HW#4

$\frac{60}{60}$

MAE 170, Introduction to control systems

UCI. Winter 2005

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HW#4
B-5-1

Thermo requires 60 seconds to indicate 98% of the response of the system to a step response. Assuming thermo to be a first order system, find τ (time constant).
If thermo placed in path, temp. is changing linearly at rate 10°/min. how much error does thermo show?

Answer

since First Order System, then

output $\leftarrow$ $\boxed{0.98 = 1 - e^{-\frac{t}{\tau}}}$ $\rightarrow$ time Constant.

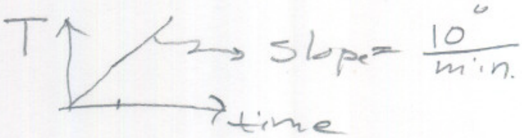
let $t = 60$ seconds. Solve for τ.

so $0.98 = 1 - e^{\frac{60}{\tau}} \Rightarrow 0.02 = e^{-\frac{60}{\tau}}$

$\Rightarrow \ln 0.02 = -\frac{60}{\tau}$

$\tau = \frac{-60}{\ln 0.02} = \boxed{15.337 \text{ seconds}}$

now the input is ramp



since 1^{st} order system, then the transfer function is $\boxed{\frac{a}{s+a}}$

so $\frac{Y(s)}{X(s)} = \frac{a}{s+a}$. when input is ramp with slope $\frac{10^\circ}{60} \Rightarrow$

secs in minute.

Laplace of input

$Y(s) = \left(\frac{10}{60}\frac{1}{s^2}\right)\frac{a}{s+a} \equiv \frac{A}{s^2} + \frac{B}{s}$

$\Rightarrow A = \frac{10}{60}, \quad B = \frac{10}{60}\frac{1}{a}$

so $Y(s) = \frac{10}{60}\frac{1}{s^2} + \frac{10}{60}\frac{1}{a}\frac{1}{s}$

so $\boxed{y(t) = \frac{10}{60}t + \frac{10}{60}\frac{1}{a}u(t)}$ $\longrightarrow$

since we found $\tau = 15.337$ seconds.

and since $\boxed{\tau = \frac{1}{a}} \Rightarrow \boxed{a = \frac{1}{15.337}}$ this is Hz. (natural freq. of system).

so temp. at time t as indicated by Thermo is given by

$$y_{measured}(t) = \frac{10}{60}t + \frac{10}{60}(15.337)$$

but <u>actual</u> temp is $y_a(t) = \frac{10}{60}t$.

so error is $\left[\frac{10}{60}t + \frac{10}{60}(15.337)\right] - \frac{10}{60}t$

$$= \boxed{2.56^{\circ}}$$

HW# 4
Problem B-5-2
Nasser Abbasi

given

when $X(s)$ is $\frac{1}{s}$ (i.e unit step), obtain rise time, peak time, maximum overshoot, settling time.

solution

Rise Time: time required for response to rise from 10% to 90% of its final value or 0% to 100% depending on system.

The closed loop transfer function is $\frac{G(s)}{1+G(s)} = \frac{\frac{1}{s(s+1)}}{1+\frac{1}{(s+1)s}}$

$= \frac{1}{1+(s+1)s} = \boxed{\frac{1}{s^2+s+1}}$

this is a second order system.

$\omega_n^2 = 1 \Rightarrow \omega_n = 1$

$2\xi\omega_n = 1 \Rightarrow \boxed{\xi = \frac{1}{2}}$, so system is underdamped. hence according to text, page 230, use time from 0% to 100%.

From page 231, Rise time $t_r = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{-\sigma}\right) = \boxed{\frac{\pi-\beta}{\omega_d}}$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$, $\sigma = \xi\omega_n$.

so $\omega_d = \sqrt{1-.5^2} = 0.866$, $\sigma = \frac{1}{2}$, $\beta = \tan^{-1}\frac{\omega_d}{\sigma} = \tan^{-1}\frac{0.866}{0.5}$

so $\beta = 1.04718$. so $t_r = \frac{\pi - 1.04718}{0.866} = \boxed{2.41849 \text{ sec}}$ $\rightarrow t_r$

Peak time $t_p = \frac{\pi}{w_d} = \frac{\pi}{0.866} = \boxed{3.6277 \text{ Sec}}$

Max. overshoot $M_p = e^{-(\sigma/w_n)\pi} = e^{-\left(\frac{0.5}{1}\right)\pi} = 0.2078.$

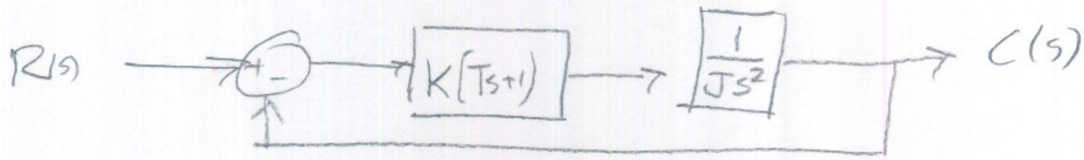
so max percent overshoot is $\boxed{20.78\%}$

Settling time using 2% criterion.

$$t_s = \frac{4}{\sigma} = \frac{4}{0.5} = \boxed{8 \text{ sec}}$$

HW#4
Problem B-5-4
Nasser Abbasi

This is a block diagram of space-vehicle attitude control system. assuming time constant T of the controller to be 3 sec and the ratio $\frac{K}{J}$ to be $\frac{2}{9}$ rad^2/sec^2, find damping ratio of system.



Solution
need to find ξ.

open loop transfer function $G(s) = \frac{K(Ts+1)}{Js^2} = \frac{2}{9}\frac{(3s+1)}{s^2}$

so closed loop transfer function $= \frac{G}{1+G} =$

$$\frac{C(s)}{R(s)} = \frac{\frac{2}{9}\frac{(3s+1)}{s^2}}{1+\frac{2(3s+1)}{9s^2}} = \boxed{\frac{6s+2}{9s^2+6s+2}}$$

so $9s^2+6s+2 \rightarrow s^2+\frac{2}{3}s+\frac{2}{9}$

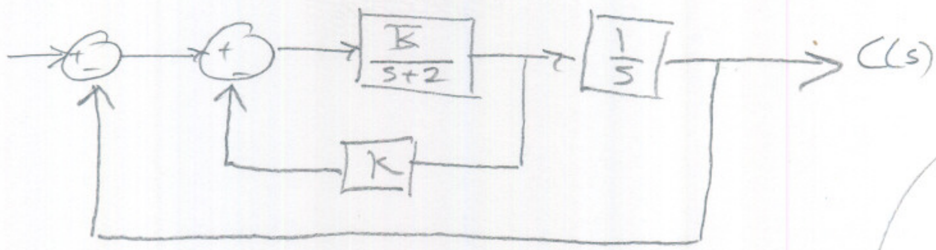
i.e $\omega_n^2=\frac{2}{9}$, $2\xi\omega_n=\frac{2}{3}$

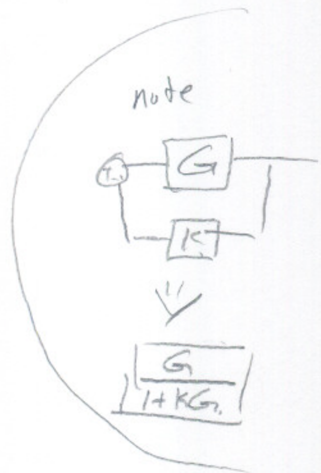
$\omega_n=\frac{\sqrt{2}}{3}$. so $\xi=\frac{1}{3}\frac{3}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

so $\boxed{\xi=\frac{1}{\sqrt{2}}} = 0.707$

HW# 4
Problem B-5-10
Nasser Abbasi

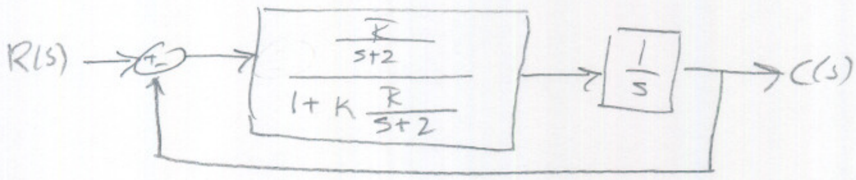
ref system in fig 5-84, determine K and k such that system has $\xi = 0.7$ and $\omega_n = 4$ rad/sec.

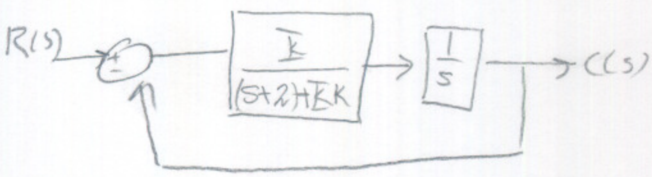




Solution

First simplify system using block diagram reduction.





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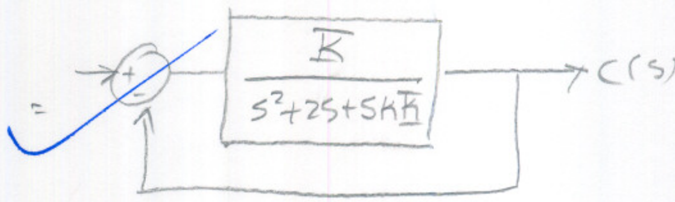
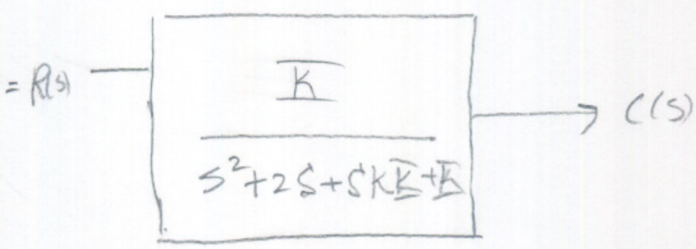




hence $G(s) = \dfrac{K}{s^2 + s(2 + kK) + K}$ $\Rightarrow$ $\boxed{\begin{aligned} 2\xi\omega_n &= 2 + kK \\ \omega_n^2 &= K \end{aligned}}$

Closed loop

$\Rightarrow \boxed{K = 16}$ so $k = \dfrac{2\xi\omega_n - 2}{K}$ $\Rightarrow k = \dfrac{2(.7)(4) - 2}{16} = \boxed{0.225}$

HW # 4

Problem B-5-21

Nasser Abbasi

Obtain the unit acceleration response curve of the unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{10(s+1)}{s^2(s+4)}.$$

Unit acceleration is defined by

$$r(t) = \frac{1}{2}t^2 \qquad t \geq 0.$$

Solution

The Laplace transform of input is $\frac{1}{s^3}$

The closed loop transfer function is $\frac{G}{1+G}$

i.e

$$\frac{10(s+1)}{s^2(s+4)+10(s+1)} = \boxed{\frac{10s+10}{s^3+4s^2+10s+10}} = G_{cL}$$

hence response is $G_{CL} \cdot X(s)$

$$\boxed{Y(s) = (G_{CL})\left(\frac{1}{s^3}\right)}$$

Please see program next for the actual output

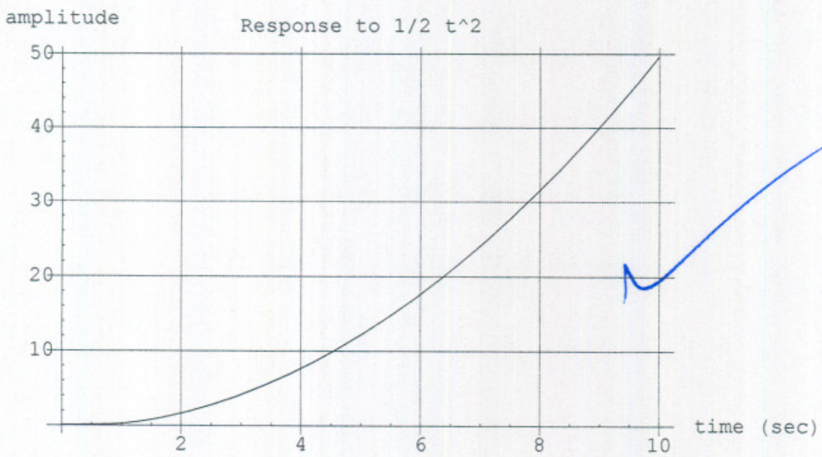
```
Remove["Global`*"];
<< ControlSystems`
```

In[127]:=

$$\texttt{input[t\_] :=}\ \frac{1}{2}\,t^2$$

$$\texttt{sys = TransferFunction}\left[s,\ \frac{10\,s+10}{s^3+4\,s^2+10\,s+10}\right];$$

```
SimulationPlot[sys, input[t], {t, 10}, PlotLabel -> "Response to 1/2 t^2",
  AxesLabel → {"time (sec)", "amplitude"}, GridLines → Automatic];
```



HW# 4
Problem B-5-31
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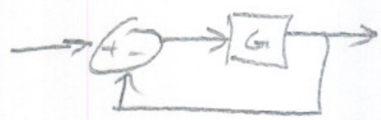
Consider unity feedback control system whose open loop tf is

$$G(s) = \frac{K}{s(Js+B)}$$

discuss the effects of varying values of K and B has on the steady state error in unit-Ramp response.
sketch typical unit-ramp response curves for a small, medium, Large values of K, assuming B is constant.

Solution

First obtain the Closed Loop TF.



so Closed Loop TF $= \frac{G}{1+G}$

$$= \frac{K}{s(Js+B)+K} = \frac{K}{s^2J+Bs+K}$$

$$= \frac{K/J}{s^2+\frac{B}{J}s+K/J}.$$

hence the natural undamped frequency $\boxed{\omega_n = \sqrt{\frac{K}{J}}}$

and $\boxed{2\xi\omega_n = \frac{B}{J}} \Rightarrow \xi = \frac{B}{2J\omega_n} = \frac{B}{2J\sqrt{\frac{K}{J}}} = \boxed{\frac{B}{2\sqrt{JK}}}$

so, for fixed B, as K increases, ω_n will increase and ξ will decrease. i.e more oscillation will result.

now to look at steady state errors $\longrightarrow$

now $Y(s) = G(s)\,X(s)$ this is $\frac{1}{s^2}$

this is $G(s)$ Found earlier.

so $E(s) = X(s) - Y(s)$

$$E(s) = \frac{1}{s^2} - \left(\frac{1}{s^2}\,\frac{K/J}{s^2+\frac{B}{J}s+\frac{K}{J}}\right) = \frac{1}{s^2} - \frac{1}{s^2}\,\frac{K}{Js^2+Bs+K}$$

using $\boxed{\text{Final value theorem}}$

$$e(\infty) = \lim_{s\to 0} s\,E(s)$$

$$= \lim_{s\to 0} \frac{1}{s} - \frac{1}{s}\,\frac{K}{Js^2+Bs+K} = \lim_{s\to 0} \frac{Js^2+Bs+K-K}{s(Js^2+Bs+K)}$$

$$= \lim_{s\to 0} \frac{Js^2+Bs}{s(Js^2+Bs+K)} = \lim_{s\to 0} \frac{Js+B}{Js^2+Bs+K} = \boxed{\frac{B}{K}}$$

so steady state error is $\frac{B}{K}$. (position error)

so for fixed B, $\boxed{\text{as K increases, the position error decrease}}$

in the next program, I will plot the response for different values of K, for different Fixed values of B.

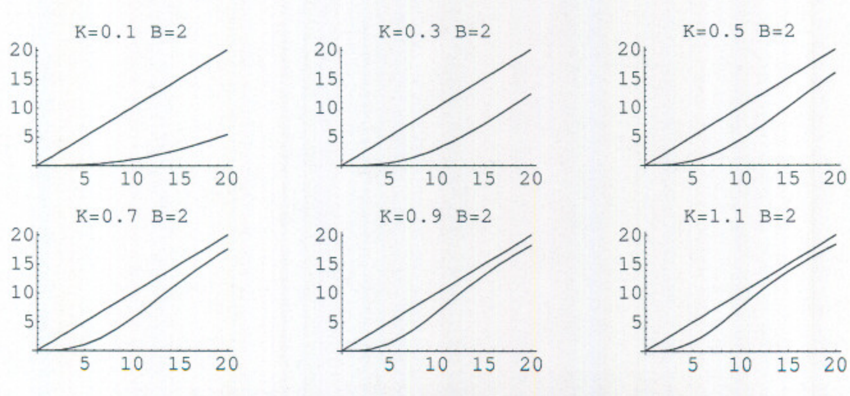
→

Conclusion

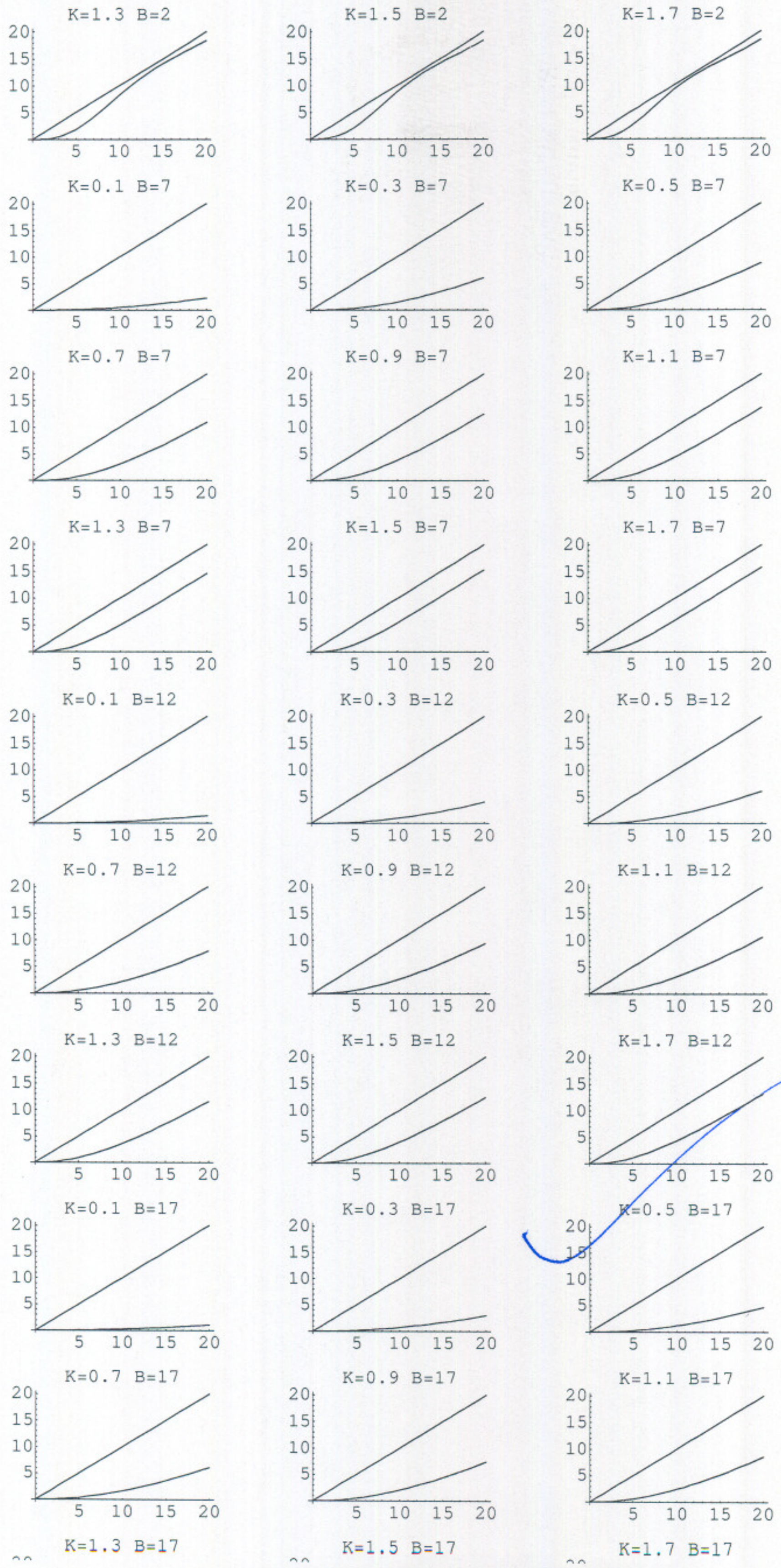
as a result of these plots, we see that as B increases, steady state error increases For the same K. For Example, for $K=1.7$, when B was 2 there was much less ss error than when B was 27.

When B is fixed, as we increase K, steady state error decreased. so Best combination is to have large K and small B.

```
In[20]:= Remove["Global`*"];
         << ControlSystems`
         << Graphics`

        input[t_] := t (*ramp input*)

        sys = TransferFunction[s, (k/j)/(s^2 + (b/j) s + k/j)];


        j = 10;
        δ = .2;
        a = Table[i, {i, 1, 100}];
        process[] := Module[{},
            i = 0;
            For[b = 2, b < 30, b = b + 5,
             {
              For[k = 0.1, k < 1.8, k = k + .2,
                {
                 i = i + 1;

                 sys = TransferFunction[s, (k/j)/(s^2 + (b/j) s + k/j)];

                 y = Re[Chop[N[OutputResponse[sys, input[t], t]]]];
                 p = Plot[{input[t], y }, {t, 0, 20},
                   PlotLabel -> "K=" <> ToString[k] <> " B=" <> ToString[b],
                   DisplayFunction → Identity];
                 a[[i]] = p;
                }
               ];
             }
            ];
          ];
        process[];
        Show[GraphicsArray[Partition[Take[a, {1, i}], 3]], DisplayFunction → $DisplayFunction];
```



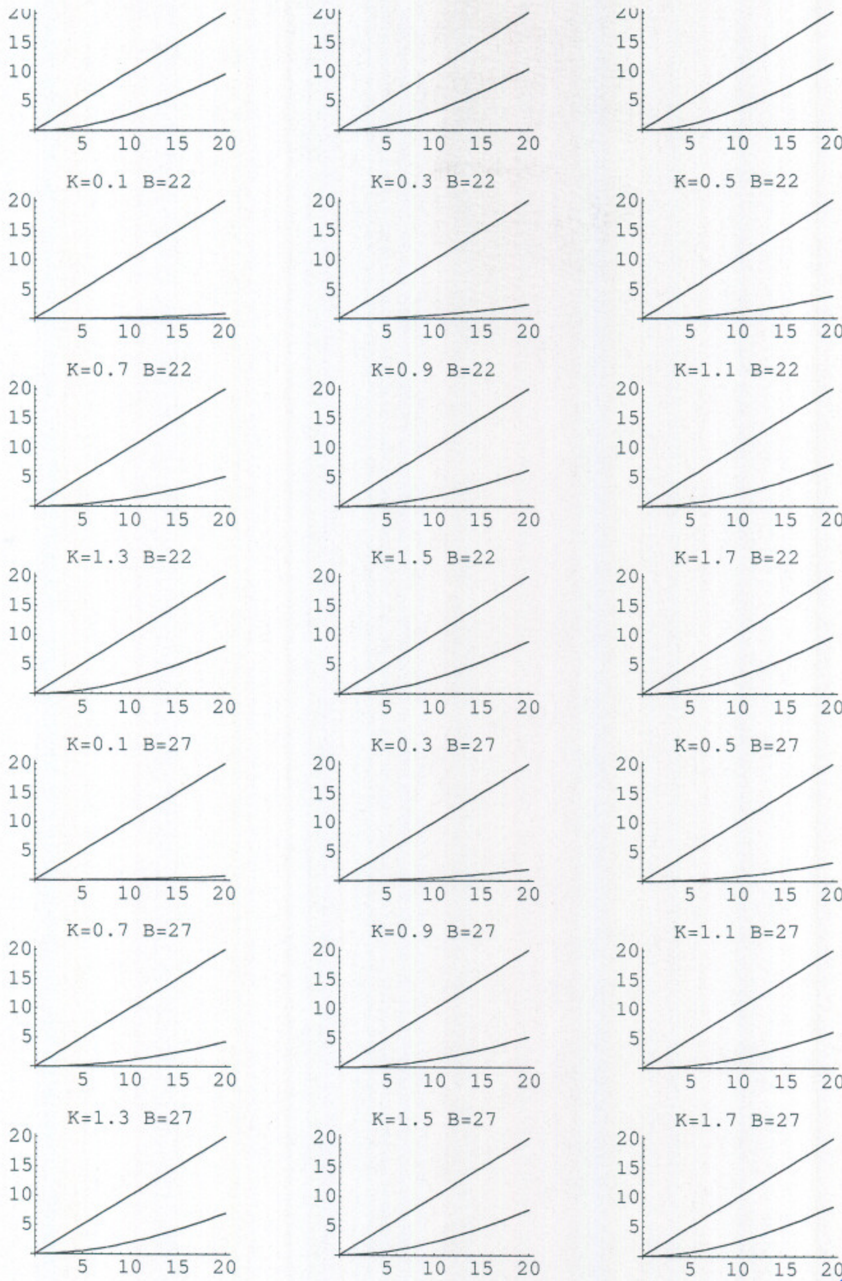
B=2


K=1.3 B=2
K=1.5 B=2
K=1.7 B=2
K=0.1 B=7
K=0.3 B=7
K=0.5 B=7
K=0.7 B=7
K=0.9 B=7
K=1.1 B=7
K=1.3 B=7
K=1.5 B=7
K=1.7 B=7
K=0.1 B=12
K=0.3 B=12
K=0.5 B=12
K=0.7 B=12
K=0.9 B=12
K=1.1 B=12
K=1.3 B=12
K=1.5 B=12
K=1.7 B=12
K=0.1 B=17
K=0.3 B=17
K=0.5 B=17
K=0.7 B=17
K=0.9 B=17
K=1.1 B=17
K=1.3 B=17
K=1.5 B=17
K=1.7 B=17

B=7

B=12

B=17


K=0.1 B=22
K=0.3 B=22
K=0.5 B=22
K=0.7 B=22
K=0.9 B=22
K=1.1 B=22
K=1.3 B=22
K=1.5 B=22
K=1.7 B=22
K=0.1 B=27
K=0.3 B=27
K=0.5 B=27
K=0.7 B=27
K=0.9 B=27
K=1.1 B=27
K=1.3 B=27
K=1.5 B=27
K=1.7 B=27

B=22

B=27