NASSER ABBASI

HW # 3

MAE 170

24/30

HW 3, MAE 170.
Computer Problem **1**, cruise Control with no engine delay
by Nasser Abbasi
UCI, Winter 2005.

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Solution

The first disturbance. 10

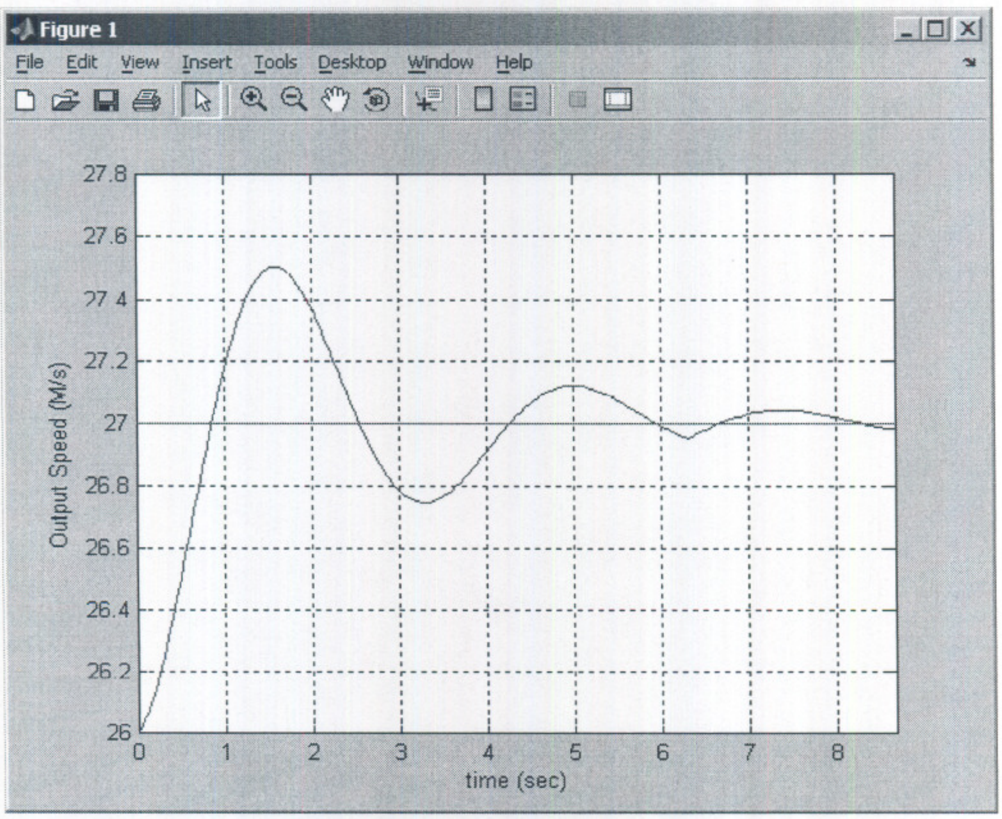
Using the input specified by $u = [27 \quad fd + 200\ sign(\sin(0.5t))]$

By trial and error, I changed k_p and k_i to find a velocity output that remained as close as possible to the reference velocity without exceeding the maximum engine output of 3381 N. I addition, I was looking for the least amount of overshoot over the reference velocity. I was not able to eliminate completely the overshoot, but had to settle with an initial of only 0.5 m/s overshoot. After trying a number of different combinations, I found the following to give the best result.

$$
\begin{aligned}
k_p &= 1500 \\
k_i &= 6500
\end{aligned}
$$

The max engine output was 3347 N

The transient values, read from this plot, are



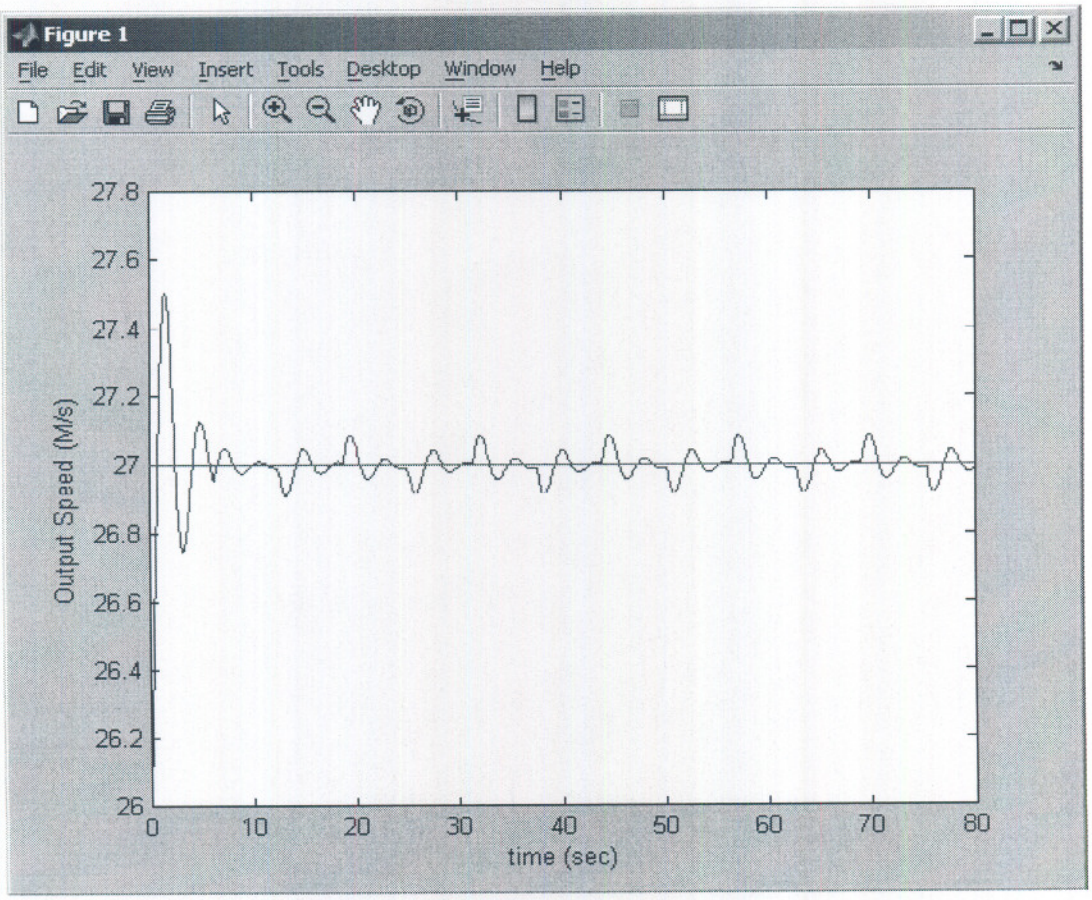
delay time=0.4 sec
rise time=0.8 sec
peak time=1.5 sec
settling time=6.2 sec
Max overshoot=0.5 m/s
(reference on page 230 of text for more explanation of these values).

The state space matrices are

$$A = \begin{bmatrix} \frac{-k_p}{m} & \frac{k_i}{m} \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{k_p}{m} & \frac{-1}{m} \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ -k_p & k_i \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ k_p & 0 \end{bmatrix}$$

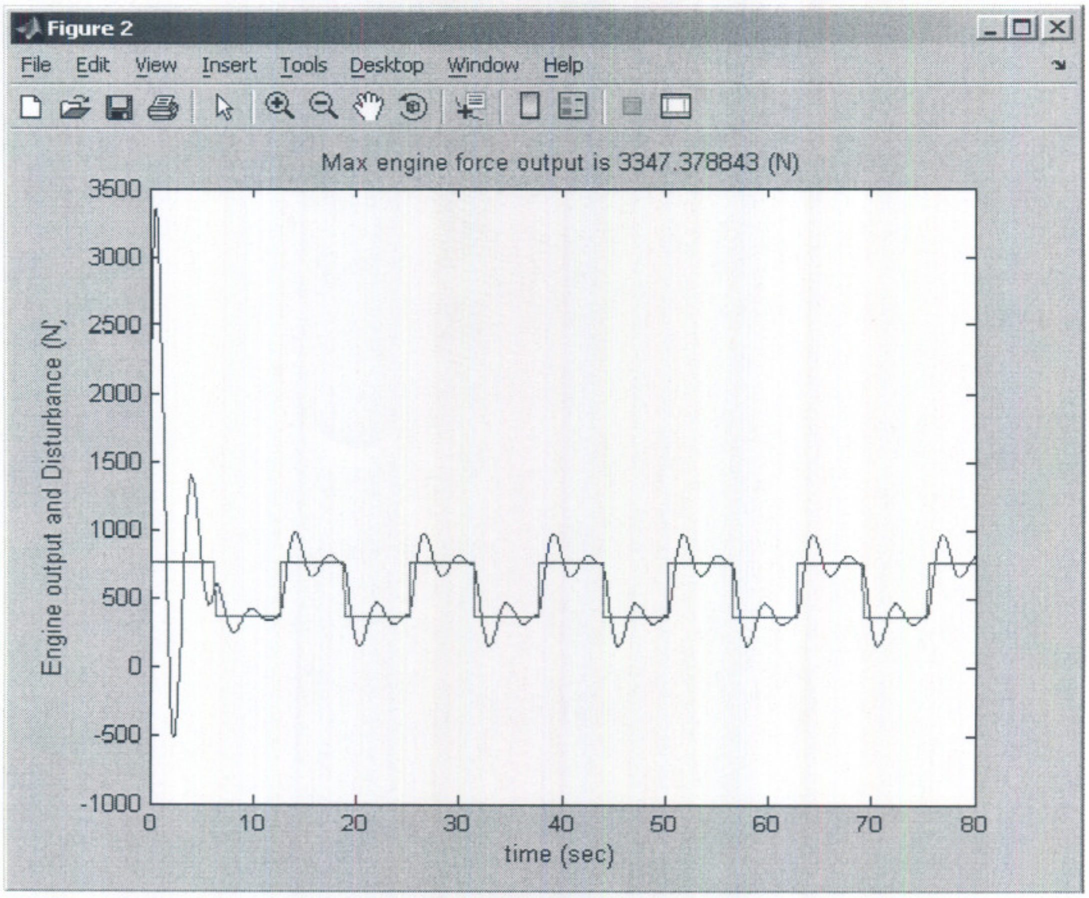
The output matrix is $\begin{bmatrix} V \\ Z \end{bmatrix}$

The output using the above values is shown in these plots



Transfer Function input velocity to output velocity is

$$\boxed{\frac{K_i + s K_p}{m s^2 + s K_p + K_i}}$$

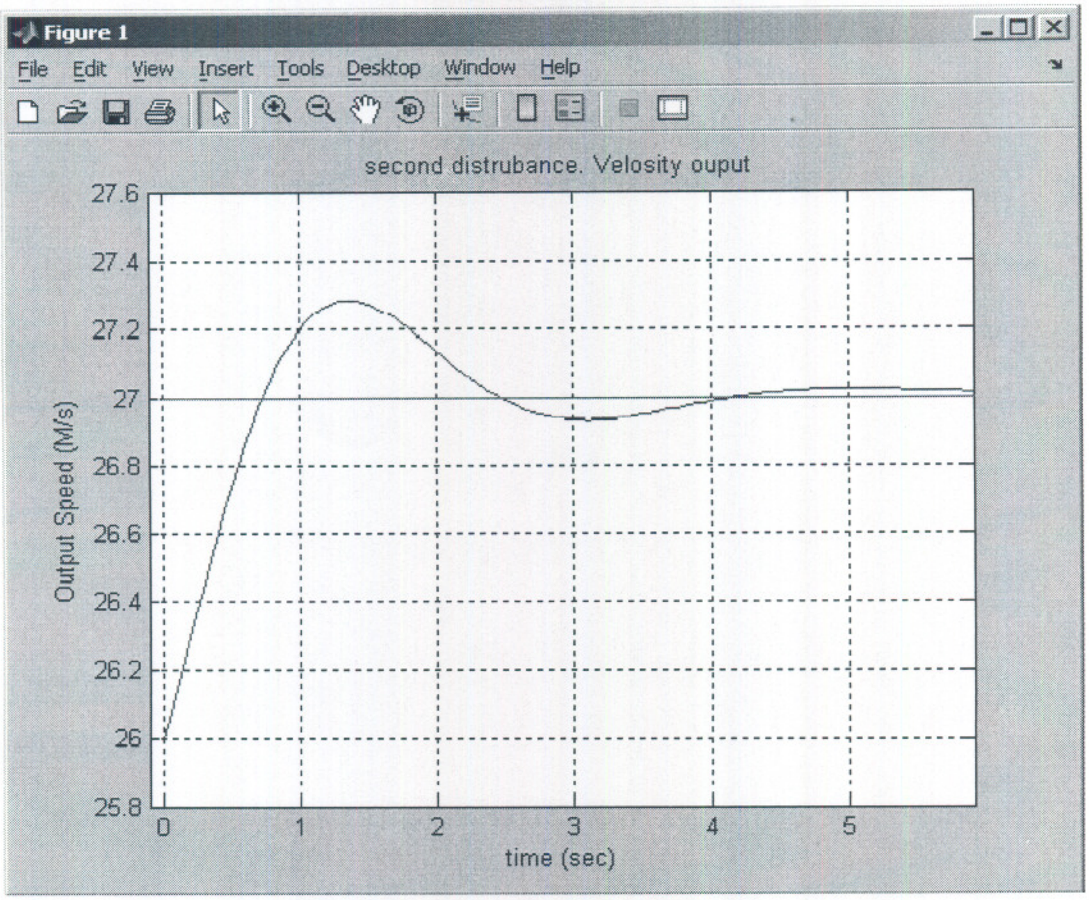


The second disturbance.

Using the input specified by $u = [27 \qquad fd + 200 \quad \sin(0.5t)]$
By trial and error, I changed k_p and k_i to find a velocity output that remained as close as possible to the reference velocity without exceeding the maximum engine output of 3381 N. I addition, I was looking for the least amount of overshoot over the reference velocity. I was not able to eliminate completely the overshoot. After trying a number of different I found the following to give the best result.

$$
\begin{aligned}
k_p &= 3100 \\
k_i &= 7000
\end{aligned}
$$

The max engine output was 3371 N



delay time=0.3625 sec
rise time=0.75 sec
peak time=1.25 sec
settling time=3.8 sec
Max overshoot=0.25 m/s
Notice that the transient response for the second disturbance is better than the first. It has a faster rise time and settling time and less max overshoot. This is due to the fact that the second disturbance is smoother than the first. The first was a square wave, and the second is a sin wave.

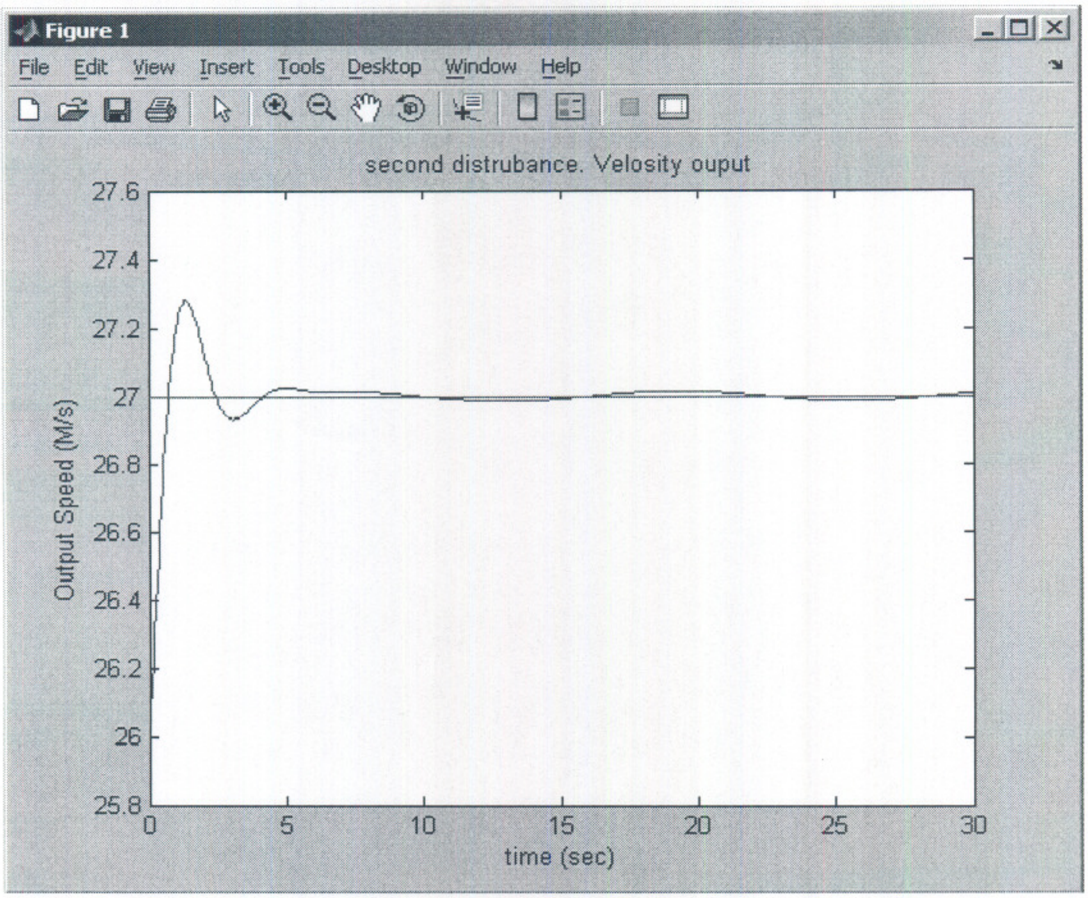


Figure 1
File Edit View Insert Tools Desktop Window Help
second distrubance. Velosity ouput
Output Speed (M/s)
27.6
27.4
27.2
27
26.8
26.6
26.4
26.2
26
25.8
0
5
10
15
20
25
30
time (sec)

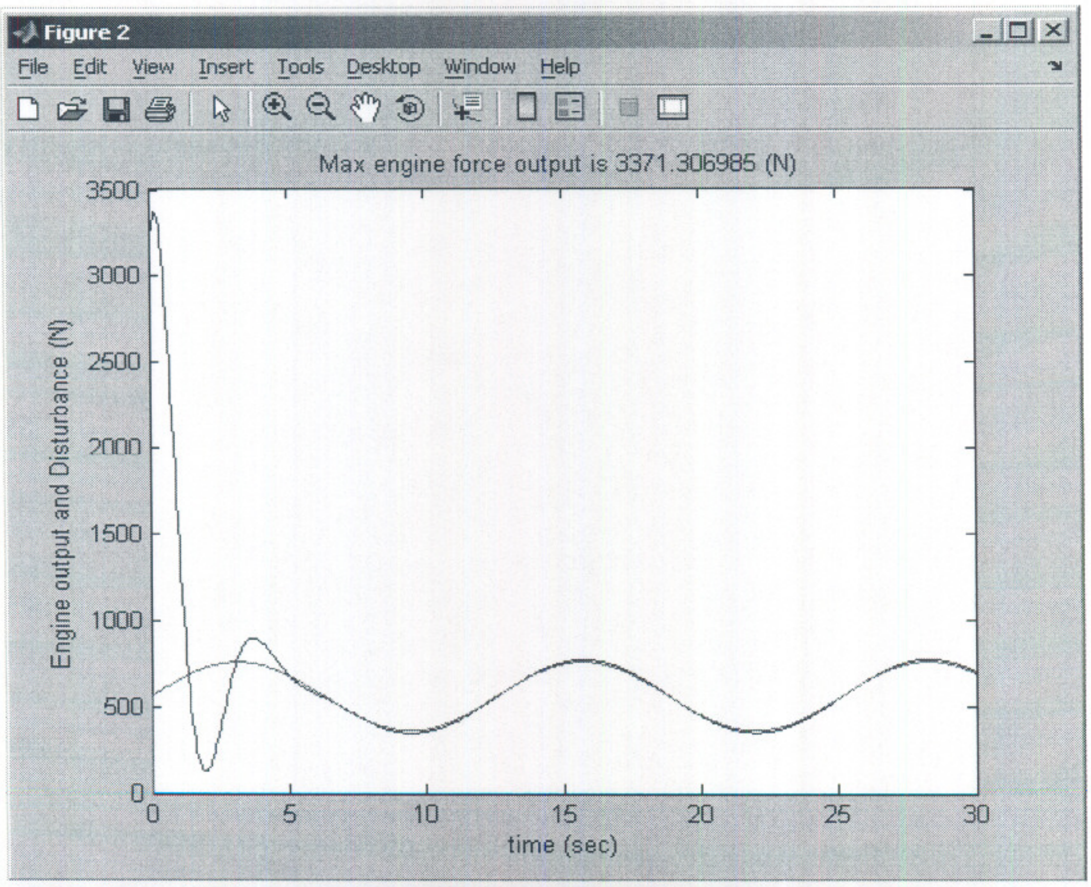


Figure 2
File Edit View Insert Tools Desktop Window Help
Max engine force output is 3371.306985 (N)
Engine output and Disturbance (N)
3500
3000
2500
2000
1500
1000
500
0
0
5
10
15
20
25
30
time (sec)

MAE 170
HW#3
second computer problem
Nasser Abbasi

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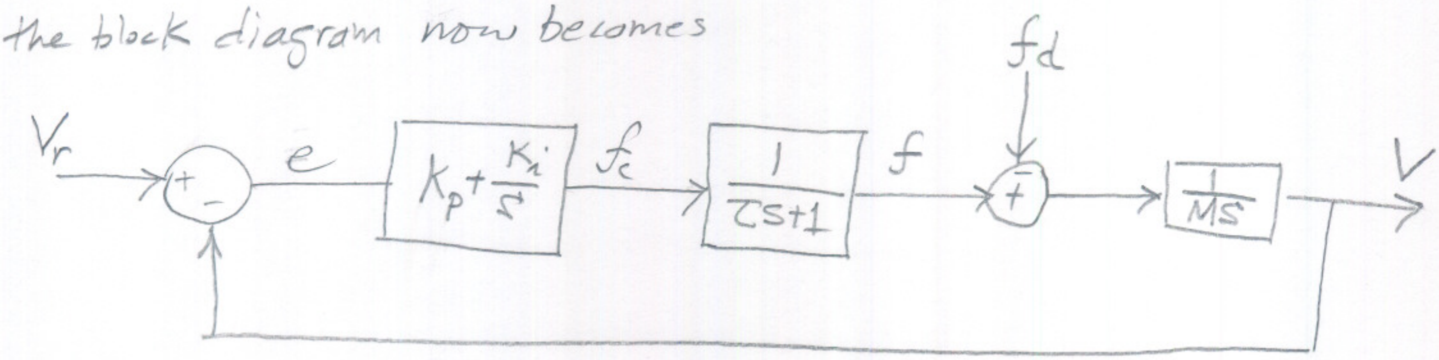
an engine, 1^{st} order, time constant τ, gain is 1 is $\boxed{\dfrac{1}{\tau s+1}}$

so the block diagram now becomes



so now $\quad f_c = K_p(V_r - V) + K_i \int (V_r - V) \quad$ —— (1)

$$F(s) = F_c(s)\,\frac{1}{\tau s+1}$$

so $\boxed{\tau \dot{f} + f = f_c}$ —— (2)

from (1), (2) $\Rightarrow \quad \tau \dot{f} + f = K_p(V_r - V) + K_i \int V_r - V$

so $\boxed{\dot{f} = \dfrac{K_p}{\tau}(V_r - V) + \dfrac{K_i}{\tau} \underbrace{\int V_r - V}_{z} - \dfrac{f}{\tau}}$ —— (3)

so Force into plant is $\quad f - f_d$.

so from $F = ma$, we set

$\boxed{f - f_d = m\dot{V}}$ —— (4)

so $\boxed{\dot{V} = \dfrac{f}{M} - \dfrac{f_d}{M}}$ —— (5)

Let $\quad z = \int V_r - V \quad \longrightarrow$

now I need to set up the 3 DE with $\dot{V}, \dot{Z}, \dot{f}$ in LHS.

$$\boxed{\begin{aligned} \dot{V} &= \frac{f}{M} - \frac{f_d}{M} \\ \dot{Z} &= V_r - V \\ \dot{f} &= \frac{k_p}{\tau}V_r - \frac{k_p}{\tau}V + \frac{k_i}{\tau}Z - \frac{f}{\tau} \end{aligned}}$$

so
$$\begin{pmatrix} \dot{V} \\ \dot{Z} \\ \dot{f} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{1}{M} \\ -1 & 0 & 0 \\ -\frac{k_p}{\tau} & \frac{k_i}{\tau} & -\frac{1}{\tau} \end{pmatrix} \begin{pmatrix} V \\ Z \\ f \end{pmatrix} + \underset{3\times 2}{\begin{pmatrix} 0 & -\frac{1}{M} \\ 1 & 0 \\ \frac{k_p}{\tau} & 0 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} V_r \\ f_d \end{pmatrix}}$$

$$\begin{pmatrix} V \\ f_c \end{pmatrix} = \underset{2\times 3}{\begin{pmatrix} 1 & 0 & 0 \\ -k_p & k_i & 0 \end{pmatrix}} \underset{3\times 1}{\begin{pmatrix} V \\ Z \\ f \end{pmatrix}} + \underset{2\times 2}{\begin{pmatrix} 0 & 0 \\ k_p & 0 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} V_r \\ f_d \end{pmatrix}}$$

you don't want to know f_c, you want to know $\underline{f}$ -1

So now I can use the above A, B, C, D matrices to solve this. please see next.

Transfer Function from $V_{reference}$ to V_{out} is

$$\boxed{\frac{k_i + s k_p}{k_i + s k_p + m s^2 (1 + s\tau)}}$$

HW 3, MAE 170.
Computer Problem **2**, cruise Control with no engine delay
by Nasser Abbasi
UCI, Winter 2005.

Solution

The state space matrices are

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{m} \\ -1 & 0 & 0 \\ \frac{-k_p}{\tau} & \frac{k_i}{\tau} & \frac{-1}{\tau} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & \frac{-1}{m} \\ 1 & 0 \\ \frac{k_p}{\tau} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ -k_p & k_i & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ k_p & 0 \end{bmatrix}$$

The output matrix is $\begin{bmatrix} V \\ \\ f_c \end{bmatrix}$ state matrix $X = \begin{bmatrix} V \\ Z \\ f \end{bmatrix}$

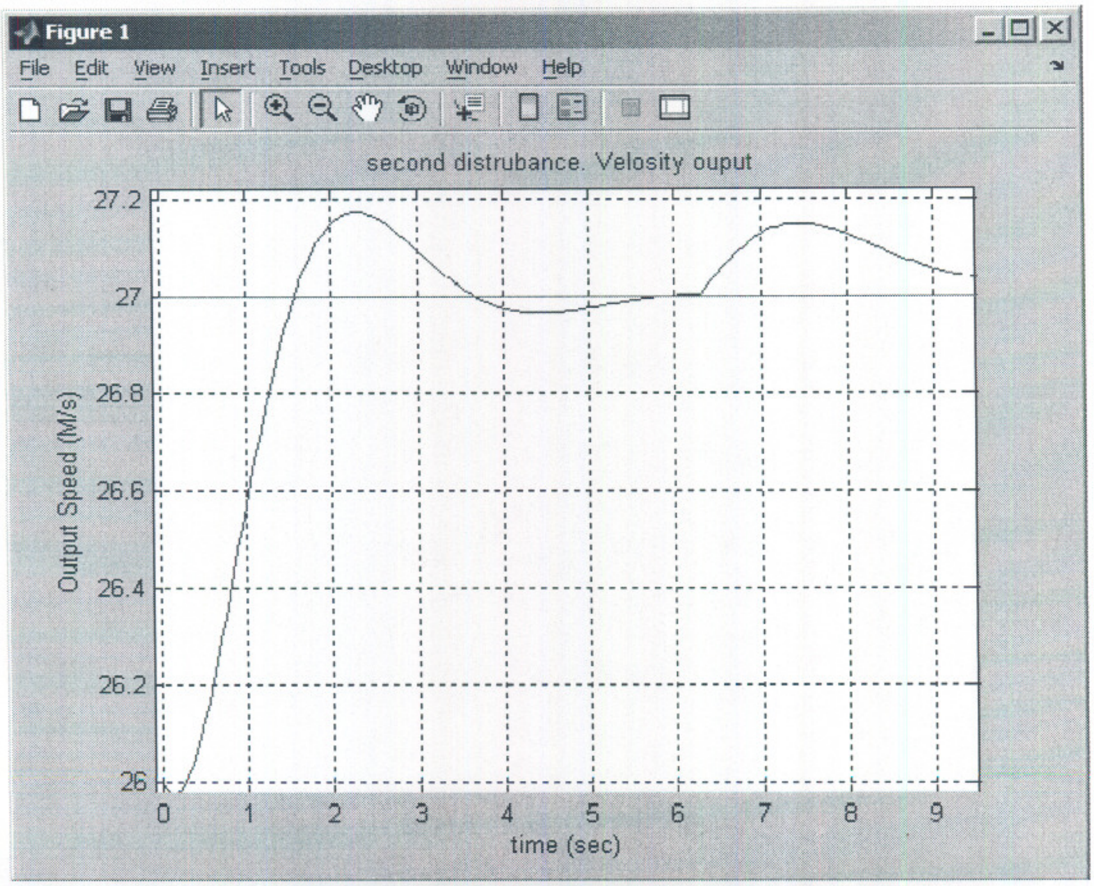
The first disturbance.

Using the input specified by $u = [27 \qquad fd + 200\ sign(\sin(0.5t))]$

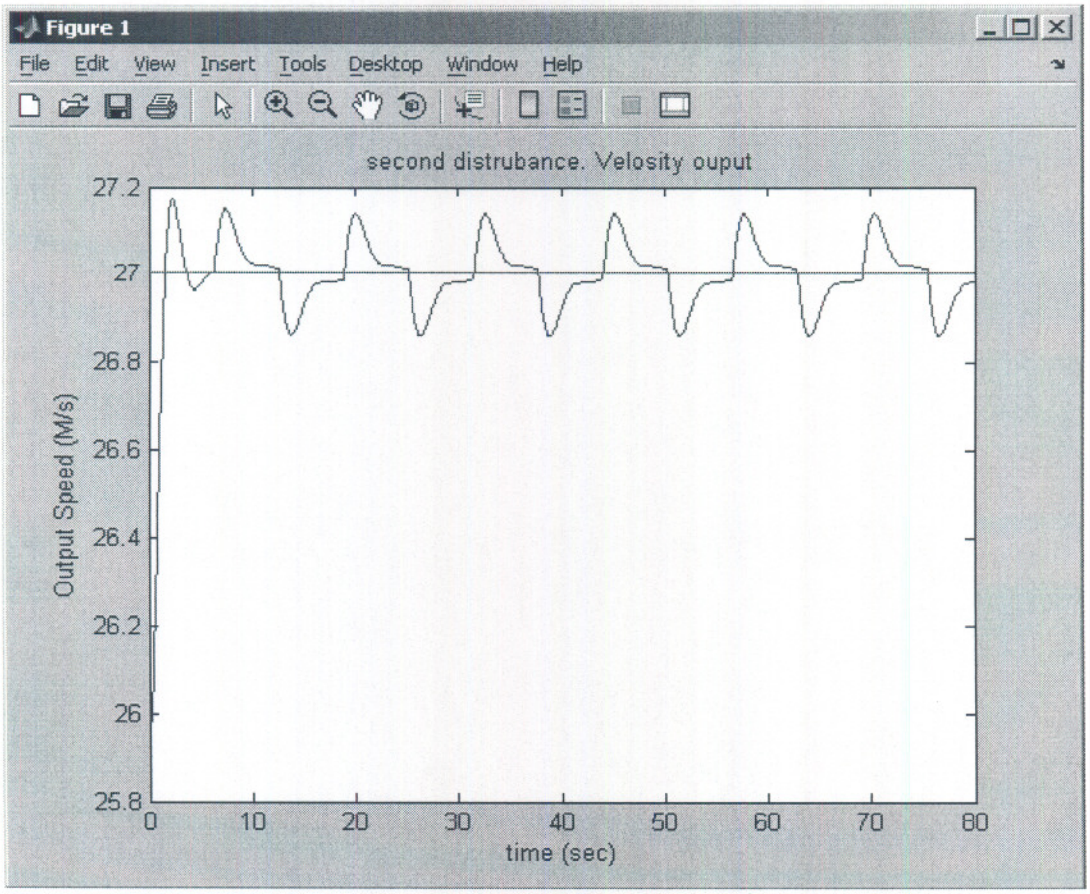
By trial and error, I changed k_p and k_i to find a velocity output that remained as close as possible to the reference velocity without exceeding the maximum engine output of 3381 N. I addition, I was looking for the least amount of overshoot over the reference velocity. I was not able to eliminate completely the overshoot, but had to settle with an initial of only 0.5 m/s overshoot. After trying a number of different combinations, I found the following to give the best result.

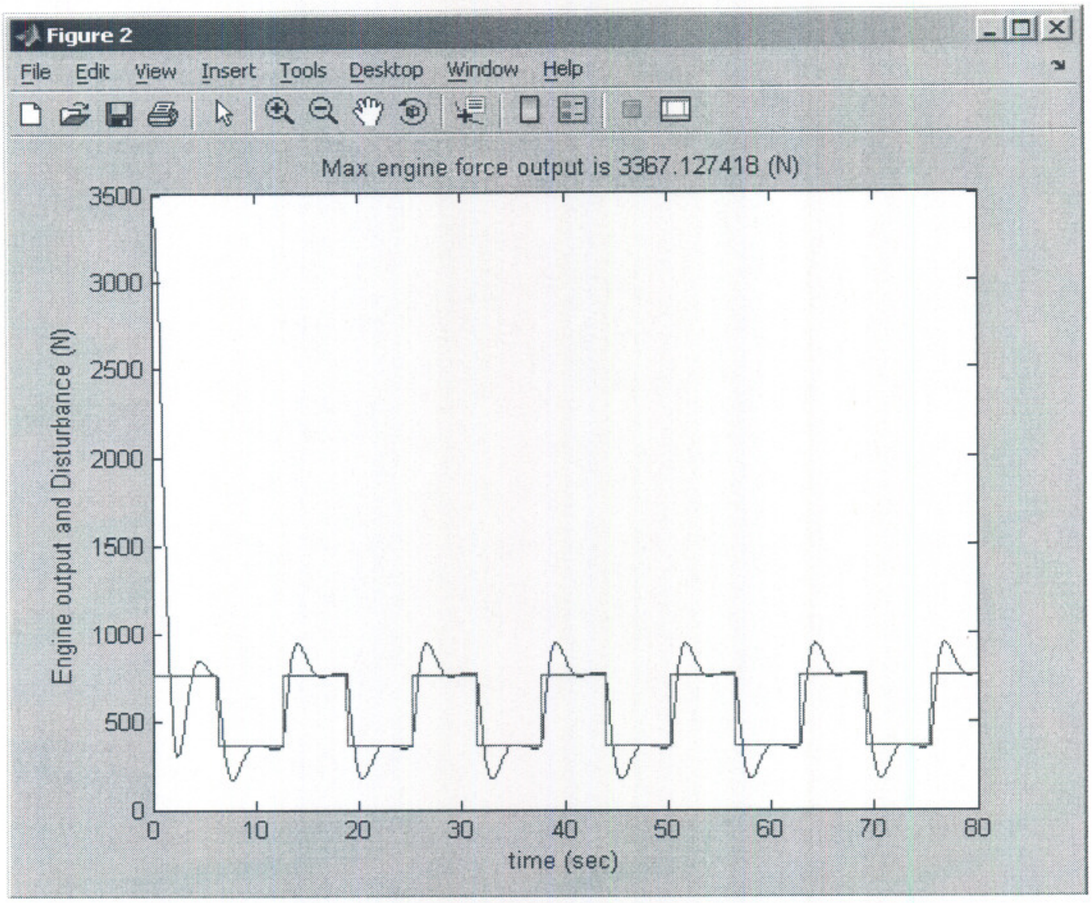
$$\begin{aligned} k_p &= 3100 \\ k_i &= 1000 \end{aligned}$$

The max engine output was 3367 N
The transient values, read from this plot, are



delay time=0.8 sec
rise time=1.6 sec
peak time=2.2 sec
settling time=5 sec
Max overshoot=0.19 m/s
(reference on page 230 of text for more explanation of these values).
The output using the above values is shown in these plots



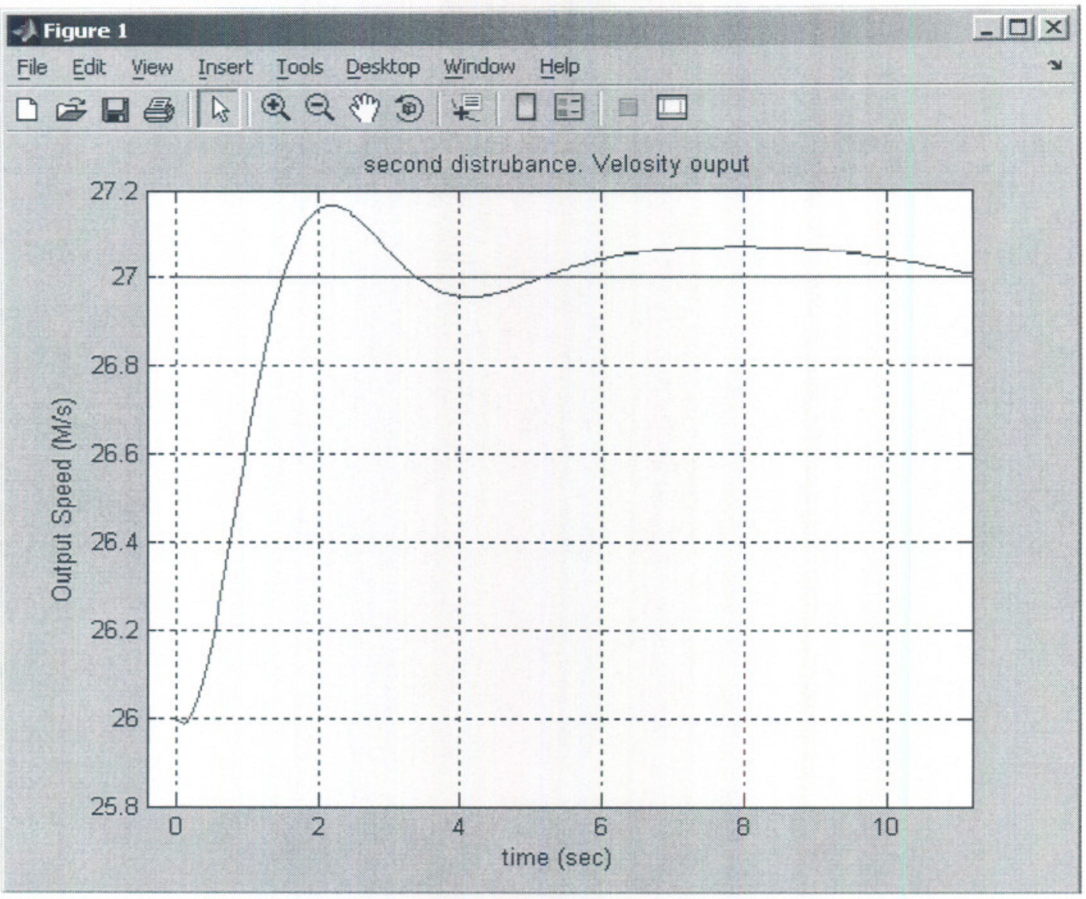


The second disturbance.

Using the input specified by $u = [27 \qquad fd + 200 \quad \sin(0.5t)]$
By trial and error, I changed k_p and k_i to find a velocity output that remained as close as possible to the reference velocity without exceeding the maximum engine output of 3381 N. I addition, I was looking for the least amount of overshoot over the reference velocity. After trying a number of different I found the following to give the best result.

$$\begin{aligned} k_p &= 3150 \\ k_i &= 1000 \end{aligned}$$

The max engine output was 3351 N



delay time=0.75 sec
rise time=1.5 sec
peak time=2.2 sec
settling time=5 sec
Max overshoot=0.18 m/s

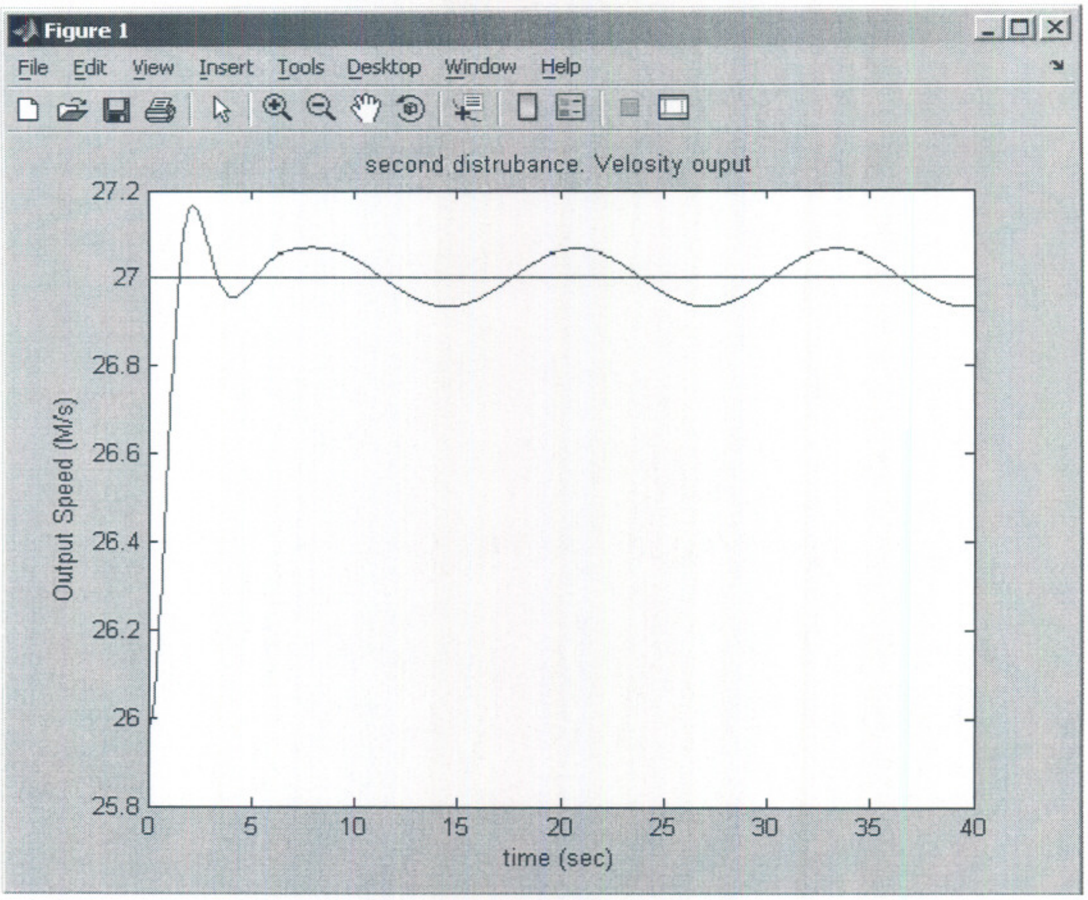


Figure 1
File Edit View Insert Tools Desktop Window Help
second distrubance. Velosity ouput
Output Speed (M/s)
27.2
27
26.8
26.6
26.4
26.2
26
25.8
0 5 10 15 20 25 30 35 40
time (sec)

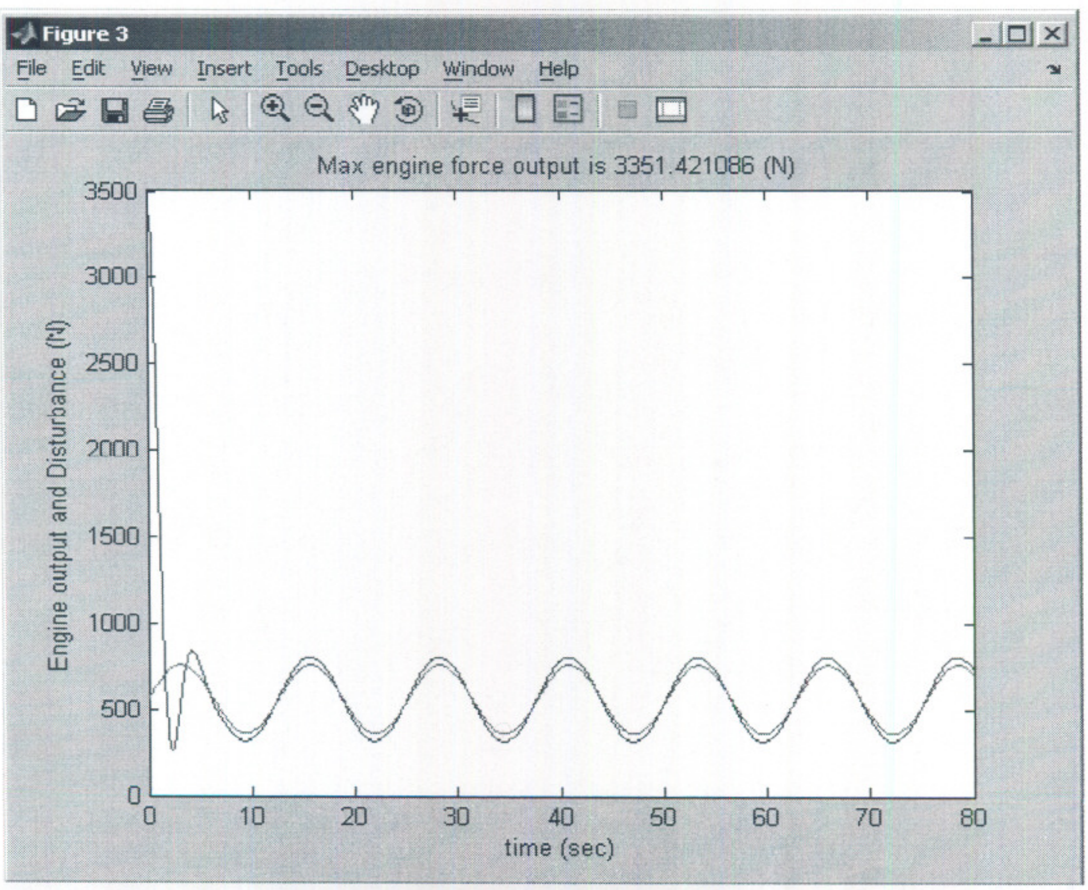


Figure 3
File Edit View Insert Tools Desktop Window Help
Max engine force output is 3351.421086 (N)
Engine output and Disturbance (N)
3500
3000
2500
2000
1500
1000
500
0
0 10 20 30 40 50 60 70 80
time (sec)

HW#3
problem 3
Nasser Abbasi

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why start here? You should have used the given diff eqns.

-5

The Closed Loop System is $X_r \rightarrow \boxed{\dfrac{ka}{ms^3+cs^2+bs+ka}} \rightarrow X$

use $a=1$, $b=350$, $m=1$, $c=20$.

Find gain K that gives fast step response.

states: position, velocity, force.

$$\frac{X}{X_r} = \frac{ka}{ms^3+cs^2+bs+ka} \Rightarrow \boxed{m\dddot{x}+c\ddot{x}+b\dot{x}+kax = kax_r}$$

$\boxed{F=m\ddot{x}}$

$\boxed{V=\dot{x}}$

divide by m

$$\dddot{x}+\frac{c}{m}\ddot{x}+\frac{b}{m}\dot{x}+\frac{ka}{m}x = \frac{ka}{m}x_r$$

tion

let $x_1 = x$

$V \leftarrow x_2 = \dot{x} = \dot{x}_1$

$F \leftarrow x_3 = m\ddot{x} = m\dot{x}_2 \Rightarrow \ddot{x} = \frac{x_3}{m}$

?

Then the DE can be written as

$$\frac{\dot{x}_3}{m}+\frac{c}{m}\frac{x_3}{m}+\frac{b}{m}x_2+\frac{ka}{m}x_1 = \frac{ka}{m}x_r$$

i.e.

$$\frac{\dot{x}_3}{m} = \frac{ka}{m}x_r - \frac{c}{m}\frac{x_3}{m} - \frac{b}{m}x_2 - \frac{ka}{m}x_1$$

or $\boxed{\dot{F} = kax_r - \frac{c}{m}F - bV - kax}$

I replaced $\dot{x}_3$ by $\dot{F}$, x_3 by F, x_2 by V and x_1 by x

$\rightarrow$

$\boxed{\dot{x} = V}$

$\boxed{\dot{V} = \frac{F}{m}}$

hence.

$$\begin{bmatrix} \dot{F} \\ \dot{x} \\ \dot{V} \end{bmatrix} = \overbrace{\begin{bmatrix} -\frac{c}{m} & -Ka & -b \\ 0 & 0 & 1 \\ \frac{1}{m} & 0 & 0 \end{bmatrix}}^{A} \begin{bmatrix} F \\ X \\ V \end{bmatrix} + \overbrace{\begin{bmatrix} Ka \\ 0 \\ 0 \end{bmatrix}}^{B} \begin{bmatrix} x_r \end{bmatrix}$$

For output, use X

so

$$\begin{bmatrix} X \end{bmatrix} = \underbrace{\underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}}_{1\times3}}_{C} \underbrace{\begin{bmatrix} F \\ X \\ V \end{bmatrix}}_{3\times1} + \underbrace{\begin{bmatrix} \text{null} \end{bmatrix}}_{D} \begin{bmatrix} x_r \end{bmatrix}$$

The following is computer result to find best value of K to give fast step response $\longrightarrow$

?

```
% Problem 3
% The parameters of the pneumatic system discussed in class are a=1,
% m=1, b=350, and c=20.  Find a proportional gain k that gives a fast
% step response.
% Model the system in state-space form using the states: position,
% velocity, and force.  Repeat the step response simulation with a
% disturbance force of Fd=-10 pushing to the left throughout the motion.
% Simulate the response of the system using the gain you found above, and
% a 1 Hz square wave input (+- 1 amplitude), a 1 Hz sin wave, a 5 hz sin
% wave, and a 10 hz sin wave.  Show your plots, and try to explain your
% results.

%by Nasser Abbasi

clear all;
close all;

m = 1;  %Kg
c = 20;
b = 350;
a = 1;
nIter=10;
t=0:0.1:100;
y=zeros(length(t),nIter);
k=0;

while nIter>0
    k=k+10;

    A = [-c/m      -k*a    -b
         0         0       1
         1/m       0       0];

    B = [k*a
         0
         0];

    C = [0    1     0];

    D = [ 0 ];

    Gss = ss(A,B,C,D);
    [y(:,nIter),t]=step(Gss,t);
    legendStr(nIter)={sprintf('%d',k)};
    nIter=nIter-1;
end
plot(t,y)
title('step response as function of changing gain k');
xlabel('time (sec)');
ylabel('position x');
legend(legendStr);
```

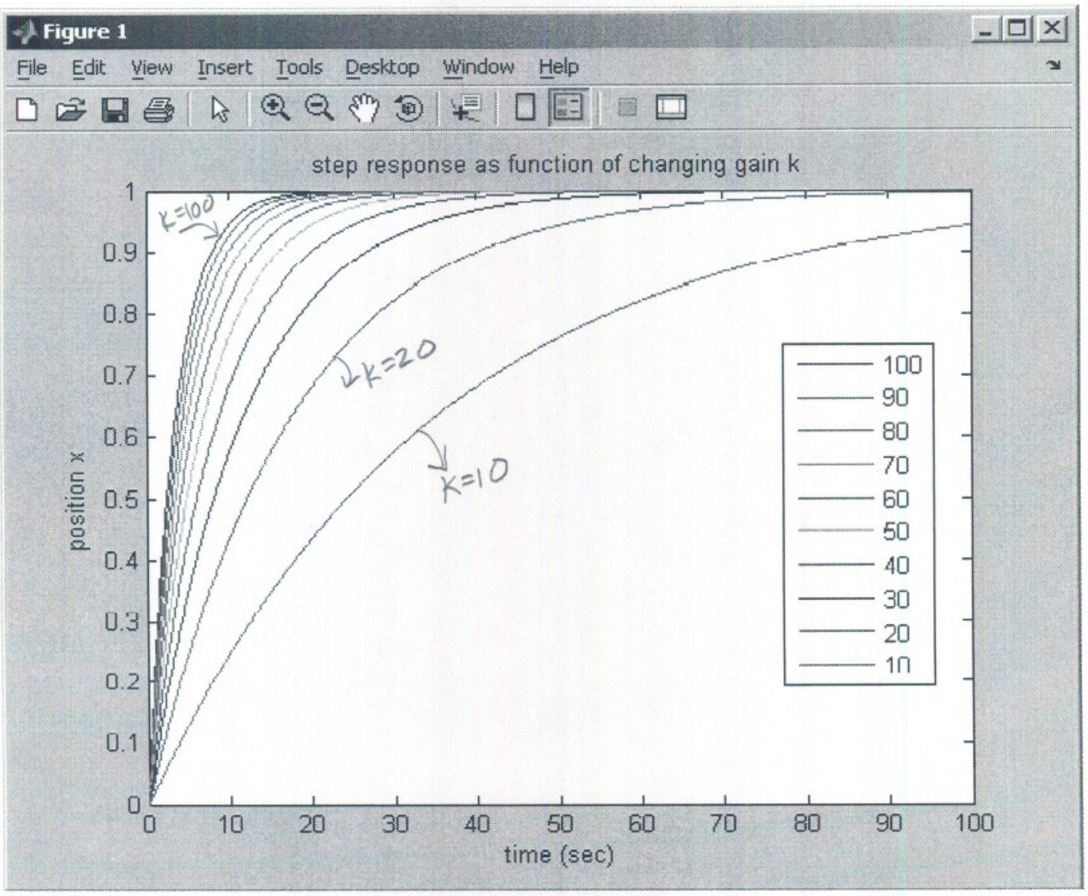
output on next page

HW 3, MAE 170.
Computer Problem **3**
by Nasser Abbasi
UCI, Winter 2005.

Solution

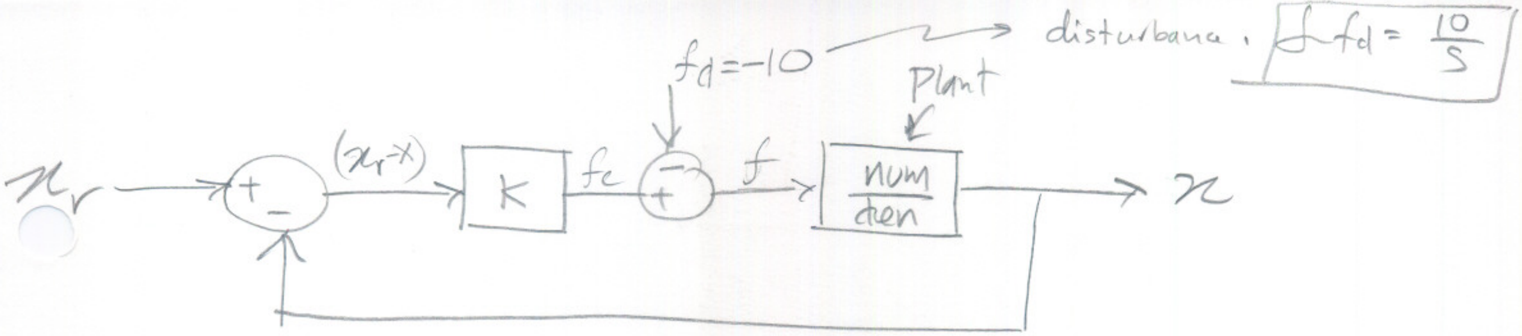
This is the result of running the step response as k is changed.

This shows that as k increases, the response is faster. at k=100 we have a rise time of about 20 seconds. at k=10 the rise time was more than 100 seconds.



(PS. Plot is in color, but my printer is black/white)

next, I repeat with $f_d = -10$ →



so $G = \left[K - F_d\right] \dfrac{num}{den}$ where $\dfrac{num}{den} = \dfrac{a/m}{s^3 + \frac{c}{m}s^2 + \frac{bs}{m}}$

so closed Loop transfer function is

$$\frac{G}{1+G} = \frac{(K - f_d)\frac{a}{m}}{(K - f_d)\frac{a}{m} + s^3 + \frac{c}{m}s^2 + \frac{bs}{m}}$$

$$= \boxed{\frac{(K - F_d)\,a}{(K - F_d)\,a + ms^3 + cs^2 + bs}}$$

?

using $K = 100$ From First part, and

$c = 20$, $a = 1$, $m = 1$, $b = 350$

now $\dfrac{X}{X_r} = \dfrac{(K - F_d)\,a}{(K - F_d)\,a + ms^3 + cs^2 + bs}$

ie $(K - F_d)\,a x + \dddot{x} m + c\ddot{x} + b\dot{x} = (K - F_d)\,a x_r$

$Kax - aF_d x + \dddot{x} m + c\ddot{x} + b\dot{x} = Kax_r - F_d a x_r$

$\dddot{x} m + c\ddot{x} + b\dot{x} + Kax - aF_d x = Kax_r - F_d a x_r$

$\dddot{x} + \frac{c}{m}\ddot{x} + \frac{b}{m}\dot{x} + \frac{Kax}{m} - \frac{aF_d}{m}x = \frac{Ka}{m}x_r - \frac{F_d a x_r}{m}$

let $x_1 = x$ ← position

$x_2 = \dot{x} = \dot{x}_1$ ← Velocity

$x_3 = m\ddot{x} = m\dot{x}_2$ ← Force

→

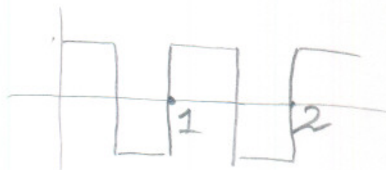
$$\frac{\dot{F}}{m} = -\frac{c}{m}\frac{F}{m} - \frac{b}{m}V - \frac{ka}{m}x + \frac{aF_d}{m}x + \frac{ka}{m}x_r - \frac{F_d a}{m}x_r$$

$$\boxed{\begin{aligned} \dot{F} &= -\frac{c}{m}F - bV - kax + aF_d x + kax_r - F_d a x_r \\ \dot{V} &= \frac{F}{m} \\ \dot{x} &= V \end{aligned}}$$

so
$$\begin{bmatrix} \dot{F} \\ \dot{X} \\ \dot{V} \end{bmatrix} = \underset{3\times3}{\begin{bmatrix} -\frac{c}{m} & -ka+aF_d & -b \\ 0 & 0 & 1 \\ \frac{1}{m} & 0 & 0 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} F \\ X \\ V \end{bmatrix}} + \underset{3\times1}{\begin{bmatrix} ka - F_d a \\ 0 \\ 0 \end{bmatrix}} \underset{1\times1}{\begin{bmatrix} x_r \end{bmatrix}}$$

$$\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} F \\ X \\ V \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} x_r \end{bmatrix}$$

now apply 1 Hz square wave for X_r



, Then 1 HZ sin wave for X_r

then 5 HZ sin wave.

then 10 HZ.

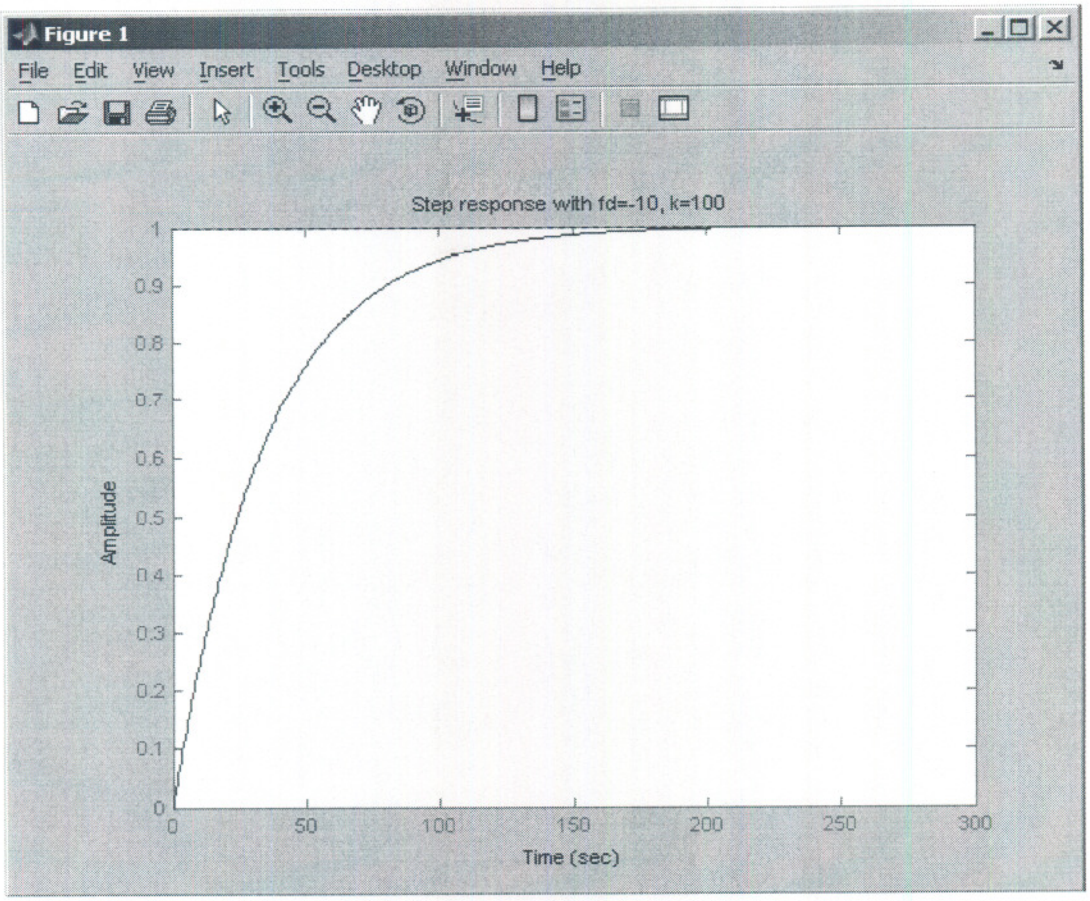
Computer solution ⟶

```
% Problem 3
% The parameters of the pneumatic system discussed in class are a=1, m=1,
b=350,
% and c=20.  Find a proportional gain k that gives a fast step response.
% Model the system in state-space form using the states: position,
% velocity, and force.  Repeat the step response simulation with a
% disturbance force of Fd=-10 pushing to the left throughout the motion.
% Simulate the response of the system using the gain you found above, and
% a 1 Hz square wave input (+- 1 amplitude), a 1 Hz sin wave, a 5 hz sin
% wave, and a 10 hz sin wave.  Show your plots, and try to explain your
% results.

%by Nasser Abbasi

clear all;
close all;

m = 1;  %Kg
c = 20;
b = 350;
a = 1;
fd=-10;
k=100;
```

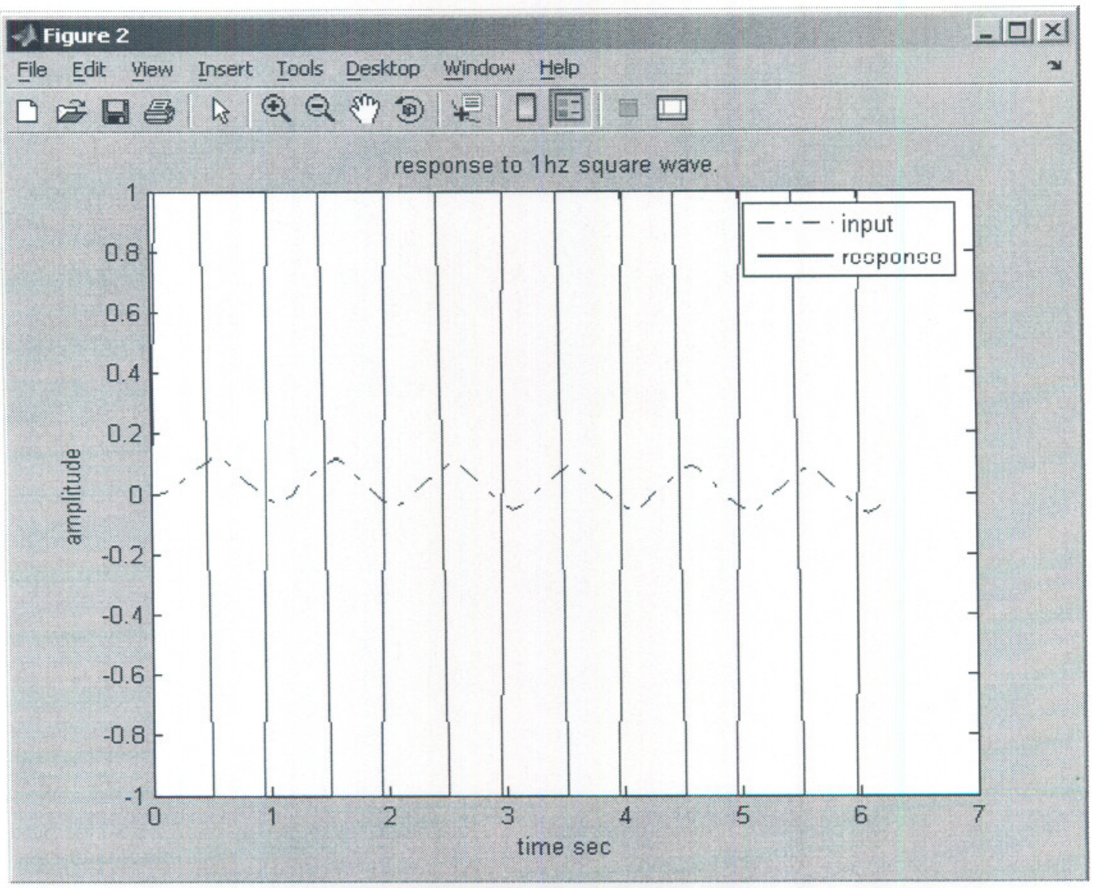
→ gain Found From earlier part

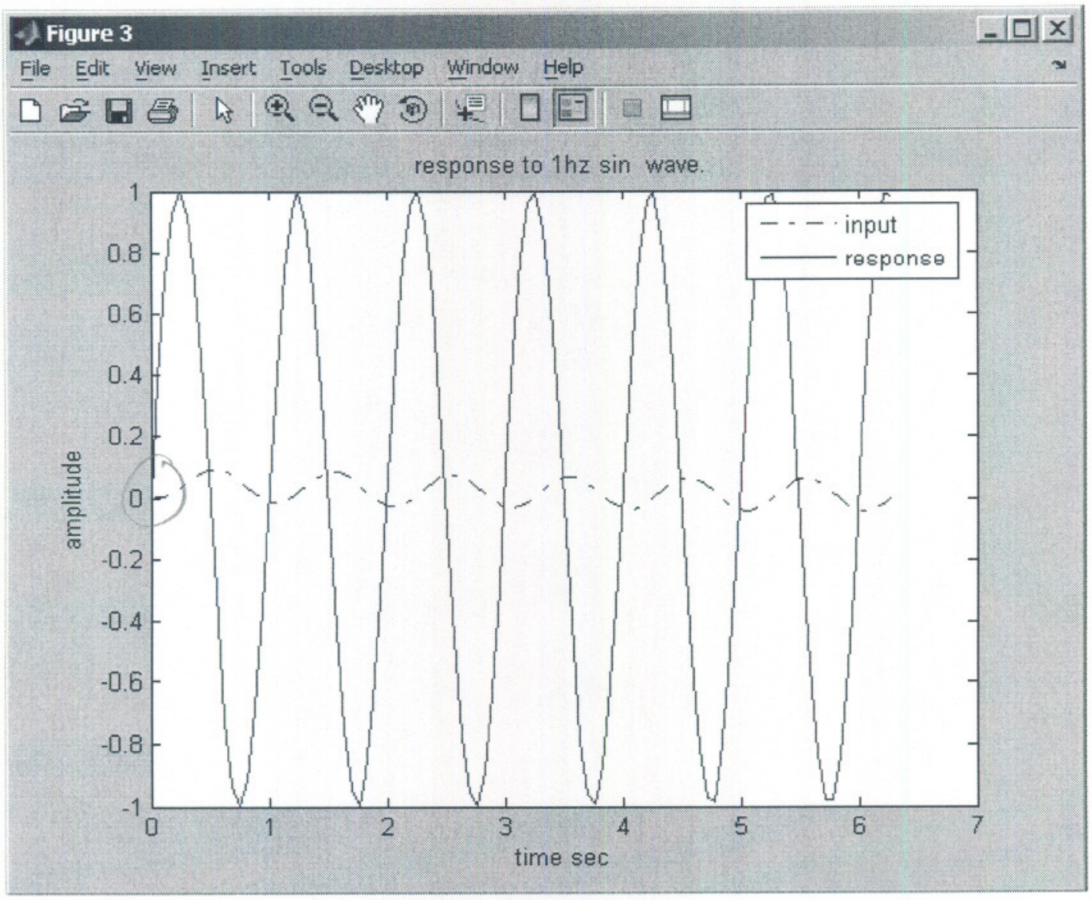
```
t=0:0.5:300;
t1=t;


A = [-c/m     -k*a+a*fd    -b
     0             0          1
     1/m           0          0];

B = [k*a-fd*a
     0
     0];

C = [0    1     0];

D = [ 0 ];


step(ss(A,B,C,D),t);
title('Step response with fd=-10, k=100');

t=0:pi/50:2*pi;
t1=t;
figure
u=sign(sin(2*pi*1*t)); %1hz square wave
[y,t]=lsim(A,B,C,D,u,t);
plot(t1,y ,'-.');
hold on;
plot(t1,u);
title('response to 1hz square wave.');
ylabel('amplitude');
xlabel('time sec');
legend('input','response');
```

```
figure;
t=0:pi/50:2*pi;
t1=t;
u=sin(2*pi*1*t); %1hz sin wave
[y,t]=lsim(A,B,C,D,u,t);
plot(t1,y ,'-.');
hold on;
plot(t1,u);
title('response to 1hz sin  wave.');
ylabel('amplitude');
xlabel('time sec');
legend('input','response');


figure;
t=0:0.001:1;
t1=t;
u=sin(2*pi*t*5); %5hz square wave
[y,t]=lsim(A,B,C,D,u,t);
plot(t1,y ,'-.');
hold on;
plot(t1,u);
title('response to 5 hz sin  wave.');
ylabel('amplitude');
xlabel('time sec');
legend('input','response');

figure;
t=0:0.001:1;
t1=t;
u=sin(2*pi*t*10); %10hz sin wave
[y,t]=lsim(A,B,C,D,u,t);
plot(t1,y ,'-.');
hold on;
plot(t1,u);
title('response to 10 hz sin  wave.');
ylabel('amplitude');
xlabel('time sec');
legend('input','response');
```



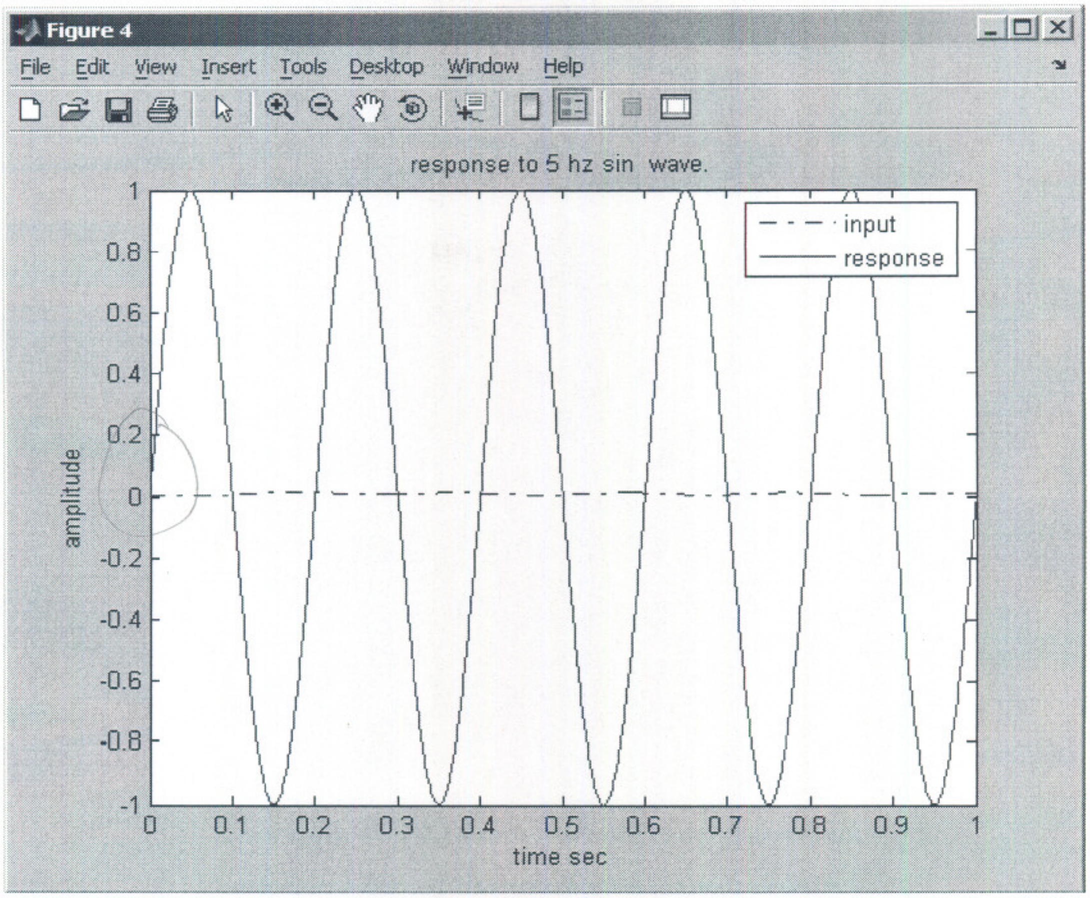
notice that now the rise time has increased when fd is present. This is expected.



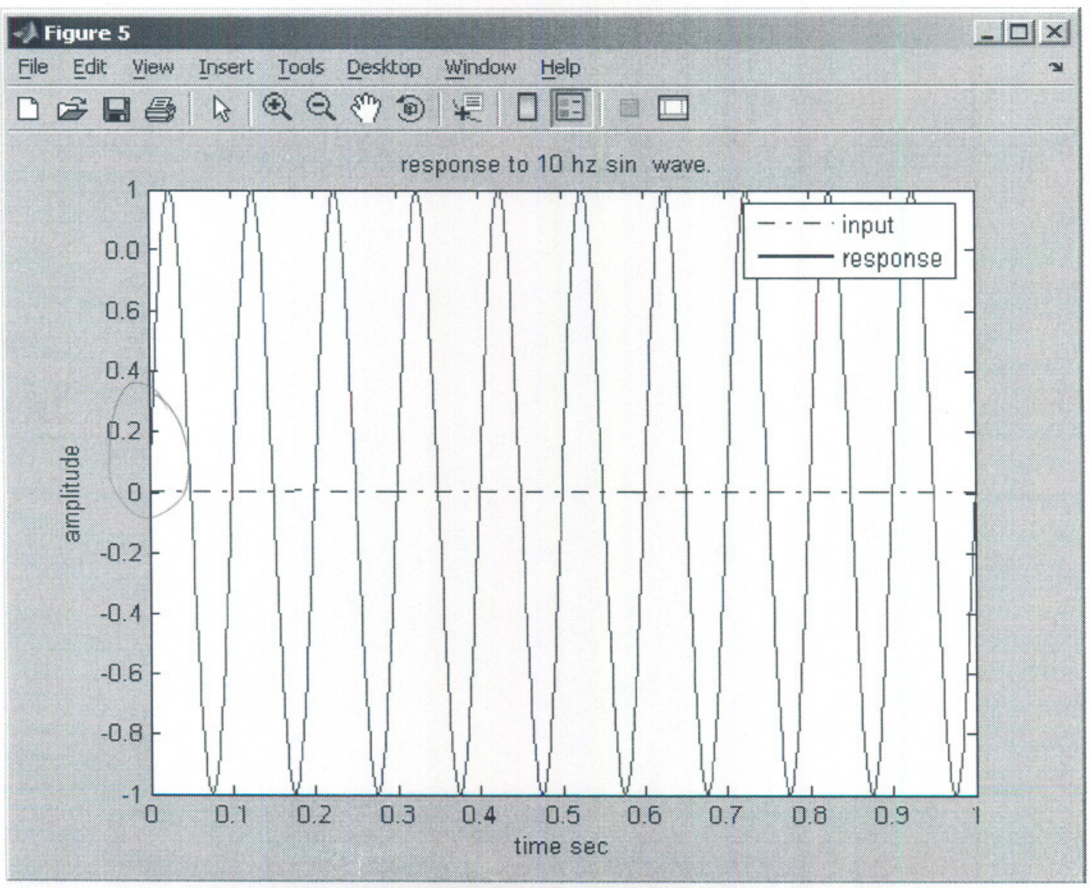


notice The response has the same frequency as the input, but different amplitude (scaling) and phase shift!

this is a property of linear systems.



response same freq. as input. different phase and scaling.



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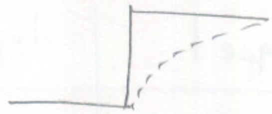
Tuesday

$K_p : \lim_{s \to 0} GH$



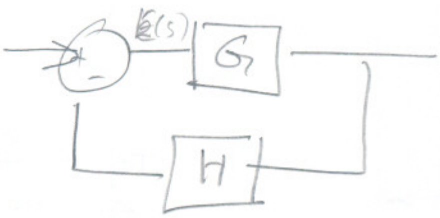
$e_{ss} = \frac{1}{1+K_p}$

$K_v = \lim_{s \to 0} s\,GH$

$e_{ss} = \frac{1}{K_v}$

$K_a = \lim_{s \to 0} s^2\,GH$

$e_{ss} = \frac{1}{K_a}$



System type

$GH = \frac{N(s)}{D(s)}$ if no s in $D(s)$ $\Rightarrow$ type 0

$GH = \frac{N(s)}{s\,D(s)}$ Type 1

extra s $\leftarrow$

$GH = \frac{N}{s^2 D}$ type 2.

so for type 2, to find $K_a = s^2 \frac{N}{s^2 D} = \frac{N}{D}$

so for type 1, $K_a = 0$ (since s in Numerator. so $e_{ss} \to \infty$)