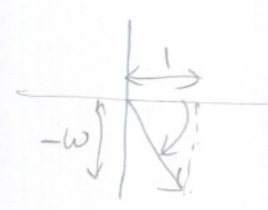
which the result given by Matlab. but $-360^\circ = 0^\circ$ also.

now I consider what happens when $\omega \to \infty$.

$$\lim_{\omega\to\infty} \angle G(j\omega) = \tan^{-1}\omega_{\omega\to\infty} - \tan^{-1}\left(\frac{-\omega^3}{-\omega^2+1}\right)_{\omega\to\infty}$$

$$= 90^\circ - \tan^{-1}(-\omega)_{\omega\to\infty}$$

$$= 90^\circ - (-90^\circ)$$



$$= 180^\circ$$

This explains the asymptotic behavior given.