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HW#2

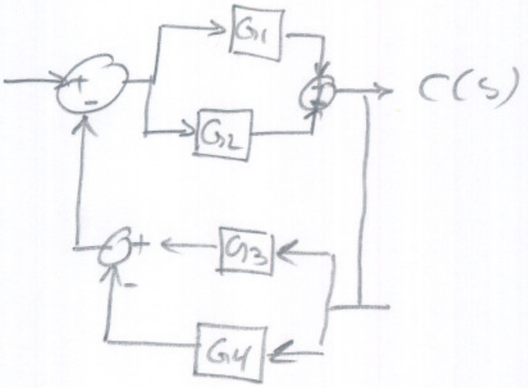
MAE 170, Introduction to control systems

UCI. Winter 2005

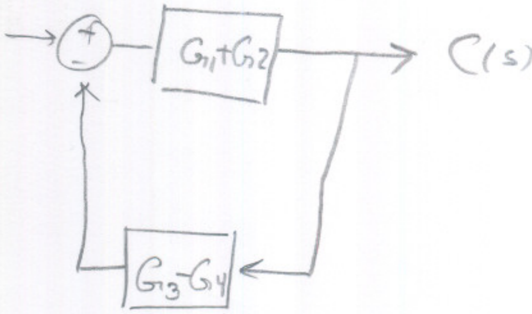
By Nasser Abbasi

Nasser Abbasi
HW#2

B-3-1



=

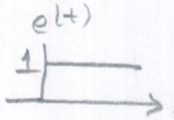


$$= \quad R(s) \longrightarrow \boxed{\frac{G_1+G_2}{1+(G_1+G_2)(G_3-G_4)}} \longrightarrow C(s)$$

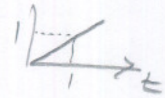
so Transfer function is $$\boxed{\frac{G_1+G_2}{1+(G_1+G_2)(G_3-G_4)}}$$

HW# 2
problem B-3-4
Nasser Abbasi

Please note: I'll call $1(t)$ as $u(t)$ and the output (response) as $y(t)$ instead of as $u(t)$ as book. I find $u(t)$ more clear than $1(t)$

$e(t) =$ unit step 

$\Rightarrow \quad E(s) = \frac{1}{s}$

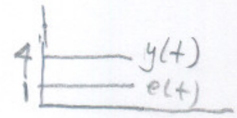
$e(t) =$ unit ramp 

$\Rightarrow \quad E(s) = \frac{1}{s^2}$

For unit step $e(t)$:

proportional

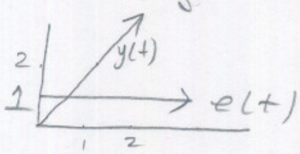
$$\frac{Y(s)}{E(s)} = k_p = 4 \Rightarrow Y(s) = 4E(s) \Rightarrow \boxed{y(t) = 4\,1(t)} \Rightarrow$$



integral

$$\frac{Y(s)}{E(s)} = \frac{k_i}{s} = \frac{2}{s} \Rightarrow sY(s) = 2\frac{1}{s} \Rightarrow \frac{dy}{dt} = 2\,u(t) \Rightarrow y(t) = 2\int_0^t dt = 2t$$

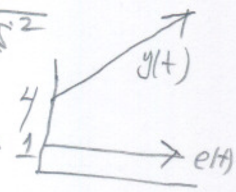
so $\boxed{y(t) = 2t}$ i.e.



proportional + integral

$$\frac{Y(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right) \Rightarrow \frac{Y(s)}{E(s)} = 4\left(1 + \frac{1}{2s}\right) \Rightarrow Y(s) = \frac{4}{s} + \frac{4}{2s^2}$$

so $\boxed{y(t) = 4u(t) + 2\,\text{unitRamp}(t)}$ i.e for $t>0$, $\boxed{y(t) = 4 + 2t} \Rightarrow$

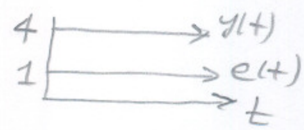


eq of line, slop=2, intersection=4

Propertional + derivative

$$\frac{Y(s)}{E(s)} = K_p(1 + T_d s) \Rightarrow \frac{Y(s)}{E(s)} = 4(1 + 0.8s) \rightarrow Y(s) = \frac{4}{s} + 3.2$$

so $y(t) = 4u(t) + 3.2\,\delta(t)$ so for $t>0$ $\boxed{y(t) = 4}$



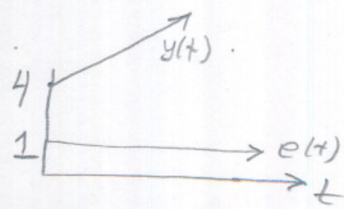
notice that P+D has same response as proportional since $e(t)$ is unit step, whose derivative is zero for $t>0$.

PID

$$\frac{Y(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = 4\left(1 + \frac{1}{2s} + 0.8s\right) \Rightarrow Y(s) = \frac{4}{s} + \frac{1}{s^2} + 0.8$$

so $y(t) = 4u(t) + \text{unitramp}(t) + 0.8\,\delta(t)$.

so for $t>0$, $\boxed{y(t) = 4 + t}$



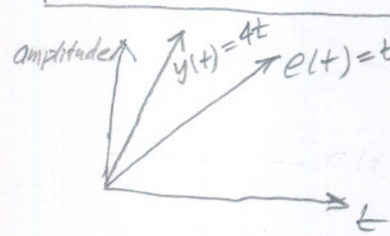
Notice that PID response same as PI, sine the "D" effect is cancelled due to $e(t)$ having zero derivative for $t>0$. $\longrightarrow$

Problem B-3-4 (Cont.)

Now for $e(t) =$ unit ramp $\quad E(s) = \frac{1}{s^2}$

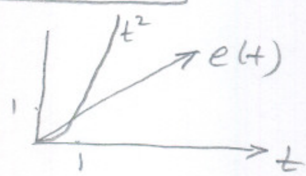
Proportional

$$\frac{Y(s)}{E(s)} = K_p \Rightarrow Y(s) = 4E(s) \Rightarrow Y(s) = \frac{4}{s^2} \Rightarrow \boxed{y(t) = 4 \text{ unitramp}(t)}$$



integral

$$\frac{Y(s)}{E(s)} = \frac{K_i}{s} \Rightarrow Y(s) = \frac{2}{s}\frac{1}{s^2} = \frac{2}{s^3} \text{ From Tables } \Rightarrow \boxed{y(t) = t^2}$$

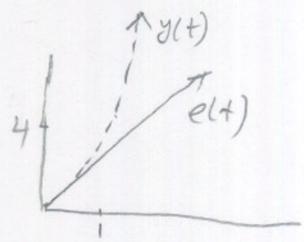


P+I

$$\frac{Y(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right) \Rightarrow Y(s) = 4\left(1 + \frac{1}{2s}\right)\frac{1}{s^2} \Rightarrow \frac{4}{s^2} + \frac{1}{2s^3}$$

$\frac{4}{s^2} \to 4 \text{ Unitramp}(t)$ $\qquad \frac{1}{2s^3} \to \frac{1}{4}\frac{2}{s^3} \to \frac{1}{4}t^2$

so $\boxed{y(t) = 4t + \frac{1}{4}t^2}$ for $t > 0$.

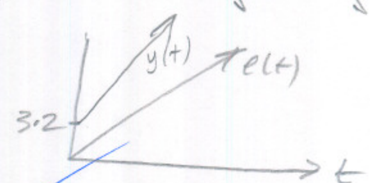


P+D $\quad \frac{Y(s)}{E(s)} = K_p(1 + T_d s) \Rightarrow Y(s) = (K_p + K_p T_d s)\frac{1}{s^2} \Rightarrow Y(s) = \frac{4}{s^2} + \frac{3.2}{s}$

so $y(t) = 4 \text{ unitramp}(t) + 3.2\, u(t)$

so for $t > 0$, $\boxed{y(t) = 4t + 3.2}$

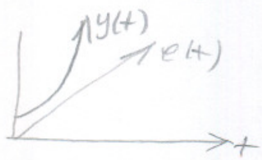
$\underbrace{4t}_{\text{slop}} + \underbrace{3.2}_{\text{intersection.}}$



PID $\quad \frac{Y(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \Rightarrow Y(s) = \left(4 + \frac{4}{2s} + (4)(0.8)\, s\right)\frac{1}{s^2}$

$$Y(s) = \frac{4}{s^2} + \frac{2}{s^3} + \frac{3.2}{s} \Rightarrow y(t) = 4 \text{ unitramp}(t) + t^2 + 3.2 \text{ unitstep}(t).$$

so for $t > 0$, $\quad \underbrace{\boxed{y(t) = 4t + t^2 + 3.2}}_{\text{quadratic equation}}$ $\qquad t = 0,\ y(t) = 3.2$; $\ t = 1,\ y(t) = 8.2$



P.S. Just For Fun, I wrote the Following small program which displays output of these controllers, which can be used for any $e(t)$ function.

Nasser Abbasi . HW #2
Problem B 3-4.

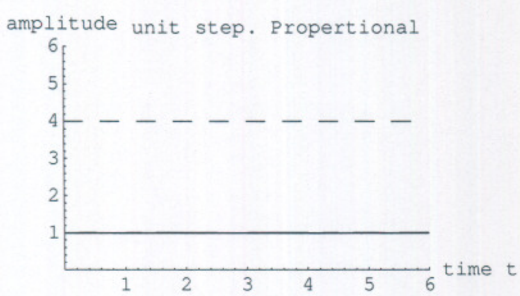
(Plots are on page 3)

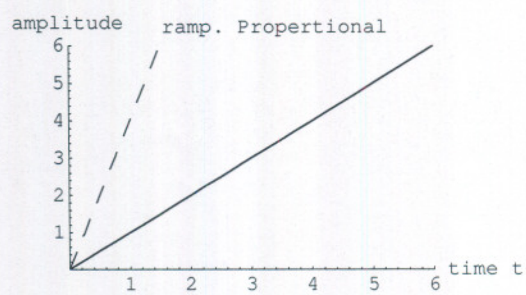
```
(*By Nasser Abbasi. To solve HW 2, Problem B-3-4*)

Clear["Global`*"];
<< Graphics`Legend`
Kp = 4
Ki = 2
Ti = 2 (*sec*)
Td = 0.8 (*sec*)
tf = 6 (*for how many seconds to run the response*)


SetOptions[Plot, PlotRange → {{0, tf}, All},
 PlotStyle → {Thickness[.005] , Dashing[{0.05, 0.05}], Dashing[{0.01, 0.01}]},
 AxesLabel → {"time t", "amplitude"}, DisplayFunction → Identity,
 PlotRange → {{0, tf}, {0, 2 tf}}
]
 SetOptions[Legend, LegendPosition → {-1, -.4}]

Out[988]=
  4

Out[989]=
  2

Out[990]=
  2

Out[991]=
  0.8

Out[992]=
  6

In[1056]:=
  p1 = Plot[{UnitStep[t], Kp UnitStep[t]}, {t, 0, tf},
    PlotRange → {{0, tf}, {0, tf}},
    PlotLabel → "     unit step. Propertional"]

  p2 = Plot[{t, Kp t}, {t, 0, tf},
    PlotRange → {{0, tf}, {0, tf}},
    PlotLabel → "    ramp. Propertional"]

  Show[GraphicsArray[{p1, p2}]];

  (*I*)
  p1 = Plot[{UnitStep[t], Evaluate[Ki ∫_0^t UnitStep[x] dx]}, {t, 0, tf},
    PlotLabel → "    unit step. Integral"]

  p2 = Plot[{t, Evaluate[Ki ∫_0^t x dx]}, {t, 0, tf},
    PlotLabel → "    ramp. Integral"]
```

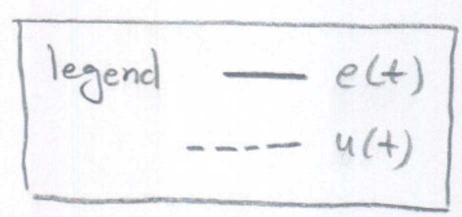
```
Show[GraphicsArray[{p1, p2}]];

(*P+I*)
p1 = Plot[{UnitStep[t], Evaluate[Kp UnitStep[t] + Kp/Ti ∫_0^t UnitStep[x] dx]}, {t, 0, tf},
  PlotLabel → "     unit step. P+I"]

p2 = Plot[{t, Evaluate[Kp t + Kp/Ti ∫_0^t x dx]}, {t, 0, tf},
  PlotLabel → "    ramp P+I"]

Show[GraphicsArray[{p1, p2}]];

(*P+D*)
p1 = Plot[{UnitStep[t],
   Evaluate[Kp UnitStep[t] + Simplify[Kp Td D[UnitStep[x], x], {x > 0}]]}, {t, 0, tf},
  PlotLabel → "    unit step. P+D"]

p2 = Plot[{t, Evaluate[Kp t + Kp Td D[x, x]]}, {t, 0, tf},
  PlotLabel → "    ramp. P+D", PlotRange → {{0, tf}, All}]

Show[GraphicsArray[{p1, p2}]];

p1 = Plot[{UnitStep[t], Evaluate[Kp UnitStep[t] + Kp/Ti ∫_0^t UnitStep[x] dx +
      Kp Td Simplify[Kp Td D[UnitStep[x], x], {x > 0}]]}, {t, 0, tf},
   PlotLabel → "   unit step. PID"];

p2 = Plot[{t, Evaluate[Kp t + Kp/Ti ∫_0^t x dx + Kp Td Simplify[Kp Td D[x, x], {x > 0}]]},
   {t, 0, tf},
   PlotRange → {{0, tf}, All},
   PlotLabel → "   ramp. PID"];

Show[GraphicsArray[{p1, p2}]];

Plot[{UnitStep[t],
  Kp UnitStep[t],
  Evaluate[Ki ∫_0^t UnitStep[x] dx],
  Evaluate[Kp UnitStep[t] + Kp/Ti ∫_0^t UnitStep[x] dx],
  Evaluate[Kp UnitStep[t] + Simplify[Kp Td D[UnitStep[x], x], {x > 0}]],
  Evaluate[Kp UnitStep[t] + Kp/Ti ∫_0^t UnitStep[x] dx +
    Kp Td Simplify[Kp Td D[UnitStep[x], x], {x > 0}]]}, {t, 0, tf},
 PlotLegend → {"e(t)", "Prop", "Integral", "P+I", "P+D", "PID"},
 LegendPosition → {1.1, -.4},
```

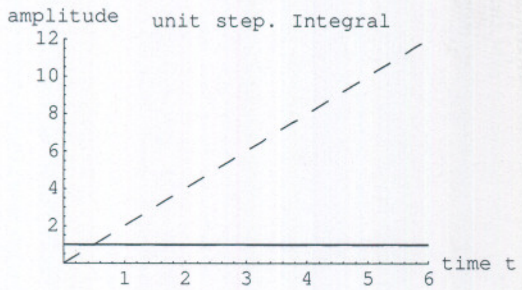
```
PlotRange → {{0, 3}, {0, 7}},
PlotLabel → "    different controllers response to unit step e(t)",
PlotStyle → {Dashing[{0.06, 0.06}], Dashing[{0.01, 0.01}], Dashing[{0.02, 0.02}],
  Dashing[{0.03, 0.03}], Dashing[{0.04, 0.04}]}, DisplayFunction → $DisplayFunction]
```

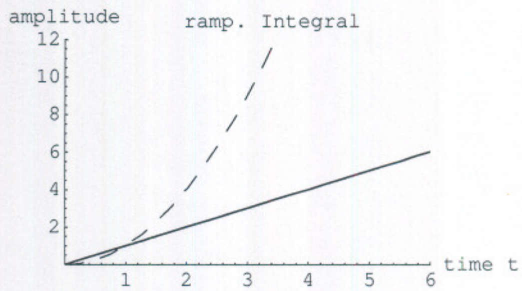
```
Plot[{t,
  Kp t,
  Evaluate[Ki ∫_0^t x dx],
  Evaluate[Kp t + Kp/Ti ∫_0^t x dx],
  Evaluate[Kp t + Simplify[Kp Td D[x, x], {x > 0}]],
  Evaluate[Kp t + Kp/Ti ∫_0^t x dx + Kp Td Simplify[Kp Td D[x, x], {x > 0}]]},
 {t, 0, tf},
 PlotLegend → {"e(t)", "Prop", "Integral", "P+I", "P+D", "PID"},
 LegendPosition → {1.1, -.4},
 PlotRange → {{0, tf}, {0, 30}},
 PlotLabel → "    different controllers response to ramp e(t)",
 PlotStyle →
  { Thickness[.008], Dashing[{0.06, 0.06}], Dashing[{0.01, 0.01}], Dashing[{0.02, 0.02}],
   Dashing[{0.03, 0.03}], Dashing[{0.04, 0.04}]}, DisplayFunction → $DisplayFunction
]
```

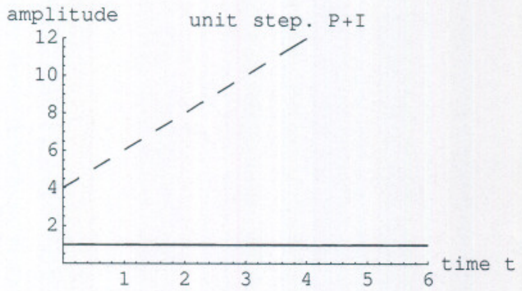


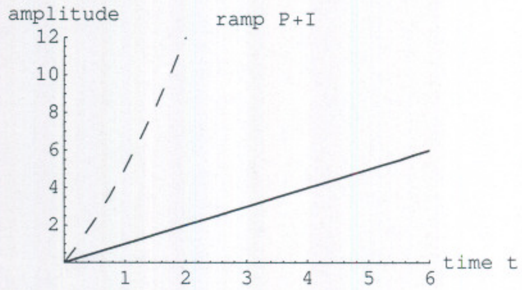


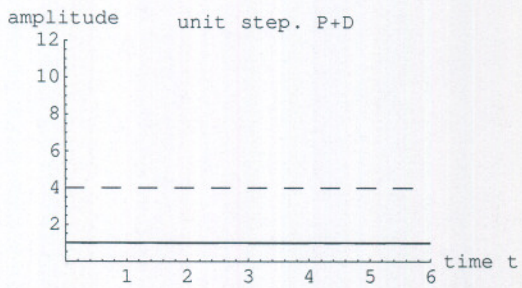


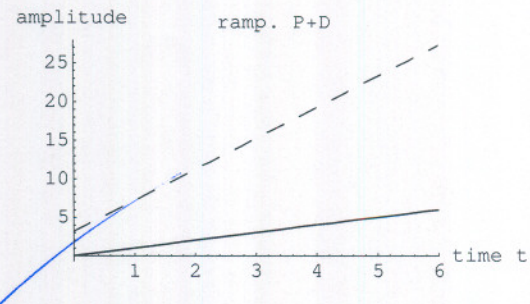

amplitude
unit step. Integral
12
10
8
6
4
2
1 2 3 4 5 6
time t

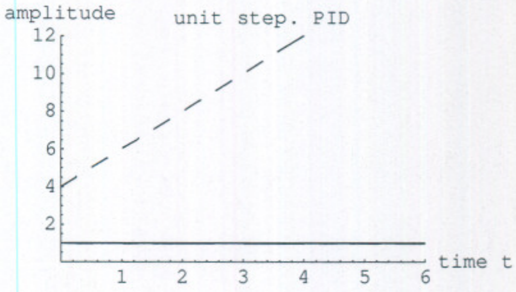

amplitude
ramp. Integral
12
10
8
6
4
2
1 2 3 4 5 6
time t

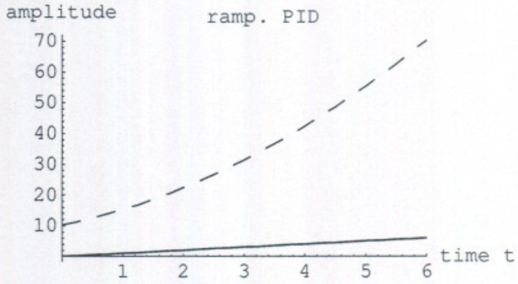

amplitude
unit step. P+I
12
10
8
6
4
2
1 2 3 4 5 6
time t

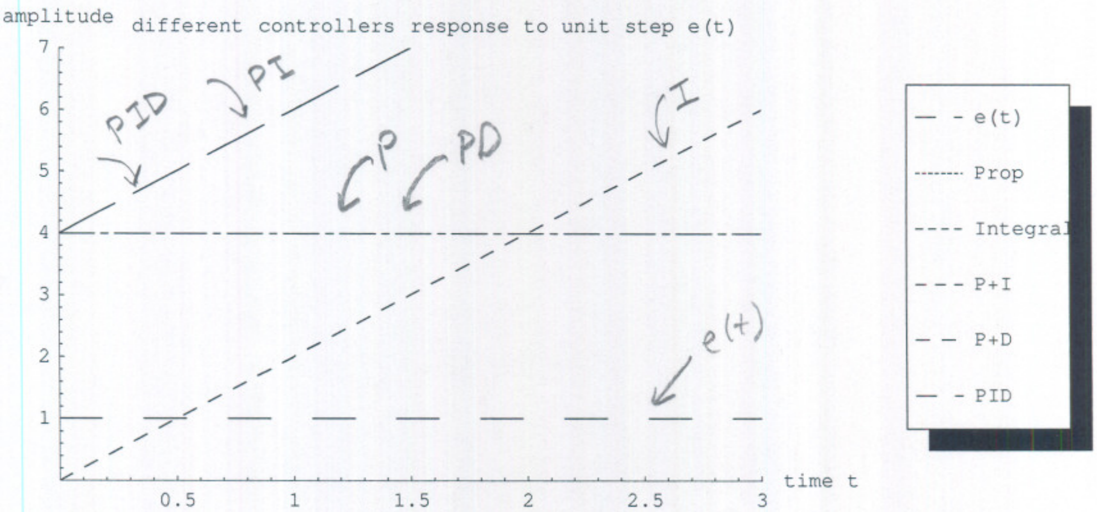

amplitude
ramp P+I
12
10
8
6
4
2
1 2 3 4 5 6
time t

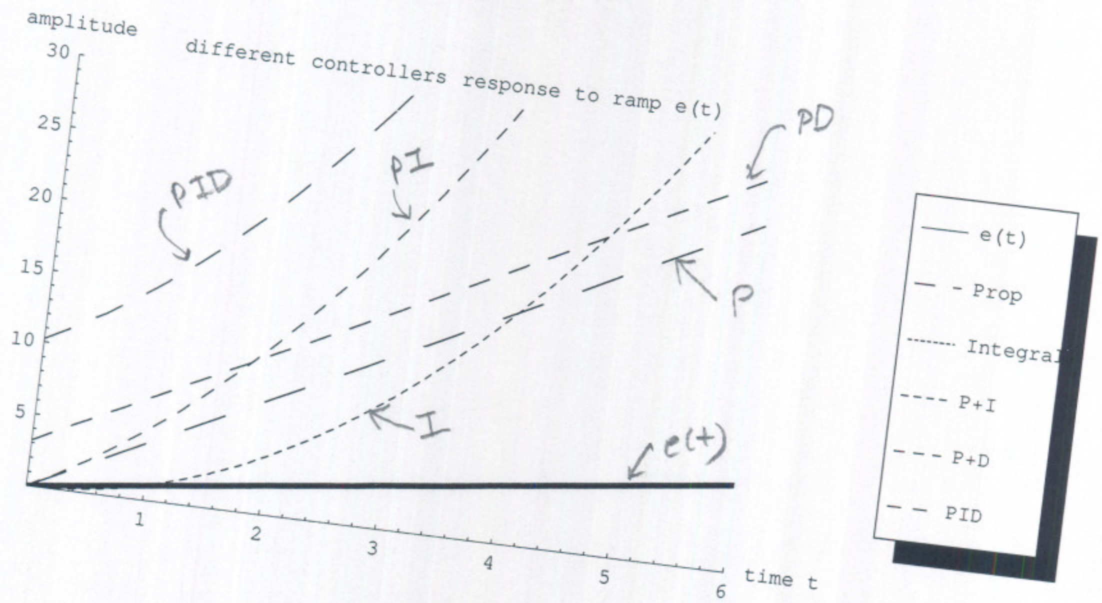

amplitude
unit step. P+D
12
10
8
6
4
2
1 2 3 4 5 6
time t


amplitude
ramp. P+D
25
20
15
10
5
1 2 3 4 5 6
time t


amplitude
unit step. PID
12
10
8
6
4
2
1
2
3
4
5
6
time t


amplitude
ramp. PID
70
60
50
40
30
20
10
1
2
3
4
5
6
time t


amplitude
different controllers response to unit step e(t)
PID
PI
P
PD
I
e(t)
7
6
5
4
3
2
1
0.5
1
1.5
2
2.5
3
time t
e(t)
Prop
Integral
P+I
P+D
PID


amplitude
different controllers response to ramp e(t)
30
25
20
15
10
5
1
2
3
4
5
6
time t
PID
PI
PD
P
I
e(t)
e(t)
Prop
Integral
P+I
P+D
PID

HW#2
Problem B-3-9
Nasser Abbasi

Consider $\dddot{y} + 3\ddot{y} + 2\dot{y} = u$
derive state space representation.

solution here $n=3$ (order of D.E.)

$$\text{state variables} \leftarrow \text{let } \begin{cases} x_1 = y \\ x_2 = \dot{y} \\ x_3 = \ddot{y} \rightarrow (n-1) \end{cases}$$

so DE can be written as $\dot{x}_3 = -3x_3 - 2x_2 + u$

and $\dot{x}_1 = \dot{y} = x_2$
and $\dot{x}_2 = \ddot{y} = x_3$.

since $\underline{\dot{x}} = A\underline{X} + B\underline{u}$ Then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} [u]$$

$$y = \underbrace{\underbrace{[1 \quad 0 \quad 0]}_{C}}_{1\times 3} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{3\times 1} + \underbrace{\underbrace{[0]}_{D}}_{1\times 1} \underbrace{[u]}_{1\times 1}$$

✓

HW #2

Problem B-3-11

Nasser Abbasi

Consider system defined by the following state space

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain the transfer function $G(s)$ for the system.

<u>answer.</u>

$$G(s) = \frac{Y(s)}{U(s)}$$

we have

$$\dot{\bar{X}} = \bar{A}\bar{X} + \bar{B}\bar{u} \quad \text{——} ①$$

$$\bar{Y} = \bar{C}\bar{X} + \bar{D}\bar{u} \quad \text{——} ②$$

Take Laplace transform of ①, we set

$$s\bar{X}(s) = \bar{A}\bar{X}(s) + \bar{B}\bar{U}(s)$$

$$s\bar{X}(s) - \bar{A}\bar{X}(s) = \bar{B}\bar{U}(s).$$

$$[s\bar{I}-\bar{A}]\bar{X}(s) = \bar{B}\bar{U}(s).$$

$$\bar{X}(s) = [s\bar{I}-\bar{A}]^{-1}\bar{B}\bar{U}(s) \quad \text{——} ③$$

take Laplace transform of ② $\Rightarrow$ $\bar{Y}(s) = \bar{C}\bar{X}(s) + \bar{D}\bar{U}(s)$ —— ④.

sub. ③ into ④ $\Rightarrow$ $\bar{Y}(s) = \bar{C}\left([s\bar{I}-\bar{A}]^{-1}\bar{B}\bar{U}(s)\right) + \bar{D}U(s)$

so $\bar{Y}(s) = \bar{C}\left([s\bar{I}-\bar{A}]^{-1}\bar{B} + \bar{D}\right)U(s).$

so $\boxed{G(s) = \bar{C}\left[[s\bar{I}-\bar{A}]^{-1}\bar{B} + \bar{D}\right]}$

now using given $\bar{C}, \bar{A}, \bar{B}, \bar{D}$ to evaluate the above $\rightarrow$

$\bar{A} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $C = [1 \quad 2]$, $D = [\,]$.

so $[s\bar{I} - \bar{A}] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}$

so $[s\bar{I} - \bar{A}]^{-1} = \dfrac{1}{(s+5)(s+1) - (1)(-3)} \begin{bmatrix} s+1 & -1 \\ 3 & (s+5) \end{bmatrix}$

$= \dfrac{1}{s^2 + 6s + 8} \begin{pmatrix} s+1 & -1 \\ 3 & (s+5) \end{pmatrix}$

so $(s\bar{I} - \bar{A})^{-1}\bar{B} = \dfrac{1}{s^2+6s+8} \underset{2\times2}{\begin{pmatrix} s+1 & -1 \\ 3 & (s+5) \end{pmatrix}} \underset{2\times1}{\begin{pmatrix} 2 \\ 5 \end{pmatrix}} = \dfrac{1}{s^2+6s+8} \begin{pmatrix} 2s+2-5 \\ 6+5s+25 \end{pmatrix}$

$= \dfrac{1}{s^2+6s+8} \begin{pmatrix} 2s-3 \\ +5s+31 \end{pmatrix}$

so $(s\bar{I} - \bar{A})^{-1}\bar{B} + \bar{D}$ = same as above since $D = [\,]$

so $\bar{C}\left[(s\bar{I} - \bar{A})^{-1}\bar{B} + \bar{D}\right] = \underset{1\times2}{[1 \quad 2]} \dfrac{1}{s^2+6s+8} \underset{2\times1}{\begin{bmatrix} 2s-3 \\ +5s+31 \end{bmatrix}}$

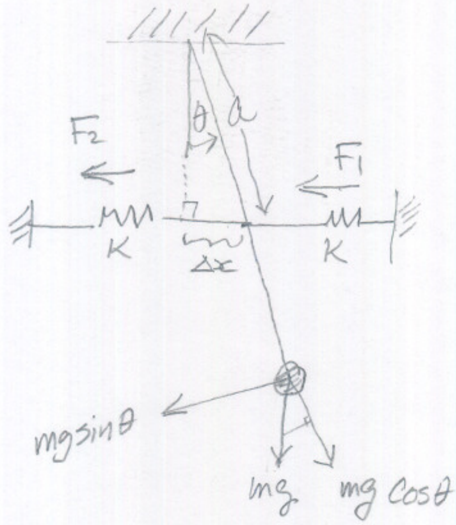
$= \dfrac{1}{s^2+6s+8} \left(2s - 3 + 62 + 10s\right)$

so $\boxed{G(s) = \dfrac{12s + 59}{s^2 + 6s + 8}}$

HW# 2

Problem B-3-16

Nasser Abbasi

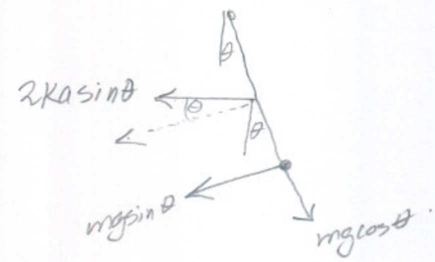


length of bar $= l$

$\sin\theta = \frac{\Delta x}{a} \Rightarrow \Delta x = a\sin\theta$. This is the amount of stretch of spring.

so total spring Force is $2(K\Delta x) = 2Ka\sin\theta$

So normal component of spring force will be $\boxed{2Ka\sin\theta\cos\theta}$



Torque produced by this force is $2Ka\sin\theta\cos\theta\, a$

$= \boxed{2Ka^2\sin\theta\cos\theta}$

Torque produced by tangential component of mg is $\boxed{mg\sin\theta\, l}$

so total torque is $-(2Ka^2\sin\theta\cos\theta + mg\sin\theta\, l)$

use eqvation of motion $\boxed{\text{Torque} = J\ddot{\theta}}$, where J is moment of inertia of bar around hinge.

$\boxed{J = ml^2}$ assume all mass in ball.

so $\boxed{ml^2\ddot{\theta} + 2Ka^2\sin\theta\cos\theta + mg\sin\theta\, l = 0}$

for small θ, since $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$, $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots$

so $\sin\theta \to \theta$, $\cos\theta \to 1$ by ignoring term with powers ≥ 2.

so $\boxed{ml^2\ddot{\theta} + 2Ka^2\theta + mg\theta l = 0}$ $\longrightarrow$

$$\ddot{\theta} + \theta\left(\frac{2Ka^2 + mgl}{ml^2}\right) = 0$$

let $\frac{2Ka^2+mgl}{ml^2} = B$.

so $\ddot{\theta} + B\theta = 0$. char. equation is $\overbrace{(D^2+B)}^{a=1,\, b=0,\, c=B}\theta = 0$

so roots of char eq. $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{\pm\sqrt{-4B}}{2} = \pm\sqrt{-B}$

$= \pm i\sqrt{B}$

so solution is $\boxed{\theta = \alpha e^{i\sqrt{B}} + \beta e^{-i\sqrt{B}}}$

or $\theta = \alpha e^{\sqrt{\frac{-2Ka^2-mgl}{ml^2}}} + \beta e^{-\sqrt{\frac{-2Ka^2-mgl}{ml^2}}}$

wher α, β are constants found from initial conditions.

we see that natural frequency is

$\sqrt{\frac{2Ka^2+mgl}{ml^2}} = \sqrt{\frac{2Ka^2}{ml^2} + \frac{g}{l}}$. we see if there were no springs, i.e. $k=0$

this simplifies to $\sqrt{\frac{g}{l}}$ as expected.

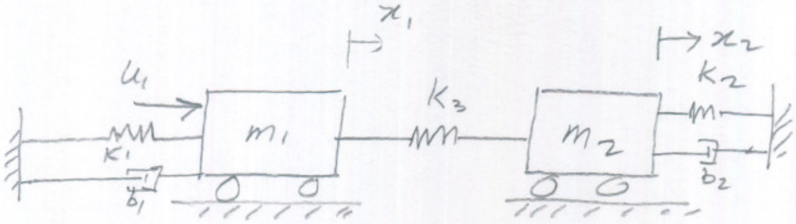
HW #2

Problem B-3-18

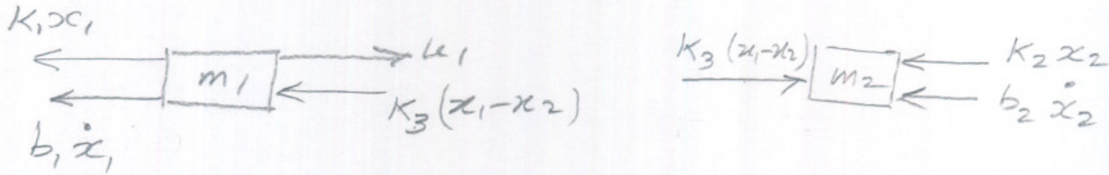
Nasser Abbasi

Obtain transfer functions $\frac{X_1(s)}{U(s)}$ and $\frac{X_2(s)}{U(s)}$ of mechanical system shown.

Solution



assume m_1 is moving faster than m_2 to the right. draw free body diagrams



so, equation of motion $\boxed{F = ma}$ applied to each mass

$$u_1 - k_3(x_1 - x_2) - b_1\dot{x}_1 - k_1 x_1 = m_1 \ddot{x}_1$$

$$k_3(x_1 - x_2) - k_2 x_2 - b_2 \dot{x}_2 = m_2 \ddot{x}_2$$

simplify, we get

$$m_1\ddot{x}_1 + b_1\dot{x}_1 + k_1 x_1 + k_3 x_1 - k_3 x_2 = u \quad \text{——— (1)}$$

$$m_2\ddot{x}_2 + b_2\dot{x}_2 + k_2 x_2 - k_3 x_1 + k_3 x_2 = 0 \quad \text{——— (2)}$$

For eq (1). take Laplace transform, assume initial conditions.

$$m_1 s^2 X_1(s) + b_1 s X_1(s) + k_1 X_1(s) + k_3 X_1(s) - k_3 X_2(s) = U(s)$$

$$X_1(s)\left[m_1 s^2 + b_1 s + k_1 + k_3\right] - k_3 X_2(s) = U(s) \quad \text{——— (3)}$$

For eq (2). take Laplace transform →

$$m_2 s^2 X_2(s) + b_2 s X_2(s) + k_2 X_2(s) - k_3 X_1(s) + k_3 X_2(s) = 0$$

→

so

$$X_2(s)\left[m_2 s^2 + b_2 s + k_2 + k_3\right] - k_3 X_1(s) = 0 \quad \text{———} \quad (4)$$

i.e $$X_2(s) = \frac{k_3}{m_2 s^2 + b_2 s + k_2 + k_3} X_1(s) \quad \text{———} \quad (5)$$

sub (5) into (3) we get

$$X_1(s)\left[m_1 s^2 + b_1 s + k_1 + k_3\right] - k_3 \frac{k_3}{m_2 s^2 + b_2 s + k_2 + k_3} X_1(s) = U(s).$$

So $$X_1(s)\left[m_1 s^2 + b_1 s + k_1 + k_3 - \frac{k_3^2}{m_2 s^2 + b_2 s + k_2 + k_3}\right] = U(s).$$

So $$G_1(s) = \boxed{\frac{X_1(s)}{U(s)} = \frac{m_2 s^2 + b_2 s + k_2 + k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}}$$

To find $G_2(s)$. sub (3) into (5) we get

$$\frac{X_2(s)}{U(s)} = \frac{k_3}{(m_2 s^2 + b_2 s + k_2 + k_3)} \frac{(m_2 s^2 + b_2 s + k_2 + k_3)}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}$$

$$\boxed{\frac{X_2(s)}{U(s)} = \frac{k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}}$$